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Verification and Refinement of an Aircraft Structural Design and Optimization Tool, ATLASS

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This paper is an update and continuation of the ATLASS paper presented in 2019. The initial application and testing of the ATLASS framework for sizing and optimization of new aircraft and structural concepts is discussed along with corresponding improvements and additional functionality. The ATLASS tool is intended to bridge the gap between conceptual and detailed design by implementing higher fidelity automated analysis tools earlier in the design process with applicability to new or novel aircraft configurations. The ATLASS tool includes fully automated modules for Outer Mold Line (OML) generation, internal structural layout, weights and C.G. generation, and Finite Element Model (FEM) generation for loads and structural analysis. All geometric and finite element based models are generated in CATIA and exported to Nastran and Hypersizer for analysis. Analysis and optimization are managed by Isight. An initial study was conducted on a legacy wing configuration to validate the tool and make improvements. Initial results were promising however additional load cases were identified as well as analyses not currently available in Hypersizer to improve weight estimations. These enhancements were integrated then applied to a full aircraft validation study.

I. Nomenclature

ATLASS = *Automated Top Level Aircraft Structural Sizing tool*
EKL = *Engineering Knowledge Language*
FMS = *Advanced Surface Meshing Workbench*
GAS = *Generative Structural Analysis Workbench*
VBA = *Visual Basic for Applications*
OML = *Outer Mold Line*
MSF = *Master Surface File*
MDF = *Master Datum File*
FEM = *Finite Element Model*
CGx = X-coordinate of the center of gravity

II. Introduction

A. Motivation

Structural research and development teams at Gulfstream regularly perform trade studies and analyses on future structural technologies, configurations and materials. Studies that include high level configuration changes or large deviations in load path can become prohibitively complex and time consuming. Changes that result in large stiffness and mass properties deviations affect the load cases for which the structure is analyzed. In order to gain a full understanding of the implications to these changes, loads must be re-run, the structure re-analyzed and repeated until sizing converges. This process is not well suited for small teams especially when new finite element models must be created and loads and mass models updated. During the early detailed design phases of a program, this process takes months and can produce large loads swings resulting in engineering rework for existing parts. This drives impacts to both program cost and schedule.

The ATLASS initiative was originally launched to reduce engineering time for performing trade studies on new materials, structures and layouts within the Advanced Structures and Materials Initiative (ASMI). A tool was needed

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to automate the design, sizing, and loads processes such that these trade studies could be performed more efficiently and with higher accuracy. The scope was later expanded to include parametric and morphable OML geometry capability which could be leveraged in the early design phase to gain a better understanding of loads and structural requirements prior to program launch, as well as preform trade studies to better optimize the aircraft.

B. ATLASS Framework Review

In order to ensure seamless data transfer and analysis model generation, a single tool was needed to create and maintain the aircraft configuration. The tool needed to support complex geometry and FEM creation for structural and loads analysis. The tool was also required to support a fully automated workflow. CATIA was chosen as the primary program for configuration management and model generation due to its extensive library of geometry tools as well as a native meshing workbench. CATIA can also be automated through VBA and a native language, EKL.

The framework of ATLASS is built on a fully parametric and morphable geometry engine. The aircraft geometry is governed by a design table spreadsheet. The aircraft parameters are entered into the design table and read into the ATLASS MSF and MDF modules to generate the OML and structural layout of the aircraft. The parametric nature of the modeling code allows for various structural configuration changes to be generated, such as changing the number of wing ribs or spars. The geometry is also morphable, able to update construction geometry dynamically through parameter input, such that changes to the shape of the wing will automatically update the internal structure.

Next, an EKL code is used to generate geometric mesh supports. This geometry is used to define the location of nodes, the shape of elements, and the mesh density of the subsequent FEM. The mesh supports use the MSF and MDF to intersect primary structure to the OML and create points and lines for nodes and element boundaries. Next additional curves are created by evenly interpitching planes between the structural intersections to define the mesh density. Mesh density is also parametric and can be defined by the user.

The surface files and mesh supports are then linked to an automated meshing tool in the FMS and GAS workbenches in CATIA. The surfaces are meshed automatically in CATIA using the support geometry to define the nodes and elements. Each surface mesh is linked directly to the parent surface and mesh support geometry such that any changes to the parent geometry will be automatically reflected in the FEM. This process is summarized in Figure 1. A detailed overview of the ATLASS framework and individual modules is discussed in the original ATLASS paper [1].

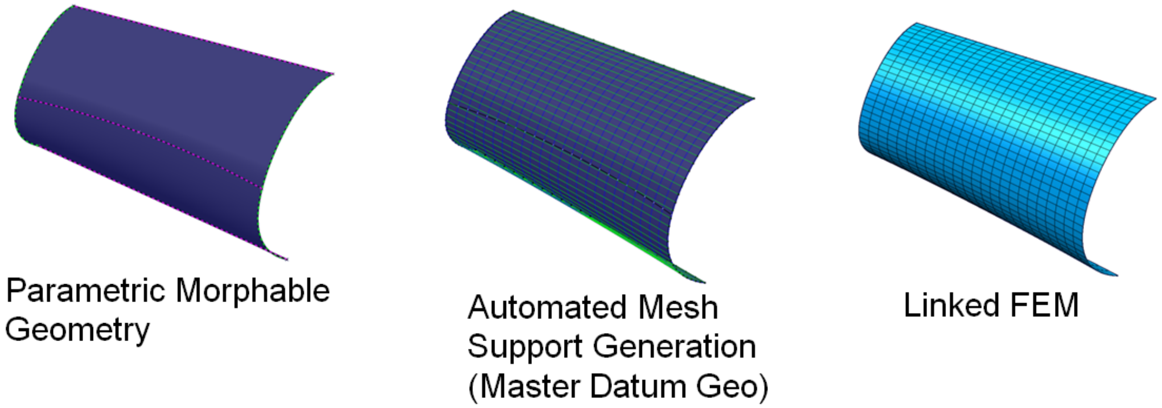


Figure 1. ATLASS Parametric Geometry to Mesh Framework

Advantages of the ATLASS framework

- Direct link between configuration geometry and mesh
- Geometry is recognized and tracked through the meshing process
- Changes to OML or structural layout will automatically update or regenerate
- Ability to add constrains, mesh density parameters and attributes to specific structures
- Exports .BDF file which can be directly read into Nastran or Hypersizer
- Single point aircraft configuration definition and model generation for all disciplines
- The framework is very scalable to include additional tools, methods and model fidelity

III. Legacy Wing Validation Study

A. Introduction

Once all initial modules were completed for model generation, loads, and structural analysis, a process was needed to gauge their performance against known baselines. A legacy wing configuration was chosen to evaluate local structural sizing capability, runtimes, and tool accuracy. The validation study was also conducted to determine the need for custom analyses, assess the sensitivities of certain load cases, and understand the role and requirements of manufacturing constrains in the optimization.

B. Legacy Data Preparation

A fully mature weights baseline was desired to compare results to a final flyaway or post-CDR structural weight. Legacy wing weights were determined using released engineering from a previously certified program. Volume measurements were taken from ‘as manufactured’ models and material density was used to determine weight. This allowed a bare structure weight to be determined in the absence of fasteners, clips, fittings and splice straps. This was done to ensure a valid comparison with Hypersizer weight results was achieved.

Secondary factors will be used, post analysis, to estimate these items such that bonded, and other integrated design concepts can be evaluated. Additional studies, currently in development, will access estimation factors for secondary structure, attachment hardware, and assembly fittings. Examples of weights data extraction are shown in Figure 2.

Similarly, mature loads data was taken from the prior certification program for comparison with the ATLASS loads module. Because ATLASS uses a reduced loads set, design limit load envelopes were used for comparison rather than individual load cases. This allows a better identification of shortcomings in the load cases that were included in the loads module.

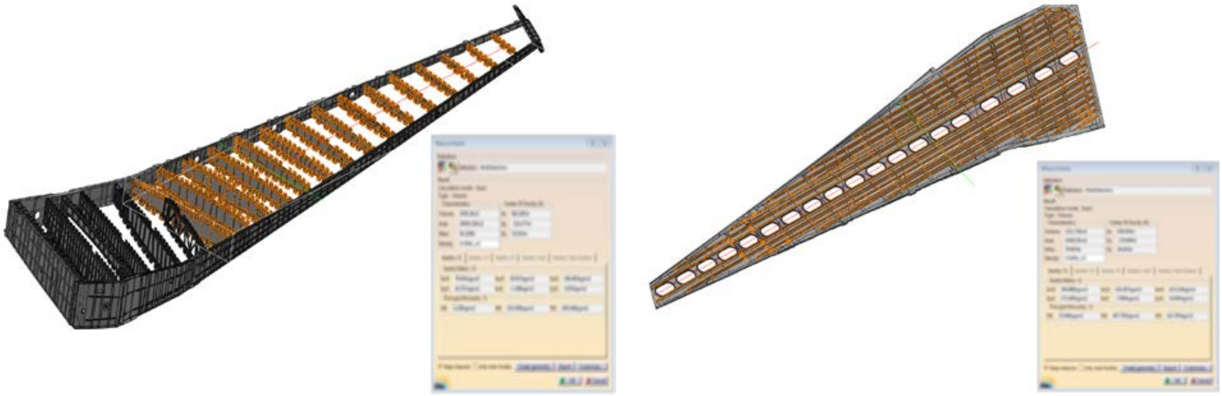


Figure 2. Weights Data Extraction

C. Loads Comparison

Before the full structural optimization was run, the loads module was run independently to determine if reasonable inputs would be produced for the sizing module. Three cases were run: 2.5g pullup, -1g pushover and 1g level cruise. The cases were initially selected to provide baseline for critical wing out-of-plane bending (Mx), recognizing this very limited load set would not generate critical forces and moments in other directions.

The initial loads run showed very good correlation with the Mx baseline with a 15% conservative maximum up-bending value and <5% delta on down-bending. As expected, the general cases did not generate any considerable loads in torque or in plane bending, shown in Figure 3. The result was considered adequate for initial sizing and the optimization continued. Additional load case requirements were added to the list of updates for the next release to address torque and in-plane bending.

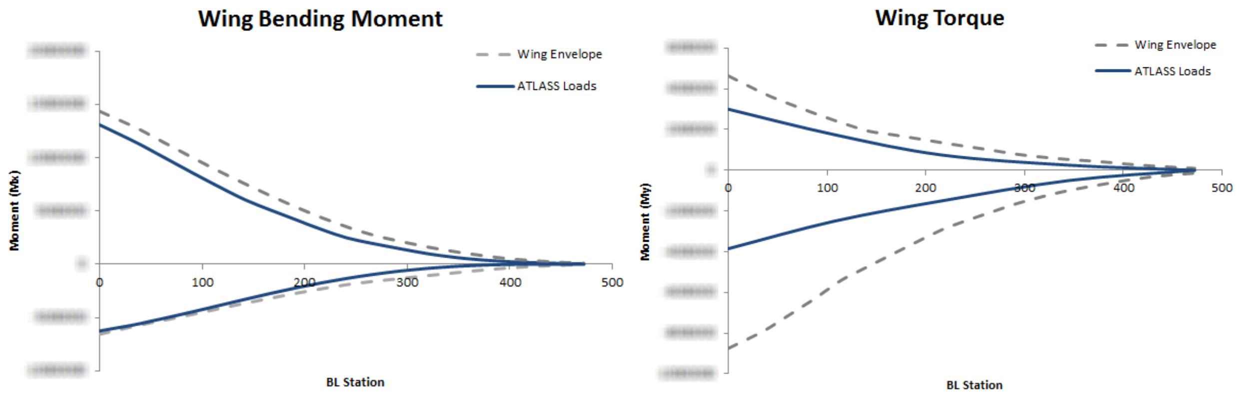


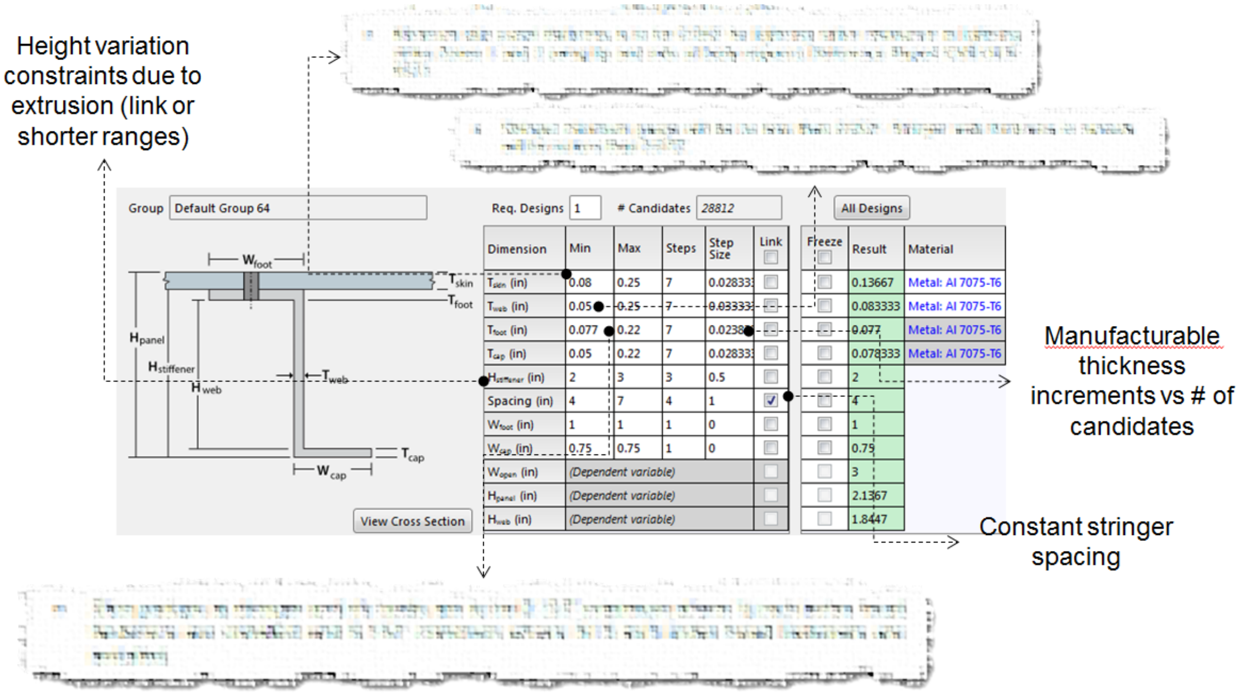
Figure 3. Mx And My Loads Comparison

D. Structural Sizing Setup

The structural optimization was set up in Hypersizer to mimic the legacy configuration used in the validation study. This required limiting many of the optimization options in Hypersizer to restrict the possible configurations. The stringer configuration was limited to the production cross-section, for example, and the ribs and spars were limited to ‘C’ cross-sections with vertically stiffened webs. Stringer spacing was also linked to prevent varying spacing between adjacent panels.

The sizing variable ranges were also set to match legacy manufacturing and design constraints, shown in Figure 4. These constraints included minimum manufacturing gage requirements, extrusion dimensional requirements and fastener installation requirements. This was done to ensure a realistic weight comparison was achieved and prevent an over-optimized solution.

Failure mode criteria were also input to match the design requirements of the program. Some compromises were made when inputting the skin buckling criterial where different requirements were imposed on a single skin panel. Hypersizer can only size to a single criteria for a given component, so for these instances the dominant requirement for the overall acreage of the panel was used. An example of this compromise was skin panels near discrete load points requiring stability to ultimate rather than limit.



Height variation constraints due to extrusion (link or shorter ranges)

Group: Default Group 64

Req. Designs: 1

Candidates: 28812

Dimension	Min	Max	Steps	Step Size	Link ☐	Freeze ☐	Result	Material
T_{skin} (in)	0.08	0.25	7	0.028333	☐	☐	0.13667	Metal: Al 7075-T6
T_{web} (in)	0.05	0.25	7	0.033333	☐	☐	0.083333	Metal: Al 7075-T6
T_{foot} (in)	0.077	0.22	7	0.02383	☐	☐	0.077	Metal: Al 7075-T6
T_{cap} (in)	0.05	0.22	7	0.028333	☐	☐	0.078333	Metal: Al 7075-T6
$H_{stiffener}$ (in)	2	3	3	0.5	☐	☐	2	
Spacing (in)	4	7	4	1	☑	☐	4	
W_{foot} (in)	1	1	1	0	☐	☐	1	
W_{cap} (in)	0.75	0.75	1	0	☐	☐	0.75	
W_{open} (in)	(Dependent variable)				☐	☐	3	
H_{panel} (in)	(Dependent variable)				☐	☐	2.1367	
H_{web} (in)	(Dependent variable)				☐	☐	1.8447	

Manufacturable thickness increments vs # of candidates

Constant stringer spacing

Figure 4. Hypersizer Sizing Form Setup with Program Requirements

E. Optimization Setup

Isight was used to manage the sizing and loads loops. The convergence criteria used for sizing was 0.5% total structural weight. The convergence criteria used for the overall optimization was 1% of converged sizing weight between loads iterations. On average this resulted in between 5-10 sizing loops and 3-5 loads loops. A process flow chart for the loads and sizing loops is presented in Figure 5. Top level optimizations were not run since the configuration was fixed. Once all modules were properly linked to Isight, the tool provided seamless module execution and data transfer.

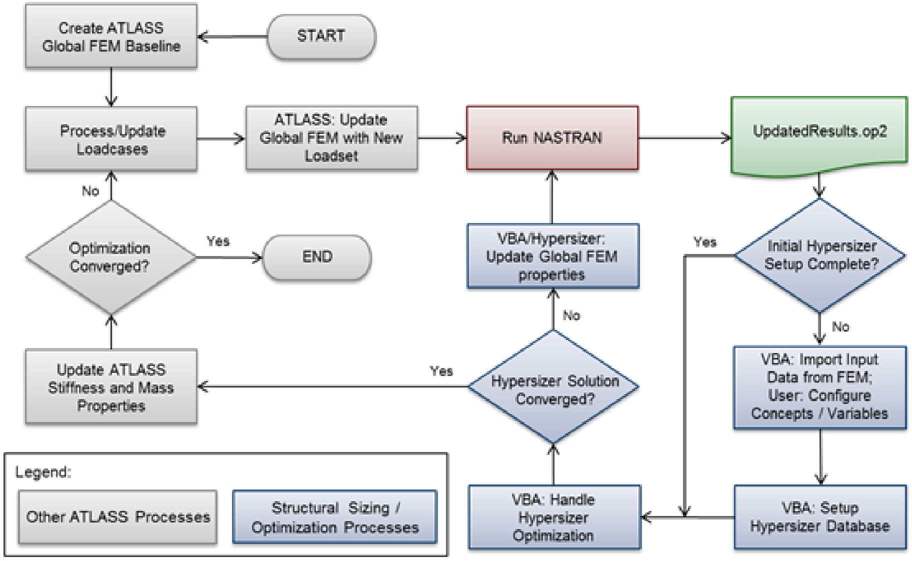


Figure 5. Sizing and Loads Loop Flow Chart

F. Results Discussion

The initial run resulted in four total loads loops and 18 Hypersizer runs, shown in Figure 6. The initial weights estimate was fairly close and resulted in small deltas between loads loops. The incremental jumps between load loops was attributed to the maneuver cases being less sensitive to mass distribution compared to dynamics cases. The total structure weight converged to within 20% of the measured weights.

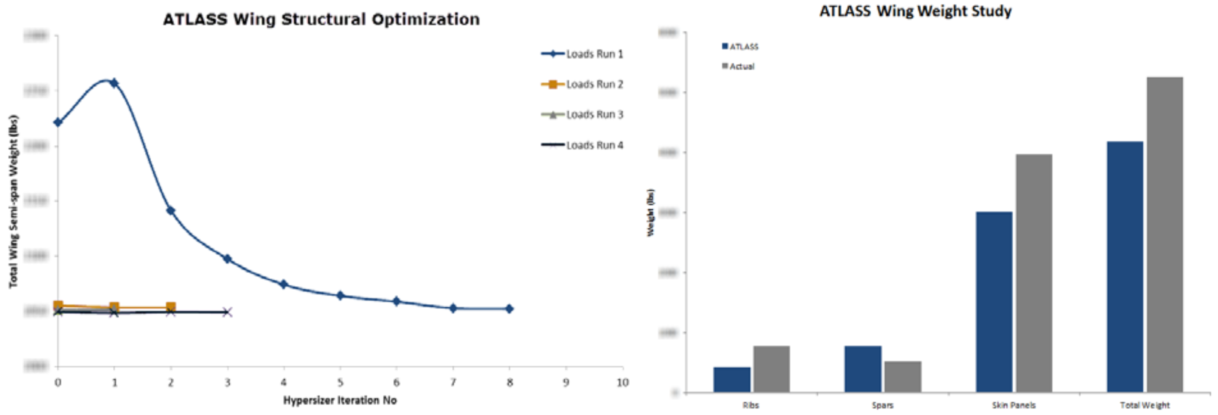


Figure 6. Initial Legacy Wing Run

The preliminary run was very promising considering all models and aircraft data were generated from scratch in ALTASS, no legacy data was used in the generation of the analysis models. It was also noted that some of the failure modes in the analysis were not being updated due to an issue with the Hypersizer API. The failure modes were updated manually and the weights converged several percent closer to actual.

G. Lessons Learned and Key Findings

- Initial results were very promising as total weight was estimated within 20%
- Weights are expected to be lower due to optimization to zero margin
- Sizing underestimated rib weights considerably
    - Critical cases were reviewed for production ribs with most being ground and dynamic cases
    - Rib crushing was not included in the analysis
    - Additional analysis and load cases are needed to accurately estimate rib weight
- Sizing favored spar stiffness over skin stiffness
    - The failure mode bug primarily effected skin buckling modes and lead to under-sizing
    - Lack of torsion in the external loads likely attributed
- Additional critical load cases are needed for other components (vertical and horizontal tail, fuselage)

IV. ATLASS Tool Updates

To address findings from the initial wing validation study, several updates were made to the ATALSS loads, sizing, and geometric modules before proceeding with a full aircraft study.

A. Loads Model Updates

In order to better correlate ATLASS loads with legacy component load envelopes, several additional load cases were added. It was decided to initially focus on maneuver loads since the existing model could be used as a starting point. However, additional model fidelity was needed to support the new cases. The model was updated from a semi-span model to a full-span model to allow for asymmetric load cases. All primary control surfaces were added to allow for more complex cases to be analyzed. The rudder, flaps, and ailerons were added by using data from the ATLASS geometry module. Lastly, a fuselage aero model was added to support fuselage air loads. The updated loads model is shown in Figure 7.

All required loads model data was incorporated into the geometric engine to allow fully parametric and morphable model data. All aerodynamic and structural model data is generated automatically and exported using Python. The module is integrated into the framework of ATLASS and runs without any intervention.

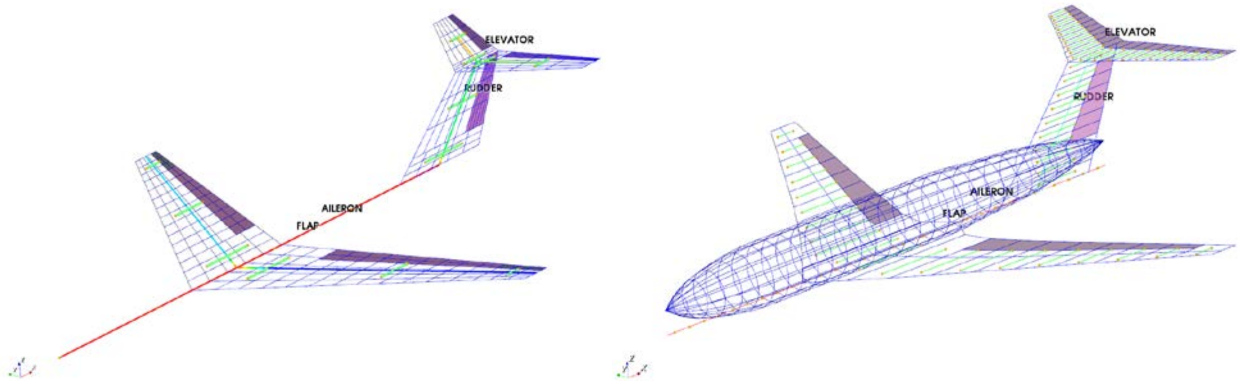


Figure 7. Updated Loads Model

When the geometric model data is exported, case control cards are written along with trim cards to set up the induvial load case runs. This is also automated through Python. For the first set of load-case updates, a total of 7 cases were added:

- 2.5G Pull up with horizontal tail mistrim (HT case)
- -1G Pushover with horizontal tail mistrim (HT case)
- 2G Pull up with Flaps: 20° (Wing torsion case)
- 2G Pull up with Flaps: 39° (Wing torsion case)
- -1G Push over with Flaps: 20° (Wing torsion case)
- -1G Push over with Flaps: 39° (Wing torsion case)
- Rudder Kick (VT case)

B. Structural Sizing Module Updates

Updating to a subsequent version of Hypersizer fixed the failure mode issue and affected skin panel weights jumped several percent. In addition, some of the manufacturing constraints were updated to better represent the legacy configuration. Lastly, there was no existing method in Hypersizer to analyze rib crushing, so a custom automated method was developed as a plugin to Hypersizer.

Integrating this functionality into an automated method was a challenge due to the analysis requiring either a deflected shape or smeared panel loads to calculate the crush load [2]. The approach used to solve this issue was to run each load case twice: once to get the panel loads and calculate crush loads then again with the crush added into the model. A separate code was developed to add rib crushing external loads to every node/element in a rib, for each load case, see Figure 8.

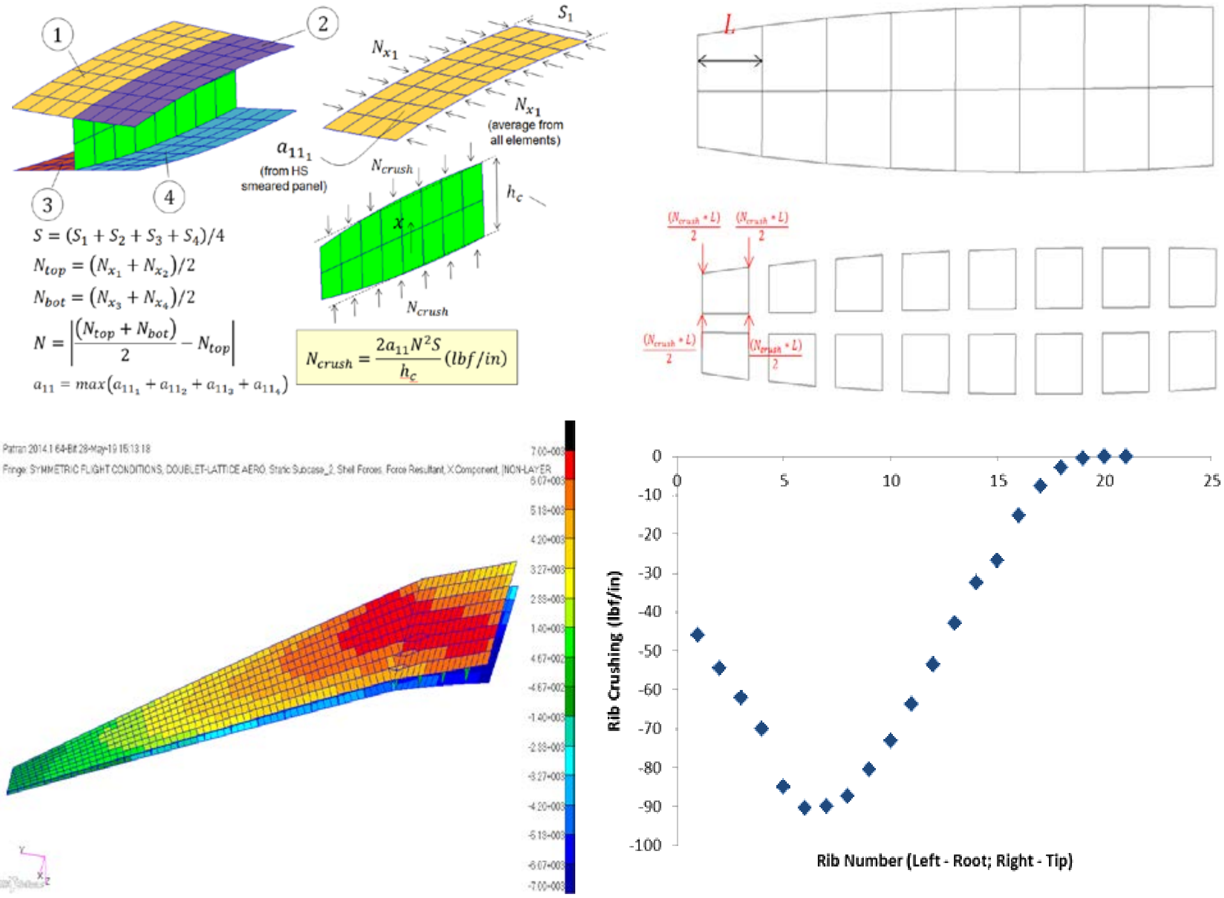


Figure 8. Automated Rib Crushing Tool Set Up and Results

The tool was fully integrated into ATLASS and interfaces with the runtime spreadsheet. The validation configuration was run through the tool and it was determined that rib crushing does contribute to margin of safety and rib weight. It did not lead to a significant weight increase for current configuration, less than 10% in most cases. However, this is a configuration sensitive analysis and will provide higher maturity rib analysis for future sizing and optimization.

C. Updated Results

After all updates to the various modules were completed, the validation study was repeated to re-correlate the results. First, the failure modes for all panels were checked and confirmed to be the correct factor of safety. The loads were also re-validated with the flap cases adding significant torque loads. The results, once again, correlated within 15% of the actual loads envelope for the wing. Next, the sizing and manufacturing constraints were checked to insure the final sized configuration properly met the requirements.

Lastly, all margins of safety were checked to ensure all panels were sized correctly. A small number of areas did generate negative margins near the spar kick. It was determined that a modeling simplification was causing a stress concentration in this area. However, by limiting the maximum values for sizing variables the effect on weight was marginal. The strategy of using the maximum values can be helpful in preventing unrealistically high weights in areas where false stress concentrations occur.

The final weights results are shown in Figure 9. As expected, the skin panel and rib weights increased while the spar weights came down slightly. The optimization still seemed to favor spar stiffness over skin panel stiffness, but did show a closer correlation. The rib weights were still being underestimated considerably. The overall wing box weight was estimated within 15% which is well within the goals of the project for a limited loads set.

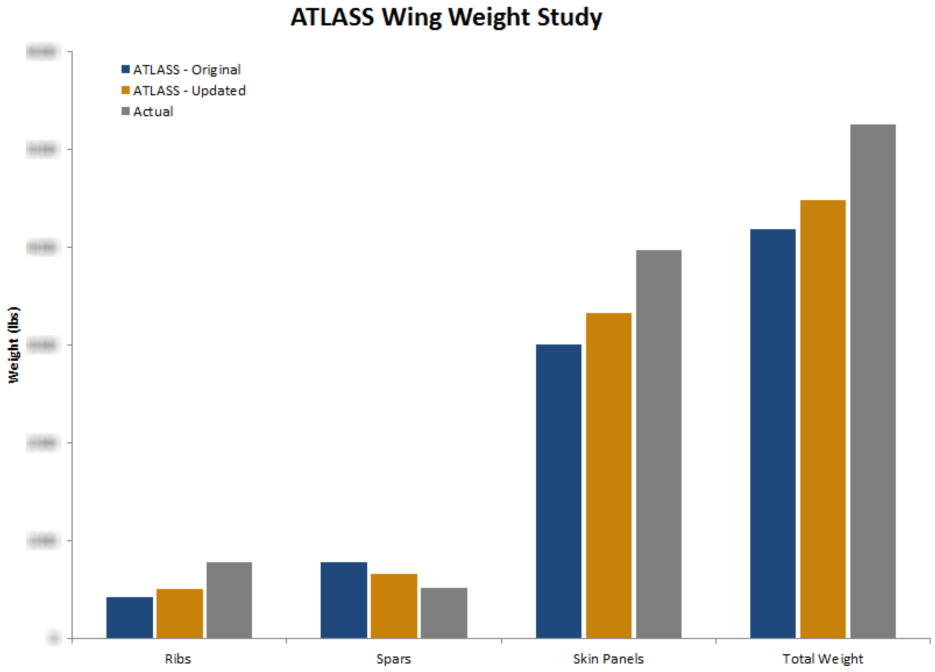


Figure 9. Updated Weights Results

In analyzing the results and sizing variables, several takeaways could be drawn from the data. As with the previous study, Hypersizer optimizes all structures to zero margin, which is not always the case in production for several reasons. Also, the Hypersizer weight estimation represents an idealized weight that does not include production features such as fillets, additional stiffeners, and interfacing details, shown in Figure 10. These features can account for over 5% of final part weight. Including this additional weight will be evaluated for future releases, and is discussed in Section VI.

In addition, the critical load cases were analyzed for a sampling of production ribs. In almost all cases maneuver loads were not the cases that lead to a critical margin. For this reason dynamics cases, ground, and crash loads will be added to the development roadmap for the loads module.

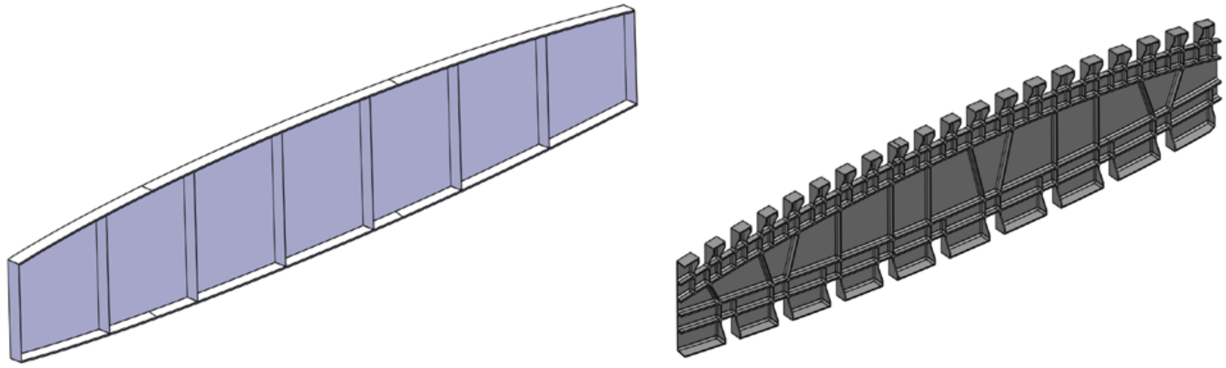


Figure 10. Idealized Hypersizer Rib and a Manufacturing Representative Rib Template

V. Full Aircraft Validation Study

A. Model Preparation

Once the wing study showed sufficiently accurate results, a full aircraft validation study was conducted. A similar approach was taken using legacy released engineering to get a baseline structural weight for comparison. For this study the wing, fuselage, horizontal and vertical tail primary structures were analyzed. The current model uses independent boundary conditions for each component to reduce model complexity. Explicit attachments are not modeled or sized, but are planned for future releases.

The global finite element model was already available within ATLASS with primary structures represented for each component, shown in Figure 11. The model uses rigid body element (RBE) spiders at each rib and frame location for load application. The code used to spread loads from the loads module to the GFEM was updated to transfer fuselage and empennage loads.

Figure 11. Global Structural Finite Element Model

The Hypersizer interface spreadsheet was setup to support all structural elements, materials and shapes used in legacy aircraft. Only the specific assemblies needed to be selected and design variable limits selected. A similar approach was used from the wing; using manufacturing and other design requirements to set these limits. Requirements such as minimum skin gage, frame height limitations, and damage tolerance were incorporated.

B. Loads Analysis

The updated loads module was also re-correlated with existing loads for the vertical and horizontal tails. The horizontal tail showed a good correlation within 15% of maximum up and down-bending (Mx). Similar to the wing, the torsion and out of plane bending were under-estimated due to being driven by other load cases. The rudder kick case showed a good correlation with the actual load case, however, was well short of the overall envelope. This was a result of the envelope being driven by non-maneuver cases. For this reason, a factor was added to the loads in this case to better correlate with the overall envelope while additional load cases are being integrated. The initial horizontal and vertical tails out of plane bending comparisons are shown in Figure 12.

The fuselage was run for all existing cases with, and without, pressurization. Pressure was added to each outer element within the fuselage pressure boundary. The pressure boundary is identified in the geometry in ATLASS by custom properties such that the proper elements can be recognized downstream. The burst pressure case was used for pressure magnitudes.

Overall, the loads correlation proved a good initial pass to provide critical bending cases for each component. However, the vertical tail further highlights the need for additional load case families in an optimization tool. While maneuver cases provide adequate load estimates for bending moment in wing-like structures, they do not provide critical applied loads in other directions. Additional loads analysis is needed such as ground loads, dynamic maneuvers, and dynamic landing and crash loads to generate a more complete external loads estimate.

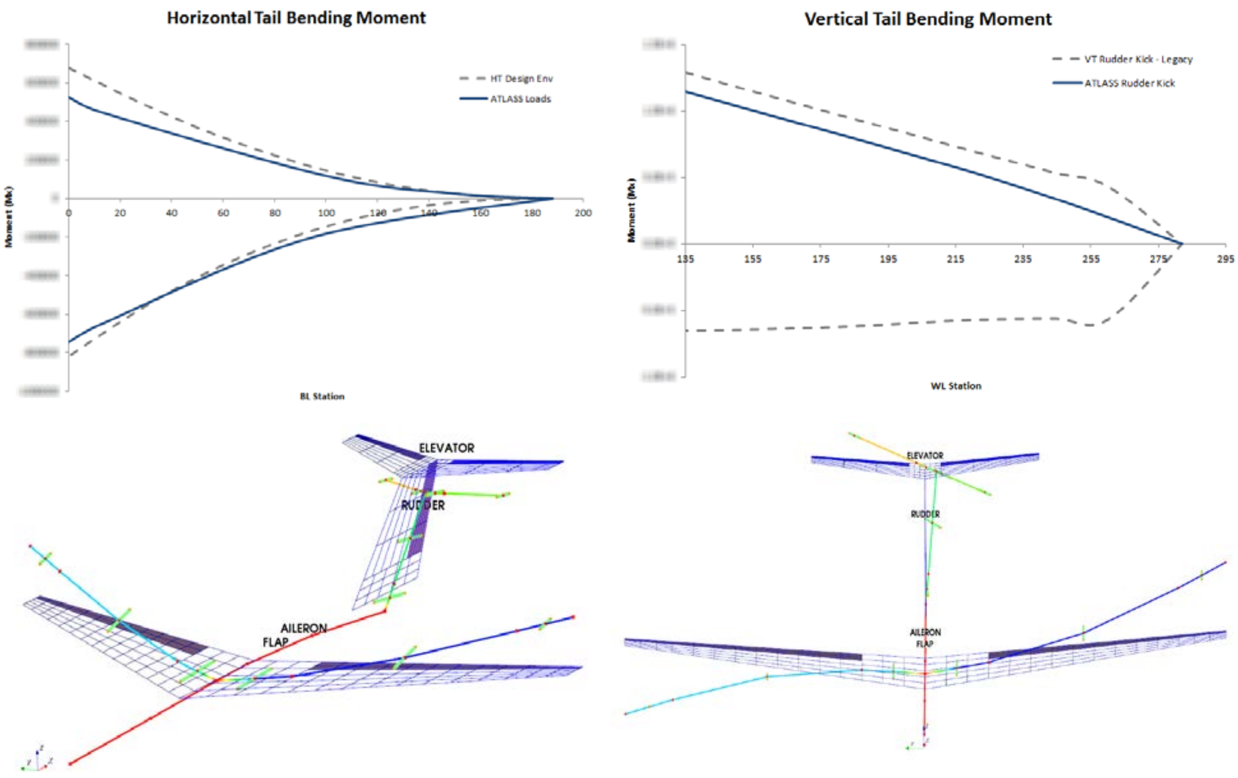


Figure 12. Loads Model Comparison

C. Results Discussion

The initial full aircraft run generated a total of 17 sizing analyses and 3 loads loops, shown in Figure 13. The total structural weight converged within 10% of the actual bare structural weight. The wing remained within 15% of actual with the HT and VT coming within 13% and 15% respectively. The HT represented a challenge in sizing setup due to the use of composites in this structure. The Sizing Module was set up to support composite structures and stiffened panels. The sizing variable set-up and constraints however, can add considerable complexity to the analysis. Manufacturing constraints such as balanced and symmetric tape layups can highly constrain the optimization. If these requirements are relaxed the potential configurations can lead to excessive run times. The constraints for this initial run were set up to allow the optimization to evaluate more configurations, but this will be revised in future releases. The VT was somewhat undersized and is likely a result of HT interaction being excluded. More detailed component to component attachment is currently in development and will be added in the next release so these interactions will be better represented. The Fuselage was sized within 13% of actual and was driven by some conservatism in the load cases.

The overall aircraft primary structural weight converged within 10%, which was well within the goals of the project for the initial run. Data generated from the runs is still being post processed, but several opportunities have been identified to increase optimization accuracy. Designing detailed ‘as manufactured’ part templates will be an area of development to address the weight disparity between the idealized estimate and the detailed model. Additional load cases will be added to the loads module to provide a more complete external loads picture, while manufacturing constraints will be further refined to address deltas in the data.

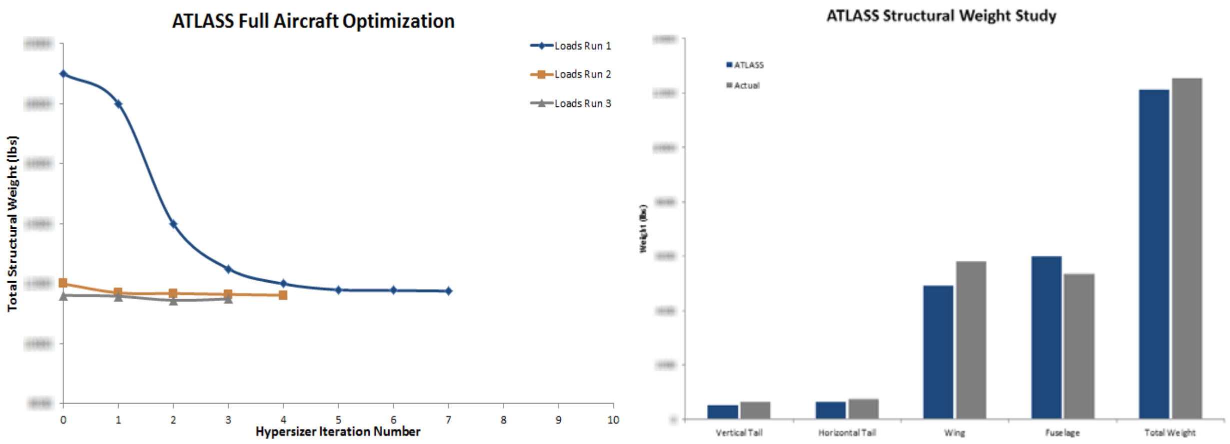


Figure 13. Full Aircraft Structural Weight Estimates

D. Lessons Learned and Key Findings

- Preliminary results were very promising, as total structural weight was estimated within 10%
- Sizing underestimated the Vertical tail considerably
    - The maneuver case included in the loads module did not provide sufficiently high loads compared to the legacy component envelope
    - Assembly effects, including that of the Horizontal tail were not included
    - Component to component attachment strategy is needed
- Fuselage structures were over-sized
    - Additional loads analysis is needed to better correlate with existing loads
    - Manufacturing constraints need to be reviewed
- Additional critical load cases are needed for all components (ground loads, dynamics, etc.)

VI. Optimization and Future Work

A template for validation studies has now been developed; additional configurations will be run to gain further correlation data. Once completed, the tool will be ready for initial use in structural trade studies and optimization analysis. In parallel several new features and improvements are planned.

A. Optimization

The current Isight run is set up to perform the loads and sizing loop to converge on an estimated weight for a given configuration. The model was also designed to perform high level design exploration and optimization, shown in Figure 14. These studies can include trading different materials, structural configuration, and wing size and shape. The process for these studies will be developed in 2020. The key limitation to optimization studies will be with tool run-time which is a primary focus of development.

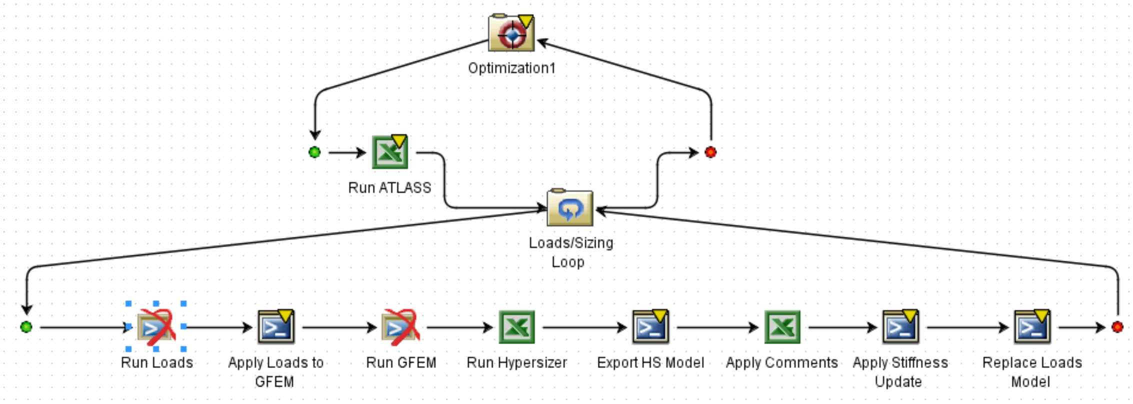


Figure 14. ATLASS Isight Model

B. Loads Module

The current ATLASS loads module represents a satisfactory starting point for aircraft loads analysis, however, the current 13 load cases do not yield critical external loads for all structural components. During detailed design, several thousand load cases are run. As previously stated, additional load cases were identified for wing ribs and other components. The process of identifying critical cases for all components being sized in Hypersizer is underway, and will drive which cases are added to the development roadmap.

Several ground, dynamics, and crash cases have been identified for possible development. In addition, flutter will be evaluated for possible integration into the loads loop to drive wing stiffness requirements. This analysis is not currently automated within ATLASS, but Hypersizer has the capability to size to stiffness targets. In the interim, a flutter check will be conducted after a design has converged.

After the integration of several additional load cases, an improved method of model setup, running the various conditions, and reducing the data will be required. A project is currently in work with the loads team to integrate these features. This will provide a more streamlined approach for initial loads estimation.

Lastly, the doublet lattice aerodynamic model currently used in the loads module does not provide accurate results in the transonic region. A method is needed to add correction factors in this region and will also be under development in the next release. Three-dimensional panel methods and other faster running analysis are being evaluated to maintain a similar run-time to the current analysis.

C. Automated Solid Model Generation

The weight deltas exhibited during the validation studies was partly attributed to Hypersizer's simplified view of the detailed parts. This was especially evident on the wing ribs. As leveraging the tool further into the detailed design process was one of the original goals, a feasibility study was conducted to evaluate detailed three-dimensional model integration into ATLASS. The goal being generation of a higher fidelity weight estimate and to provide more part data to other engineering disciplines earlier in the detailed design process.

A fully parametric part template approach was used in CATIA to provide an automated interface from sizing data to solid models. Both a rib and stringer template were developed to demonstrate the tool's capability to generate solid parts dynamically, based on the master datum files and sizing data generated in ATLASS. A code was also developed to automatically instantiate the templates at the required number of datum elements, allowing the tool to run fully autonomously and to be integrated into ATLASS, shown in Figure 15.

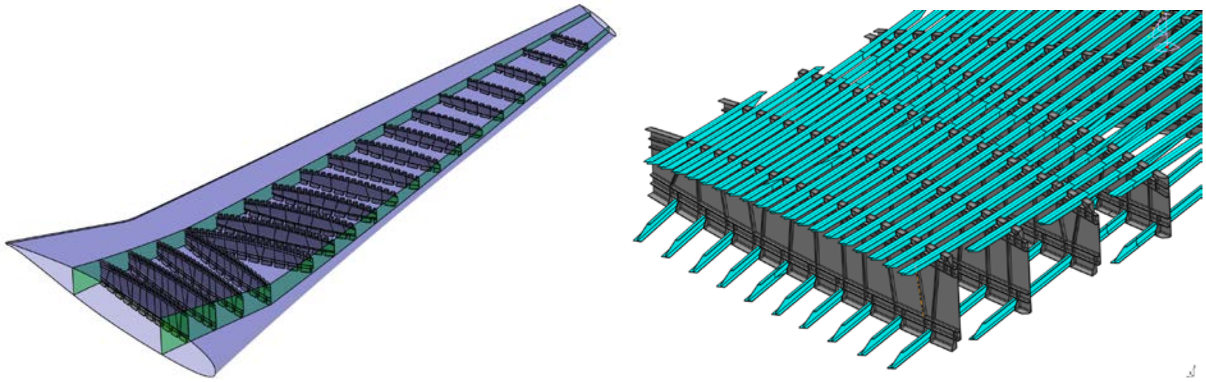


Figure 15. Detailed Solid Model Templates

The templates can be linked to ATLASS sizing data or design tables to allow for automatic updates during sizing revisions. The templates also implement a feature based design approach with design requirements and best practices built into the modeling features. Requirements such as assembly clearances, pocket depth vs radii, and edge-distance criteria are integrated into the code used to generate the feature geometry. The templates also have the ability to generate an automated three-dimensional mesh that is linked to the parent geometry. This allows for more detailed analysis and iterations to be conducted much faster during early phase design.

The initial feasibility study proved very successful and showed potential in several applications. The templates were instantiated using data completely generated in ATLASS on the left-hand wing. Total instantiation time was less than 15 minutes for over 50 unique solid parts. Template-based design methods will also be a primary area of development for future ATLASS releases and general design methods development.

VII. Concluding Remarks

The initial runs and validation studies of a fully automated aircraft structural design tool were conducted. The tool provides a new capability for performing structural trade studies as well as analyzing new or novel aircraft and structural configurations earlier in the design cycle. In addition, the tool can be leveraged to conduct multi-disciplinary optimization on any aircraft level variable to quantify its effect on weight.

ATLASS allows the ASMI group to better understand the implications of new material and structural technologies on the aircraft in a much more efficient way. As the tool matures it will also provide a much higher fidelity representation of the structure during the early design phases, as well as better integrate structures into the initial aircraft design and optimization. Lastly, the tool is built on a powerful engine and provides a platform for extensive additional development.

Acknowledgments

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