THE COMPOSITES AND ADVANCED MATERIALS EXPO

Dallas 2015

Analysis and Sizing Software for Composite and Metallic Airframe Structures

Composite Design & Analysis

Monday (10/26) 9am – 12pm

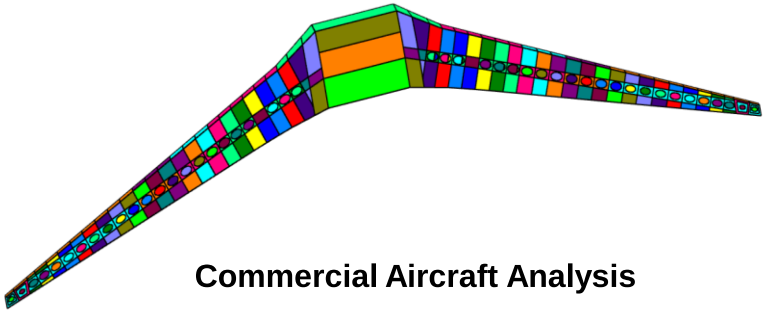
James Ainsworth
Collier Research Corporation
Hampton, VA

Let Me Introduce Myself

- Started at Collier Research in Jan 2009 (6+ years experience)
  - Title: Composite Stress Analysis, application engineer
  - Expertise: Closed-form analysis of stiffened composite structures
- Relevant Project Experience:

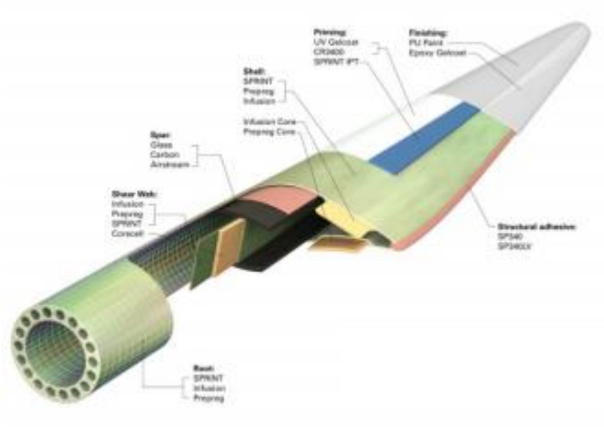
Composite Crew Module (CCM)

Ares V Launch Structures



Commercial Aircraft Analysis

Composite UAV



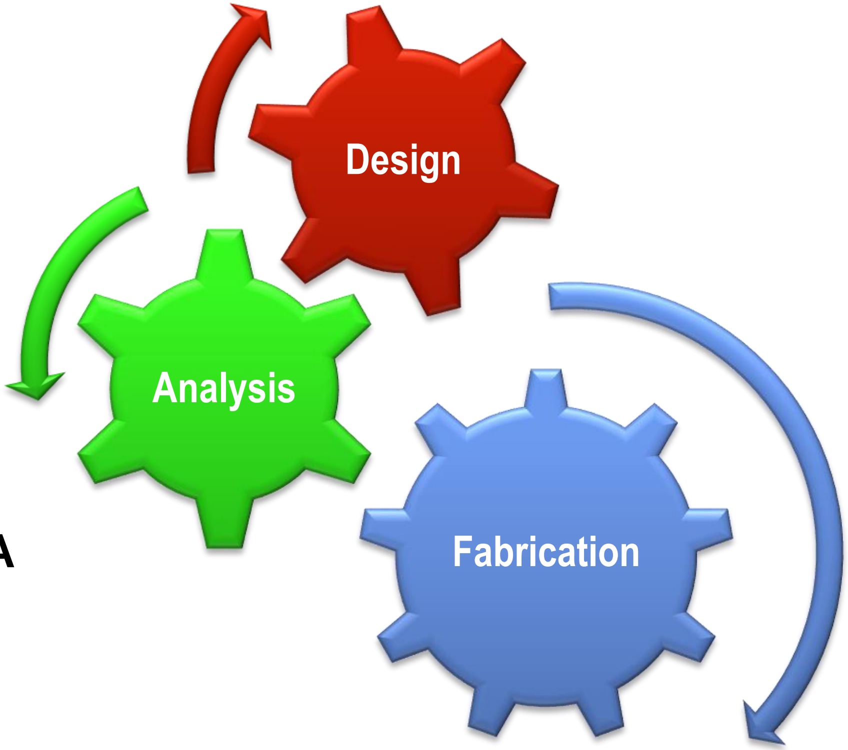
Wind Turbine Blades

Recreation

Outline for Presentation

- Composite ply properties
- Classical Lamination Theory (CLT)
- Extension of CLT to stiffened panels
- Margin of Safety
- Composite strength failure criteria
- Linear buckling
- Honeycomb panel failure
- Stiffened panel failure
- Composite joints
- Coupling analytical methods with FEA
- Stiffened panel modeling approaches
- Composite optimization
- Continuous vs. Discrete Sizing
- Designing composites for producibility and repair

Structural Design & Analysis with Composite Materials



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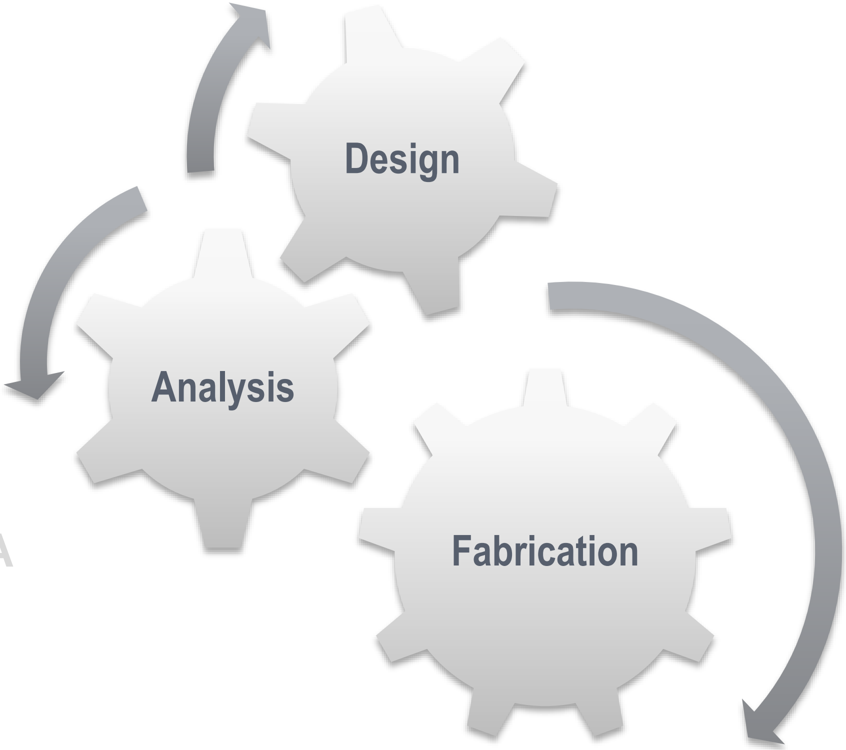
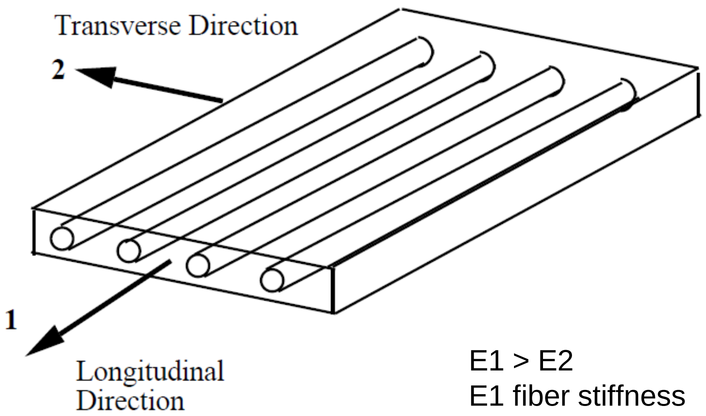


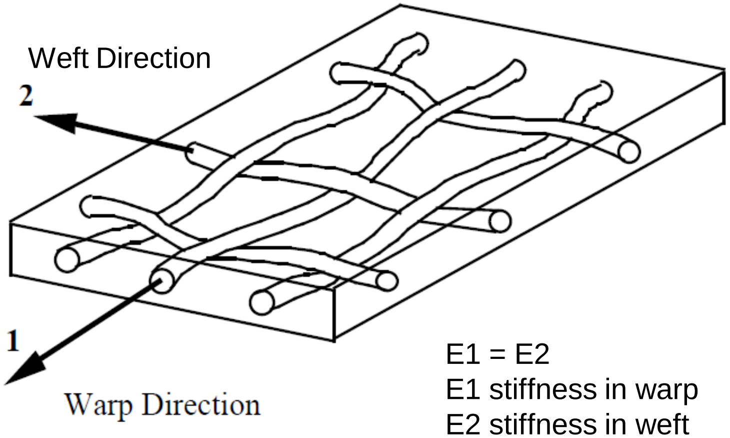
Orthotropic Material Properties

- Orthotropic materials have properties dependent on fiber (or warp) and matrix (or weft) directions (1, 2)



Unidirectional Tape

E1 > E2
E1 fiber stiffness
E2 matrix stiffness



E1 = E2
E1 stiffness in warp
E2 stiffness in weft

Woven Fabric Lamina

Sources for Composite Ply Properties

1. Coupon Testing
2. Mil-Hdbk17
3. Vendor data sheets

What does Orthotropic Mean?

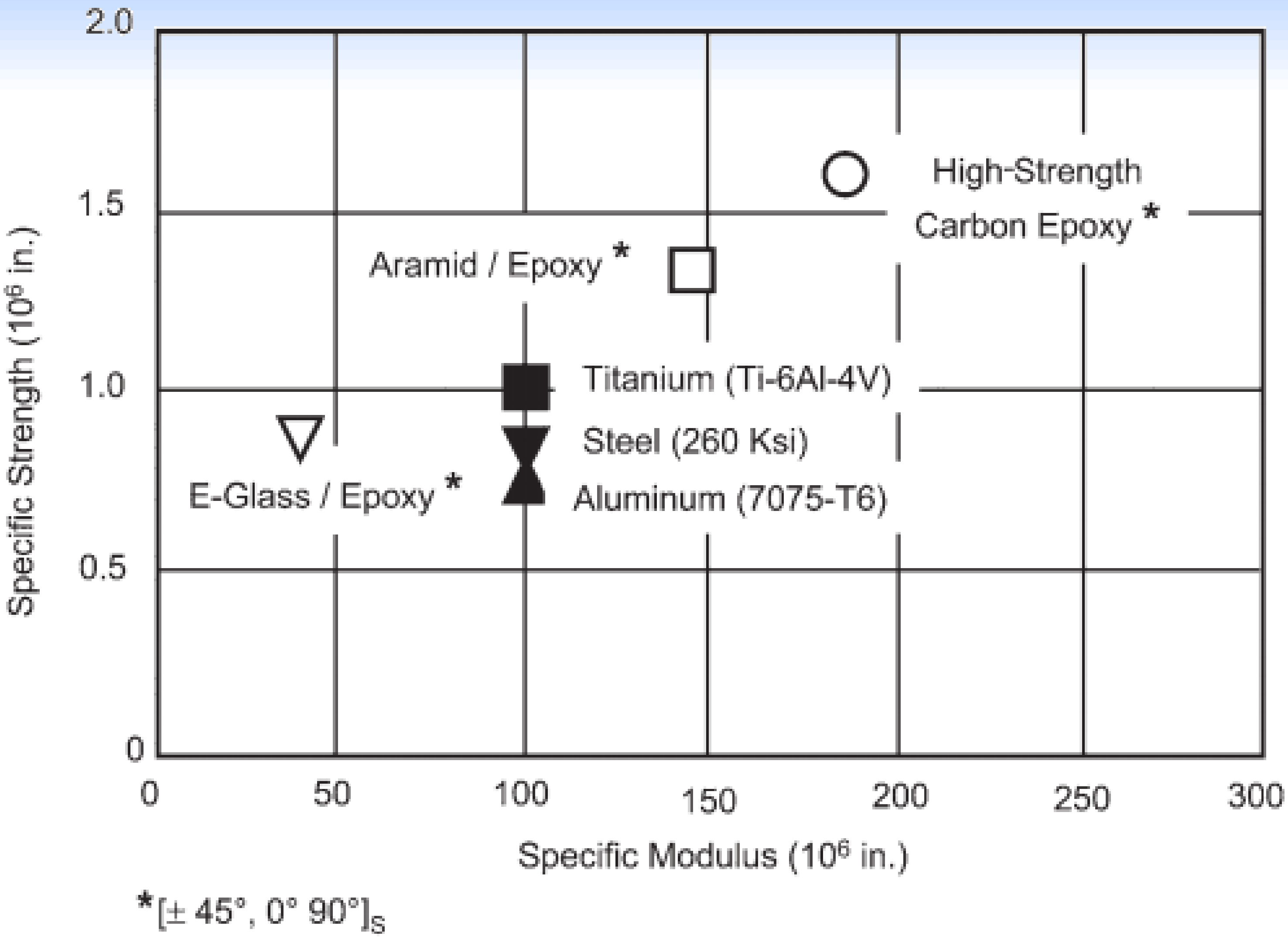
- Orthotropic
  - Properties are unique in 3 perpendicular directions

- Stiffness terms:

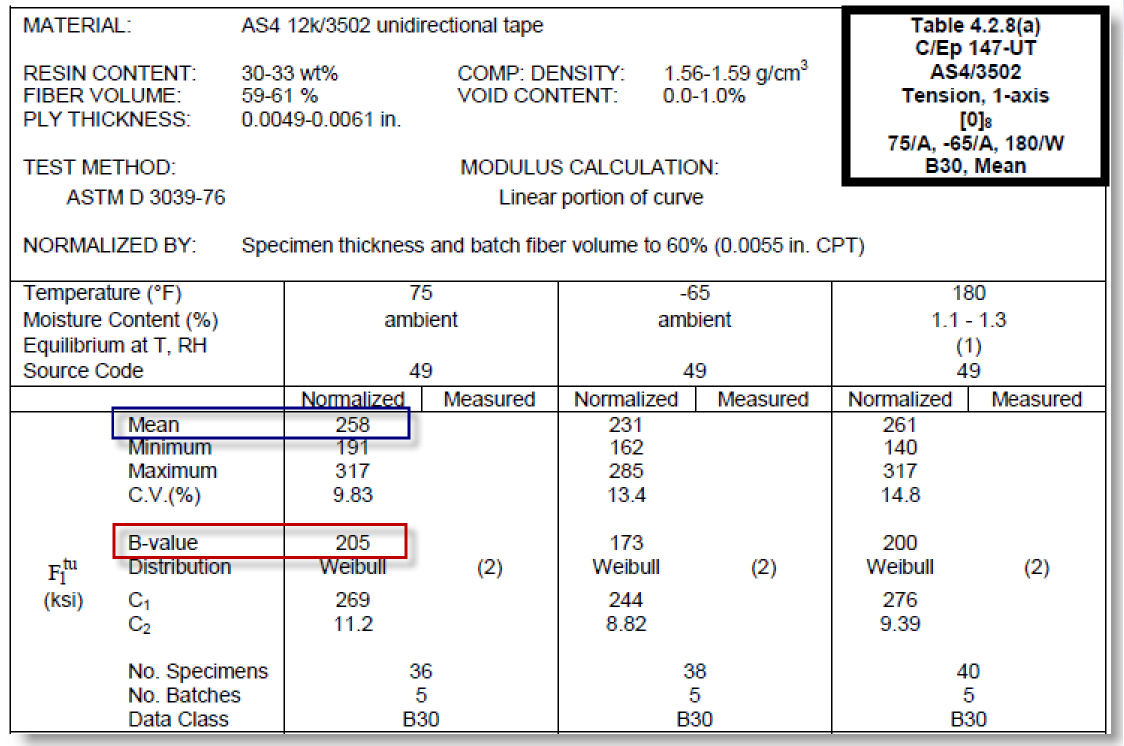
$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}$$

No normal-shear coupling terms, No Bij terms

Material Properties



Typical vs. “Basis” Properties



MATERIAL: AS4 12k/3502 unidirectional tape

Table 4.2.8(a)
C/Ep 147-UT
AS4/3502
Tension, 1-axis
$[0]_8$
75/A, -65/A, 180/W
B30, Mean

RESIN CONTENT: 30-33 wt%
FIBER VOLUME: 59-61 %
PLY THICKNESS: 0.0049-0.0061 in.
COMP: DENSITY: 1.56-1.59 g/cm^3
VOID CONTENT: 0.0-1.0%

TEST METHOD: ASTM D 3039-76
MODULUS CALCULATION: Linear portion of curve

NORMALIZED BY: Specimen thickness and batch fiber volume to 60% (0.0055 in. CPT)

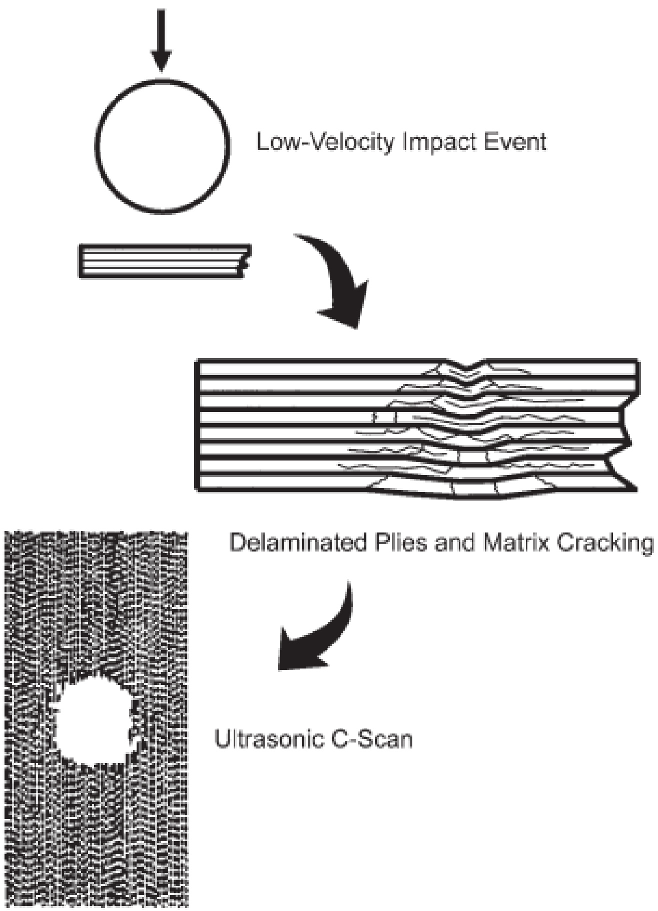
Temperature (°F)		75		-65		180	
Moisture Content (%)		ambient		ambient		1.1 - 1.3	
Equilibrium at T, RH						(1)	
Source Code		49		49		49	
		Normalized	Measured	Normalized	Measured	Normalized	Measured
	Mean	258		231		261	
	Minimum	191		162		140	
	Maximum	317		285		317	
	C.V.(%)	9.83		13.4		14.8	
	B-value	205		173		200	
F_1^{tu}	Distribution	Weibull	(2)	Weibull	(2)	Weibull	(2)
(ksi)	C_1	269		244		276	
	C_2	11.2		8.82		9.39	
	No. Specimens	36		38		40	
	No. Batches	5		5		5	
	Data Class	B30		B30		B30	

- Typical (or Mean) properties are determined as the average failure load from a series of identical tests.
- “Design-to” allowables are statistically determined such that a certain percentage of the test values will be above the allowable with a certain confidence.

- Typical = Mean of test sample
- Basis (design-to):
  - A-Basis = 99% of failure is expected to occur above allowable with 95% confidence
  - B-Basis = 90% of failure will occur above allowable with 95% confidence

Pristine vs. Damage Tolerance Properties

Barely Visible Impact Damage



- **In practical design situations pristine ply allowables are knocked down for damage tolerance.**
- **Knocked down allowable may be 40%-60% pristine value**
- **Material corrections used to account for…**
  1. **Open hole (0.25” open hole)**
  2. **BVID**
  3. **After-impact, CAI, TAI, SAI**
  4. **Filled Hole, FHT, FHC**
  5. **Ageing, Moisture**

Design-to damage tolerant ply strain allowable (AS4-3502 Gr/Ep) = 4400μin/in

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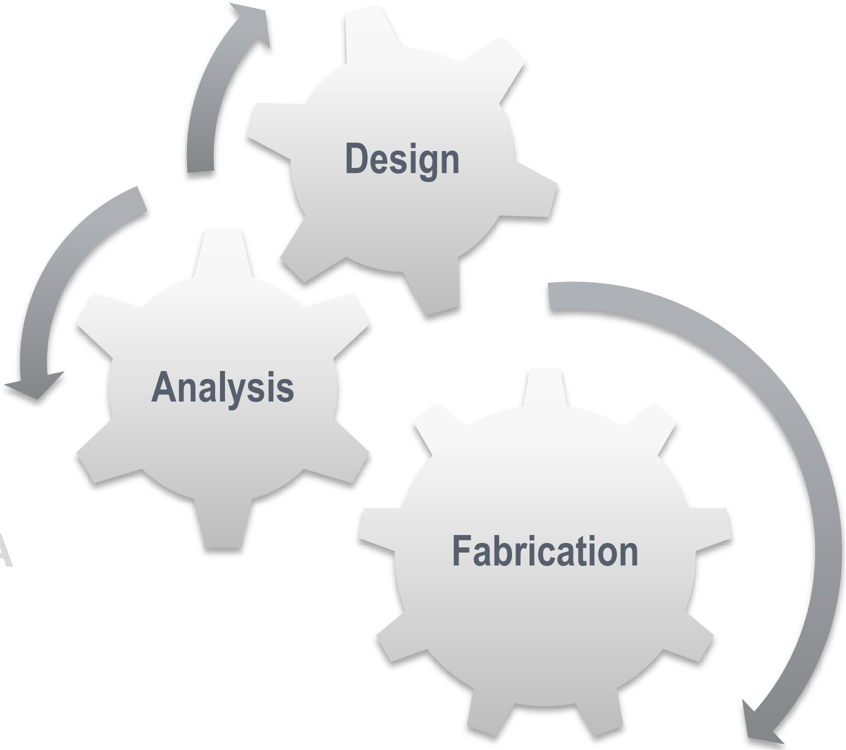
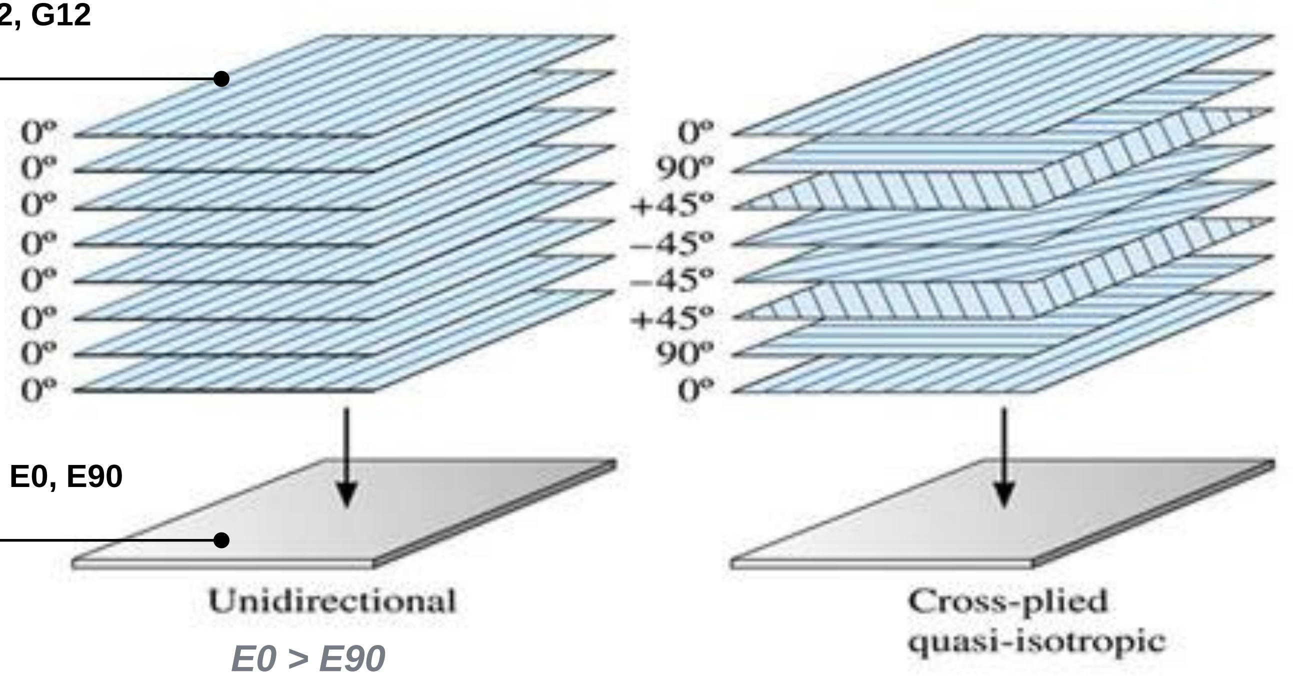


Laminate Properties

- **Laminate stiffness properties determined from Classical Lamination Theory (Laminated Plate Theory)**



Ply stiffness, E1, E2, G12

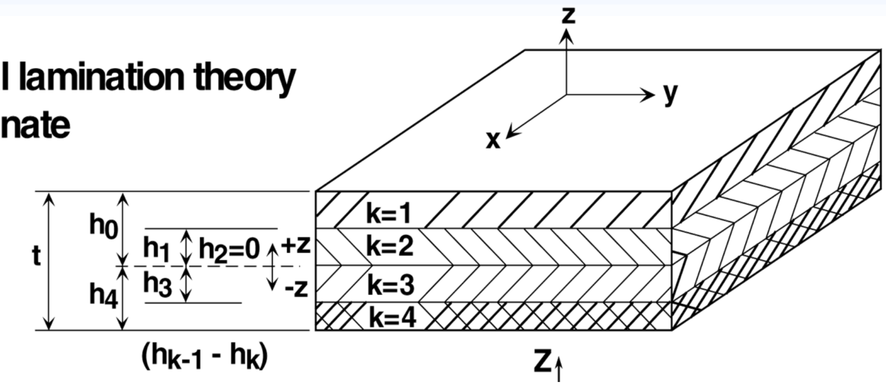
Laminate stiffness, E0, E90

E0 > E90

E0 = E90

Laminate Stiffness Formulation

Classical lamination theory of a laminate



Reduced stiffness terms based on orthotropic ply properties

$$Q_{11} = \frac{E_1}{1 - \nu_{12}^2 \frac{E_2}{E_1}} \qquad Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12}^2 \frac{E_2}{E_1}}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}^2 \frac{E_2}{E_1}} \qquad Q_{66} = G_{12}$$

$$\boldsymbol{A}_{ij} = \sum_{k=1}^{n} \{Q_{ij}\}_n (z_k - z_{k-1})$$

$$\boldsymbol{B}_{ij} = \frac{1}{2} \sum_{k=1}^{n} \{Q_{ij}\}_n (z_k^2 - z_{k-1}^2)$$

$$\boldsymbol{D}_{ij} = \frac{1}{3} \sum_{k=1}^{n} \{Q_{ij}\}_n (z_k^3 - z_{k-1}^3)$$

[A] → membrane stiffness (EA)

[D] → bending stiffness (EI)

[B] → membrane-bending coupling

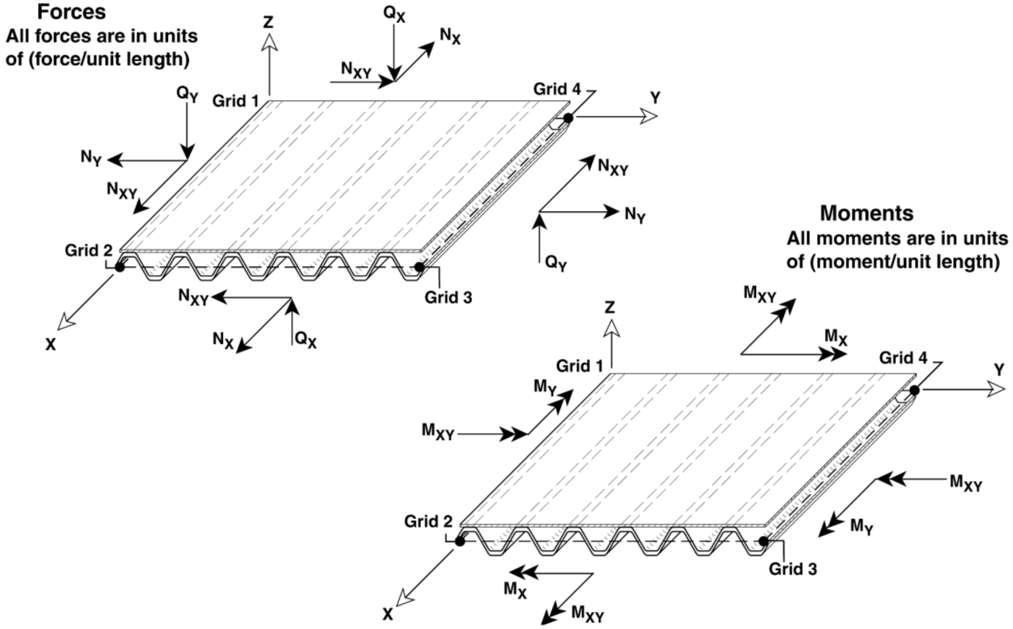
Basic Plate Theory

- Panel constitutive equation

$$\begin{bmatrix}\vec{N}\\ \vec{M}\end{bmatrix} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{D}\end{bmatrix}\begin{bmatrix}\vec{\varepsilon}\\ \vec{\kappa}\end{bmatrix} - \begin{bmatrix}\vec{N}^{T}\\ \vec{M}^{T}\end{bmatrix}$$

- **Straight-forward method for resolving uniform in-plane load and bending into laminate strains and curvatures.**

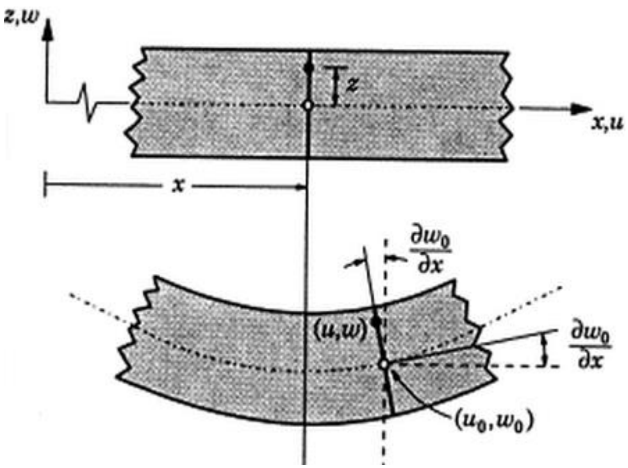
- Force Sign Convention



Forces
All forces are in units of (force/unit length)

Moments
All moments are in units of (moment/unit length)

- Kirchoff-Love Plate Assumption
  - **Straight lines normal to the mid-surface remain straight after deformation**
  - **Straight lines normal to the mid-surface remain normal to the mid-surface after deformation**
  - **The thickness of the plate does not change during a deformation.**



$$\varepsilon_x = \varepsilon_x^{\circ} + z\kappa_x$$

$$\varepsilon_y = \varepsilon_y^{\circ} + z\kappa_y$$

$$\gamma_{xy} = \gamma_{xy}^{\circ} + z\kappa_{xy}$$

Relationship Between Force and Strain

"Unknowns" **Stiffness Matrix** **"Knowns"**

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Unknowns on left, Knowns on right

Relationship Between Force and Strain

$$\begin{array}{ccc} \textbf{STRAINS} & \textbf{Inverted Matrix} & \textbf{FORCES} \\ \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = & \begin{bmatrix} A & B \\ & \\ B & D \end{bmatrix}^{-1}_{6x6} & \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} \end{array}$$

$$\varepsilon_x = A^{-1}_{11} N_x + A^{-1}_{12} N_Y + \ldots$$

When coupling analysis codes with a FEM, the FEA computed forces are imported to compute panel strains and curvatures this way.

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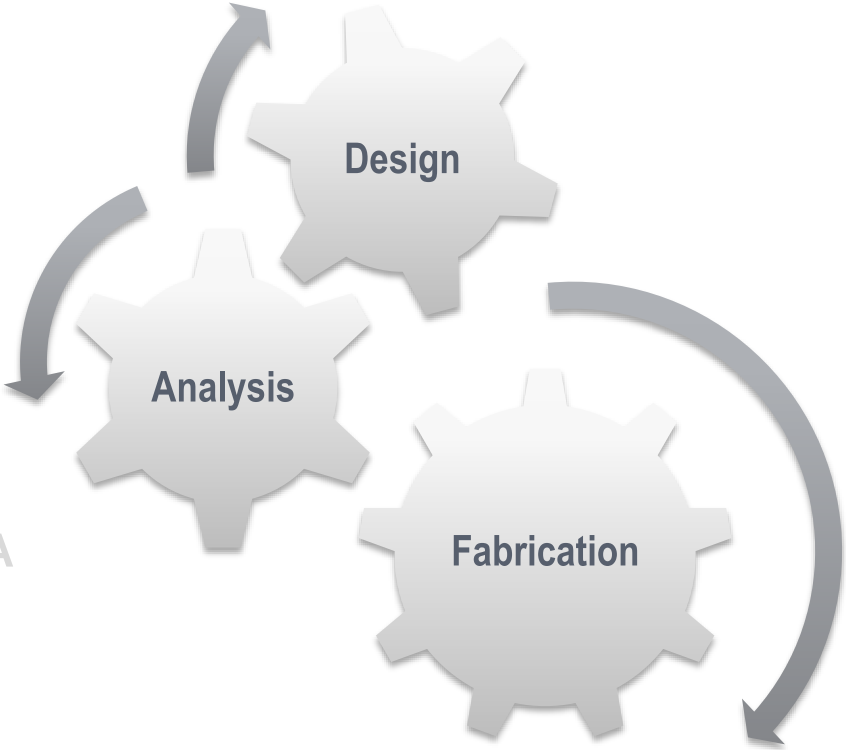
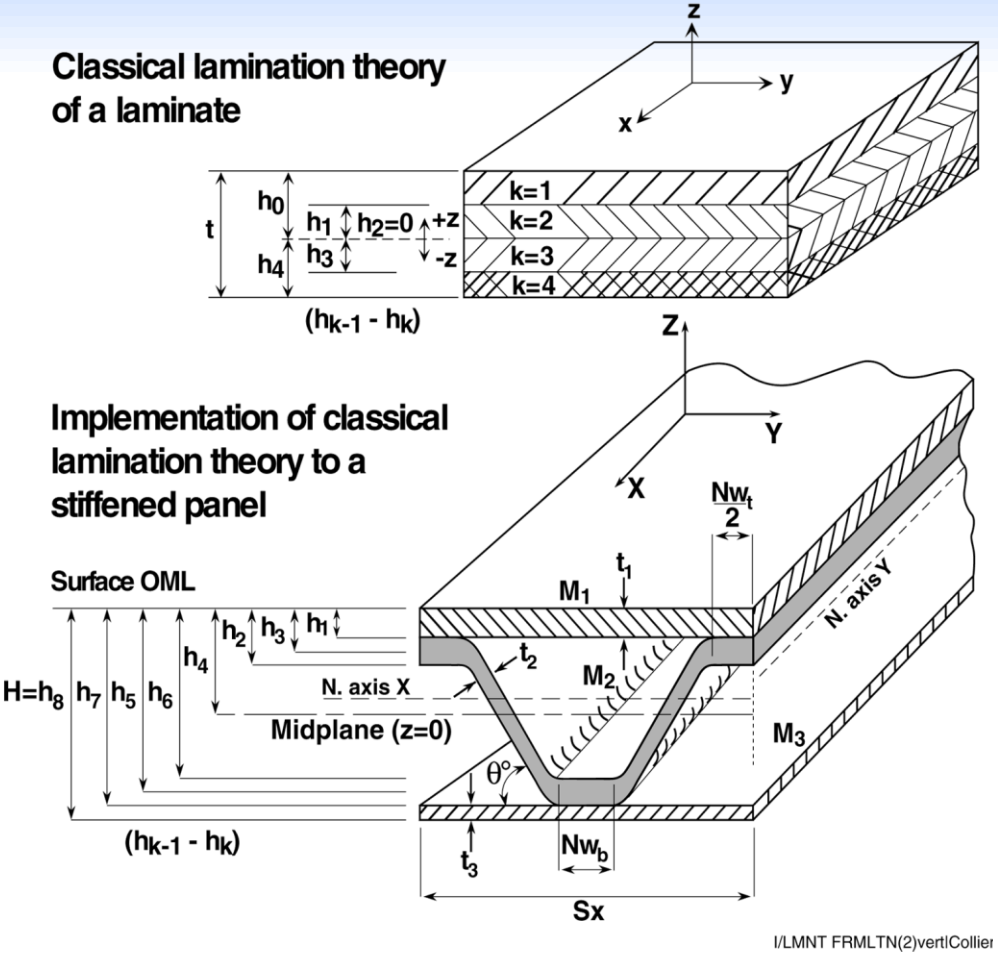


Panel Stiffness - Technical Approach

- **Classical Lamination Theory extended to a represent any stiffened cross sectional shape**

- **General panel behaviors, are quantified with:**
    - **Stiffness terms [A], [B], [D]**
    - **Thermal coefficients $[A^{\alpha}]$, $[B^{\alpha}]$, $[D^{\alpha}]$**

- **Stiffness terms must be summed about an assumed reference plane. The appropriate coupling terms must be included to represent offset of N/A from reference plane.**



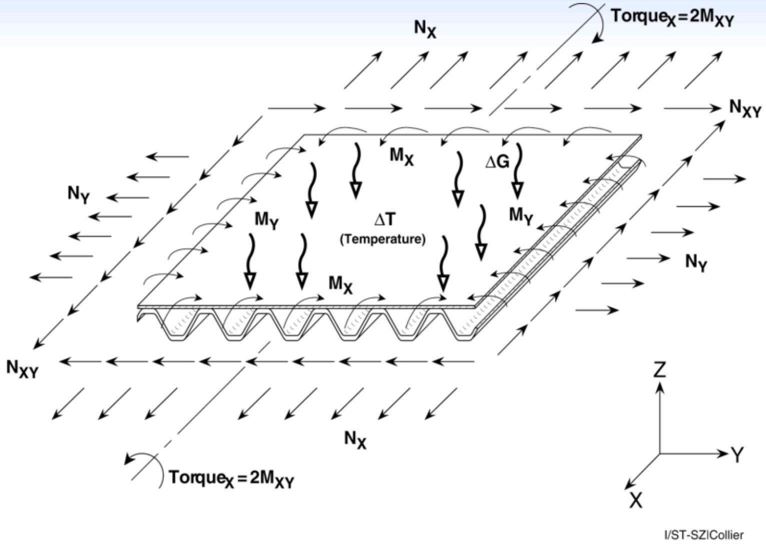
Classical lamination theory of a laminate

Implementation of classical lamination theory to a stiffened panel

I/LMNT FRMLTN(2)vert|Collier

Free Body Analysis Approach

General load definition



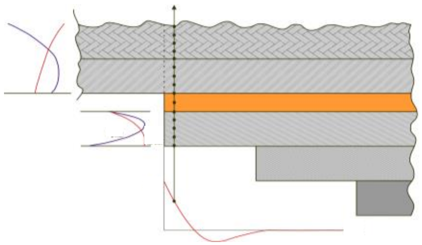
Determine Strains & Curvatures

STRAINS **Inverted Matrix** **FORCES**

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\\ \kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{Bmatrix}$$

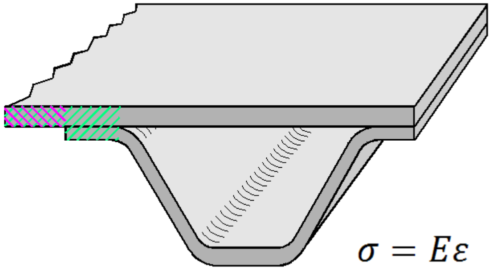
6x6

Determine Ply Stress & Strain



In-plane and out-of-plane stress and strain

Resolve Strain to Load for each panel object



$$\sigma = E\varepsilon$$

$$Nx\left(\frac{lb}{in}\right) = \sigma t$$

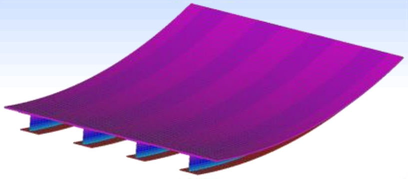
Localize Strains (through thickness)

$$\varepsilon_x = \varepsilon_x^\circ + z\kappa_x$$

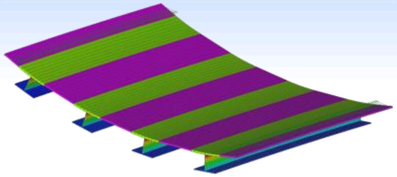
$$\varepsilon_y = \varepsilon_y^\circ + z\kappa_y$$

$$\gamma_{xy} = \gamma_{xy}^\circ + z\kappa_{xy}$$

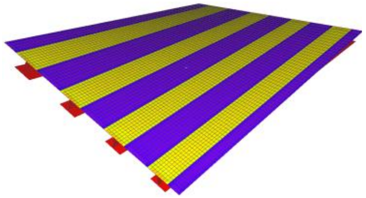
Stress Evaluation Points



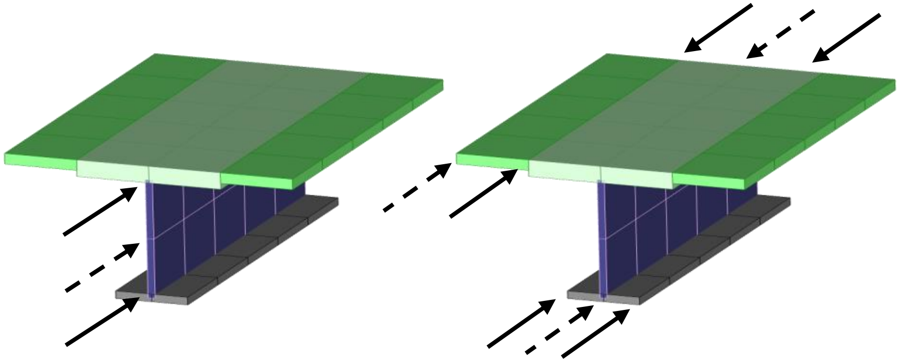
Axial Curvature



Transverse Curvature



Twisting Deformation



1. Stiffener Web

2. Stiffener Flanges and Facesheet

- A fully populated ABD stiffness matrix, with all off-diagonal coupling terms, should accurately predict stress and strain for any combination of axial curvature, transverse curvature and twisting deformation.
- Local strains may be corrected to account for evaluation points

1. Stress Evaluation Points at top, bottom and mid-plane of web
2. Stress Evaluation Points at left, right and mid-plane of flanges, bonded comb and open span.

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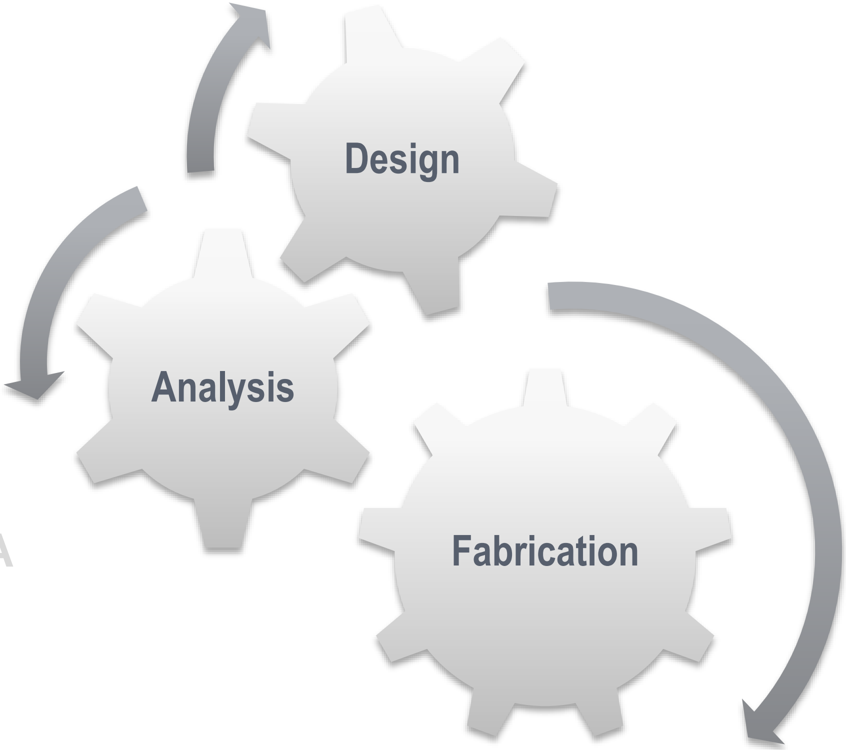

Margin of Safety, MS

- Margin of safety is generally written in the form

$$MS = \frac{P_{allow}}{P_{applied}} - 1$$

  - Above relation does not refer to load exclusively, it could refer to any criteria such as load, stress, principle strain, req. stiffness, etc.

- Interaction equations may be used to approximate the combined affect of two failure modes. Typically written using stress ratios (R), the interaction equations may be converted to margin of safety.

Interaction Equation	Margin of Safety (MS)
$\mathbf{R}_1 + \mathbf{R}_2 = 1$	$MS = \frac{1}{R_1 + R_2} - 1$
$\mathbf{R}_1^2 + \mathbf{R}_2^2 = 1$	$MS = \frac{1}{\sqrt{R_1^2 + R_2^2}} - 1$

- For higher-order interaction equations numerical methods are typically used to solve for MS.

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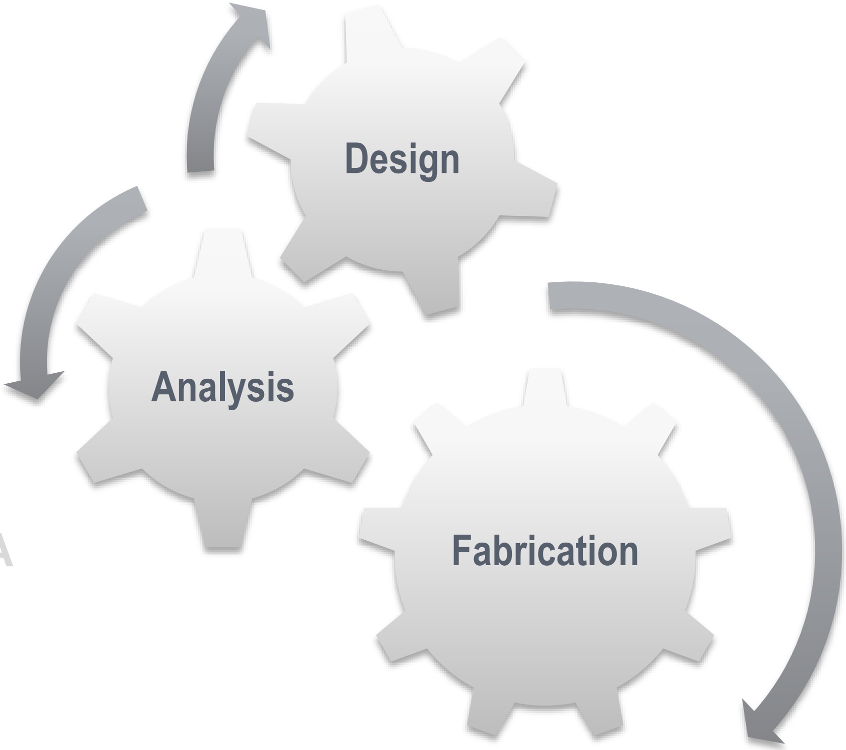


Lamina Strength Analysis

- **Ply based failure analysis**
  - **Major Advantage: Simplicity**
  - **Major Disadvantage: Lack of interaction among stress components**
- **Max Stress Predicts failure when:**

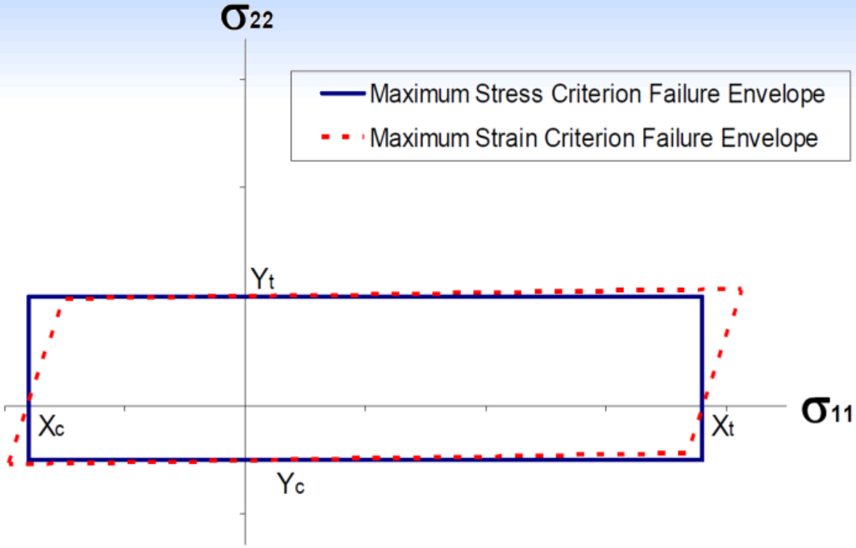
$$\varepsilon_{11} \geq X_{\varepsilon t}, \quad \varepsilon_{22} \geq Y_{\varepsilon t}, \quad |\gamma_{12}| \geq S_{\varepsilon}, \quad \varepsilon_{11} \leq X_{\varepsilon c}, \quad \varepsilon_{22} \leq Y_{\varepsilon c},$$

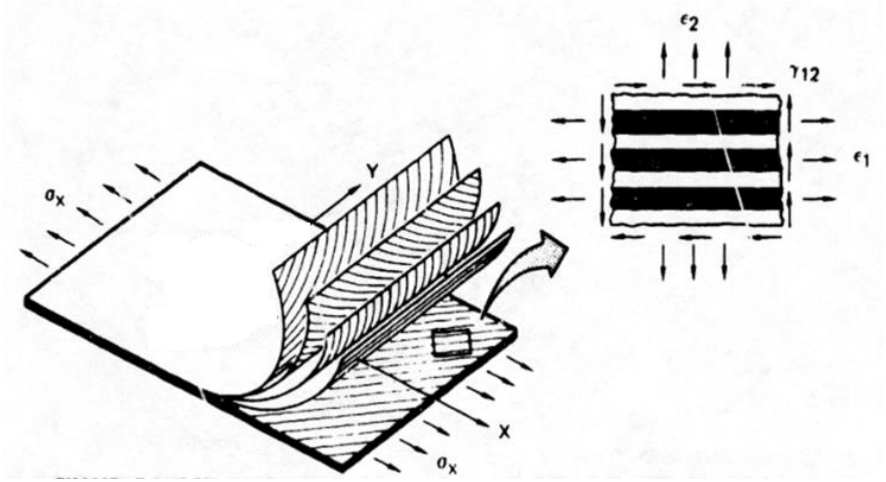
  - ***Where X_t, Y_t, X_c, Y_c, and S are the ply failure stresses in principal directions***
- **Max Strain Predicts failure when:**

$$\sigma_{11} \geq X_t, \quad \sigma_{22} \geq Y_t, \quad |\sigma_{12}| \geq S, \quad \sigma_{11} \leq X_c, \quad \sigma_{22} \leq Y_c,$$

  - ***Where $X_{\varepsilon t}$, $Y_{\varepsilon t}$, $X_{\varepsilon c}$, $Y_{\varepsilon c}$, and S_ε are the ply failure strains in principal directions***

Failure Envelope for single ply



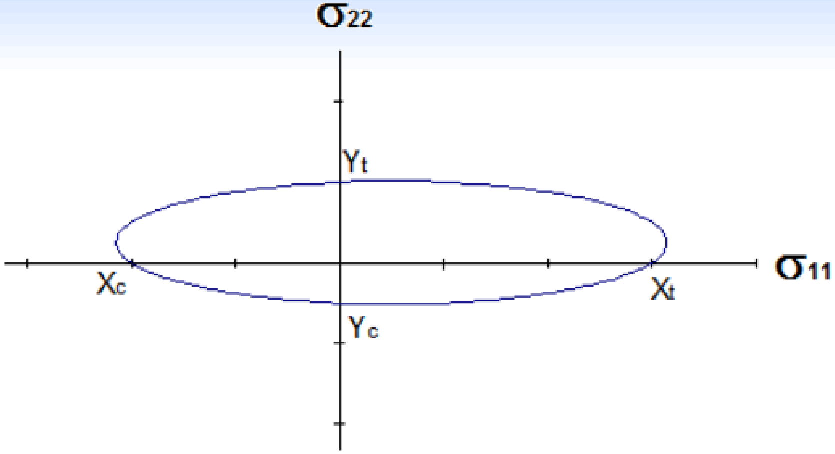


Lamina Strength Analysis

- **Quadratic ply based failure analysis predict failure when:**

$$\frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\sigma_{12}^2}{S^2} \geq 1$$

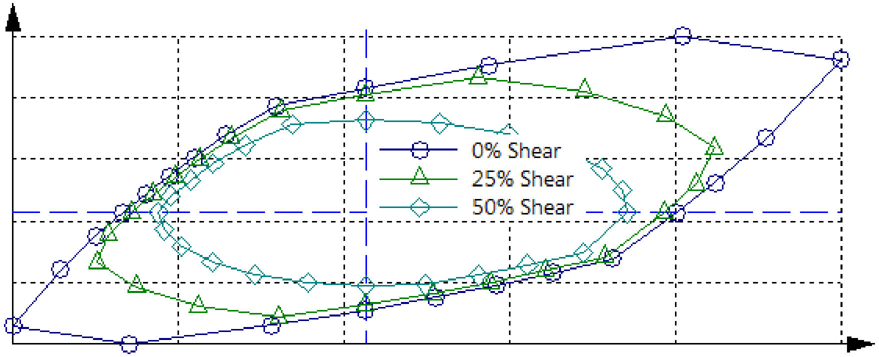
Failure Envelope for single ply



- **Advantages:**
  - **Provides interaction between stresses/strains in principle directions**

- **Stress-based quadratic failure critiera:**
  - **Hoffman Criterion**
  - **Tsai-Hill Criterion**
  - **Tsai-Wu Criterion**
  - **Tsai-Hahn Criterion (Slight modification to Tsai-Wu, F_{12} Coefficient)**
  - **Hashin Failure Theory**
  - **Inter-Fiber Failure (Matrix Cracking)**
    - **LaRC03 and Puck**

Failure Envelope for laminate (Tsai-Hahn) Based on first-ply failure



Laminate Strength Analysis

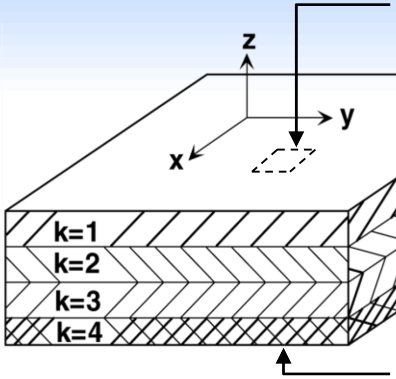
- **Laminate In-Plane Analysis**
  - **Transform laminate strains in 4 directions (-45,0,+45,+90 deg)**
  - **Use laminate-based strain allowables**
  - **Checks laminate IML, OML**

- **<u>Strains and Laminate Allowables in 0° Analysis Direction</u>**
- **The percentages of plies in this analysis direction are:**

0° Plies:	**40**
45°Plies:	**40**
90°Plies:	**20**

- **The strain allowable, interpolated from the "Laminate Based Strain Allowables" plots:**

- **Strain Allowable, e_{OHC}:** **<u>4,900 min/in</u>**



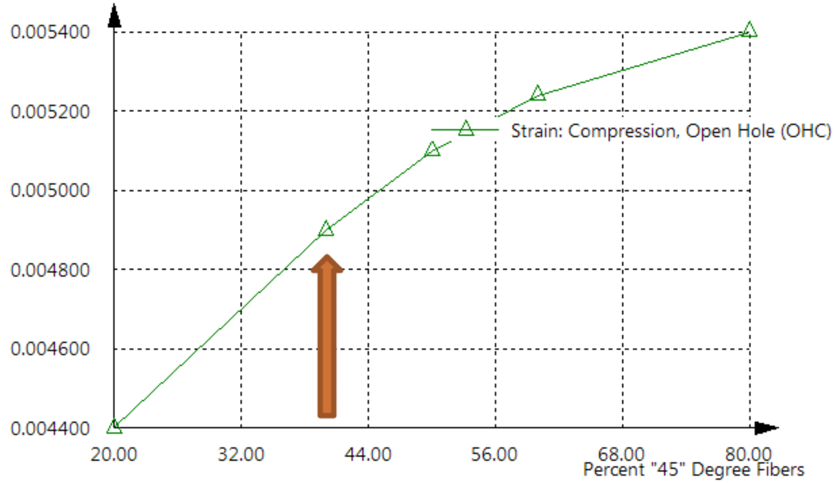
OML Strains in Laminate CS

e_{-45}, e_0, e_{+45}, e_{90}

IML Strains in Laminate CS

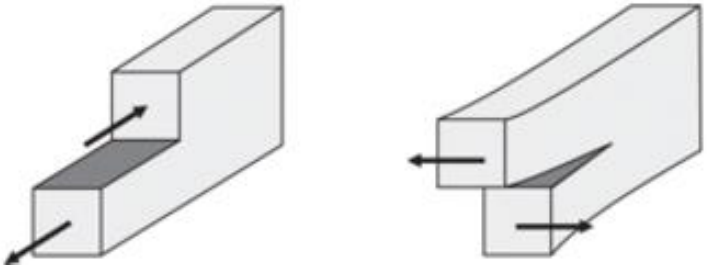
e_{-45}, e_0, e_{+45}, e_{90}

Strain allowable curves based on fiber percentage %45s, %0s, AML (%45s - %0s), etc.



Interlaminar Analysis

- **Interlaminar shear**
- **Simplified shear solution (SSS)**

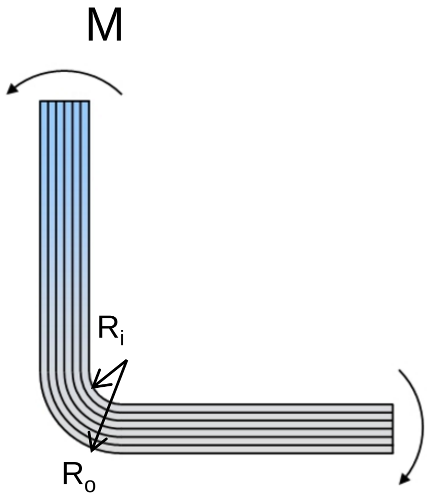


$$\tau_{iz}(k,\hat{z}) = -\frac{Q_i}{\bar{I}}\left[\frac{n_k}{2}\left(\hat{z}_c^2 - \hat{z}_k^2\right) + \frac{1}{2}\sum_{m=k+1}^{N} n_m\left(\hat{z}_{m-1}^2 - \hat{z}_m^2\right)\right]$$

$$n_k = \frac{E_i^k}{\bar{E}_i}$$

$$MS = \frac{1}{\sqrt{\left(\frac{\tau_{13}}{Fsu_{13}}\right)^2 + \left(\frac{\tau_{23}}{Fsu_{23}}\right)^2}} - 1$$

- **Interlaminar Tension**

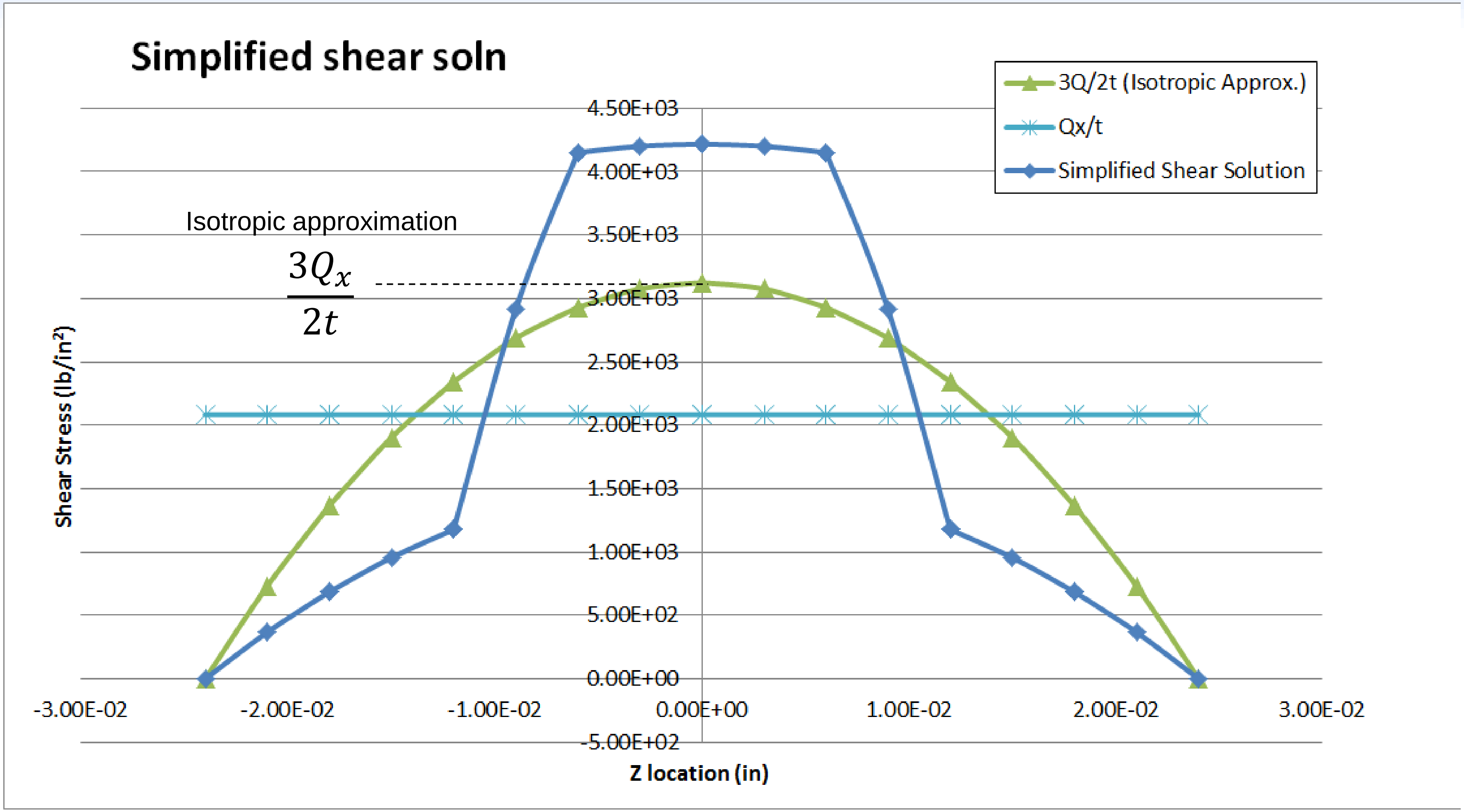


$$\sigma_{33} = \frac{3M}{2t\sqrt{R_i R_o}}$$

$$MS = \frac{F_{tu3}}{\sigma_{33}} - 1$$

Interlaminar Shear Interaction

- Interlaminar shear stress distribution through the thickness of the laminate



Example Laminate: 45/-45/0/90/90/0/-45/45

Outline for Presentation

- Composite ply properties
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- Stiffened panel failure
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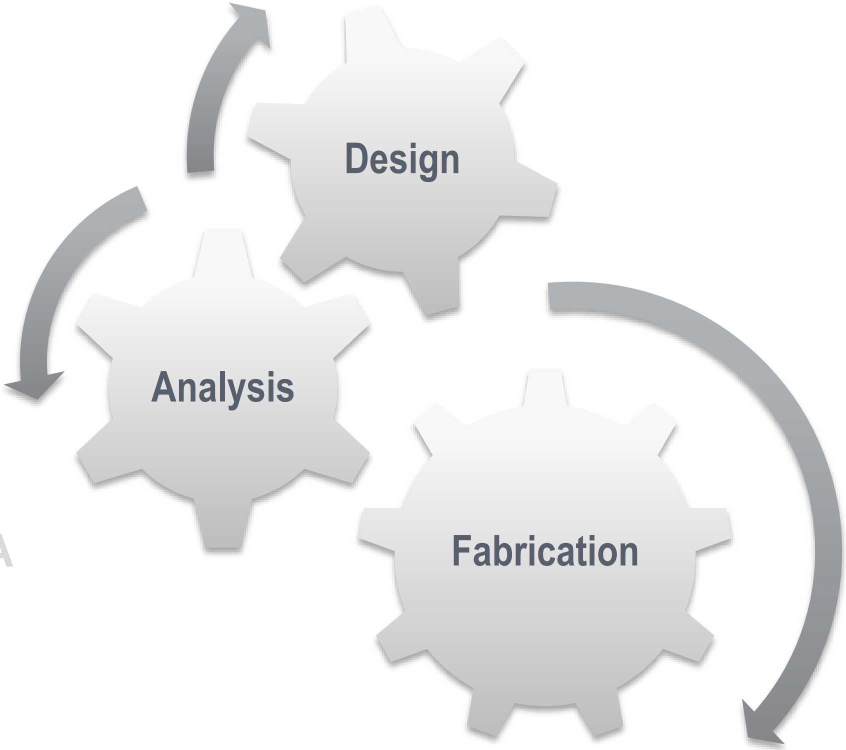
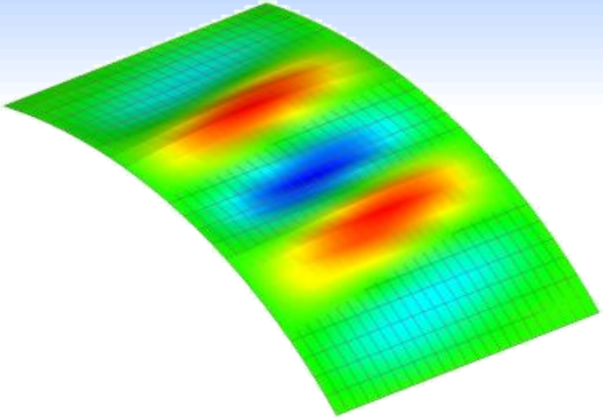


Linear Buckling – Composite Plates



- **Methods for calculating buckling margins**
  - **Numerical**
  - **Analytical**

- **<u>Numerical</u> buckling, Eigenvalue method**

$$det(A - \lambda I) = 0$$

n x n stiffness matrix → A

Where:
A = Global stiffness matrix
I = Identity matrix
λ = Eigenvalue

- **<u>Analytical</u> buckling methods for orthotropic plates are an extension of the governing equation.**

$$D_{11}\frac{\partial^2 w}{\partial x^4} + 2\left(D_{12} + 2D_{33}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2}$$

- **For SSSS boundary conditions, the common plate buckling equation is written as:**

$$N_{x,crit} = \frac{-\pi^2\left[D_{11}\left(\frac{m}{a}\right)^2 + 2\left(D_{12} + 2D_{33}\right)\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4\left(\frac{a}{m}\right)^2\right]}{1 + \left(\frac{N_y}{N_x}\right)\left(\frac{a}{b}\right)^2\left(\frac{n}{m}\right)^2}$$

Where:
a = Length of plate
b = Width of plate
n = number of half mode shapes, x direction
m = number of half mode shapes, y direction

Biasing Stacking Sequence

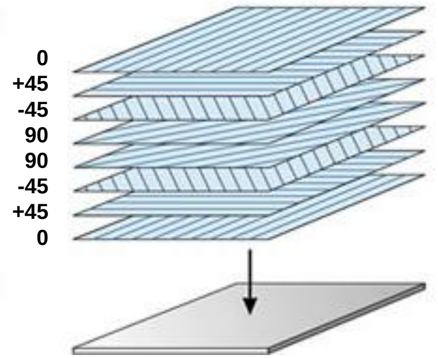
- **For short (small a), wide (large b) plates the buckling margin is most sensitive to D11.**

When a>b, Nx compression
Dominant Term

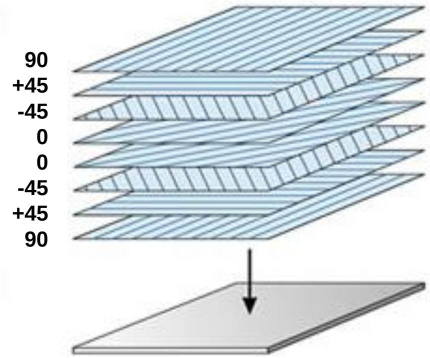
When b>a, Ny compression
Dominant Term

$$N_{x,crit} = \frac{-\pi^2\left[D_{11}\left(\frac{m}{a}\right)^2 + 2\left(D_{12}+2D_{33}\right)\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4\left(\frac{a}{m}\right)^2\right]}{1+\left(\frac{N_y}{N_x}\right)\left(\frac{a}{b}\right)^2\left(\frac{n}{m}\right)^2}$$

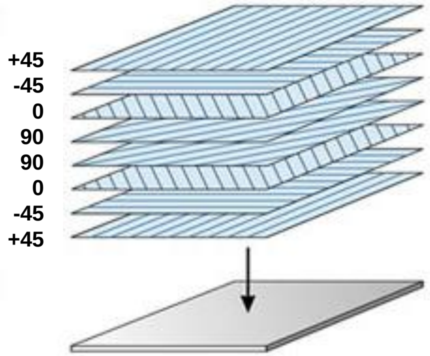
- **Laminate bending stiffness may be biased to provide buckling stability.**



A11 = A22
Bij = 0
D11 > D33 > D22
Best for compressive Nx

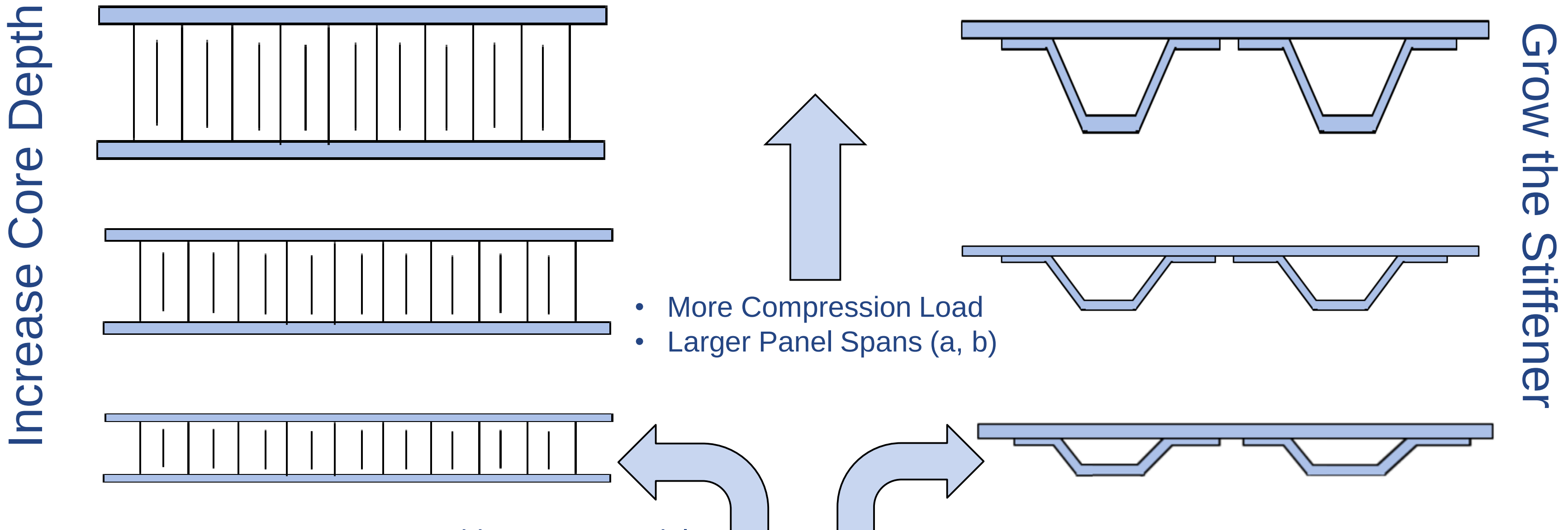


A11 = A22
Bij = 0
D22 > D33 > D11
Best for compressive Ny



A11 = A22
Bij = 0
D33 > D11 > D22
Best for high Nxy

How to add Stability?



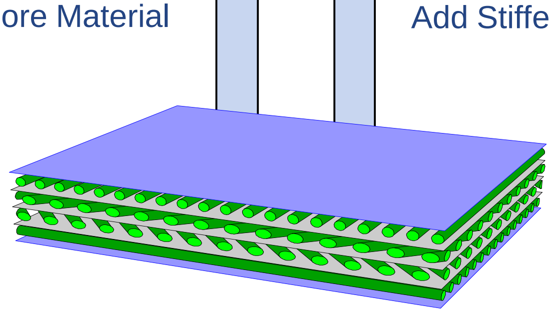
Increase Core Depth

Grow the Stiffener

- More Compression Load
- Larger Panel Spans (a, b)

Add a Core Material

Add Stiffeners



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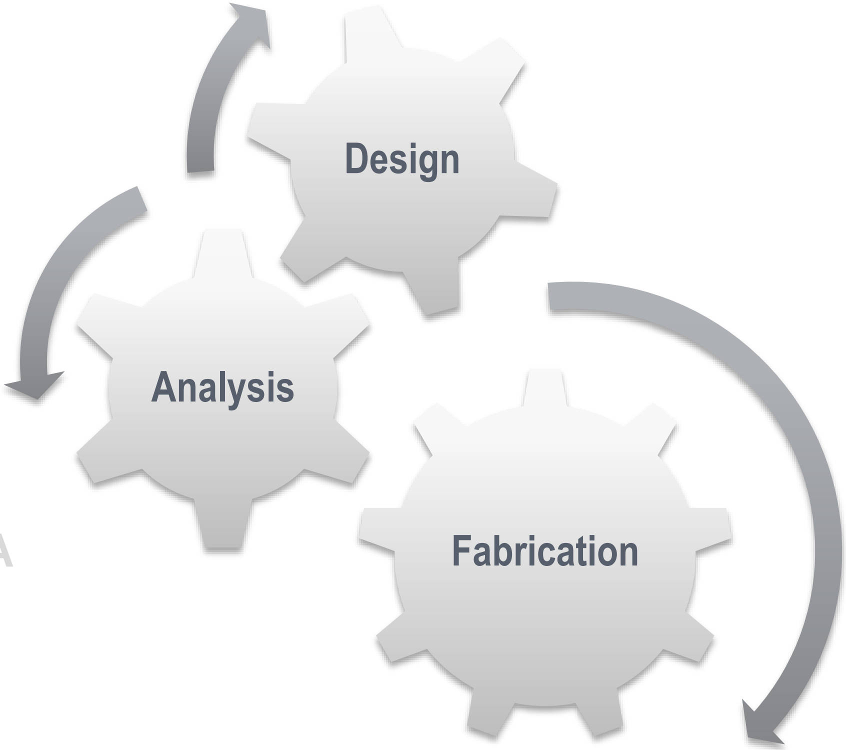

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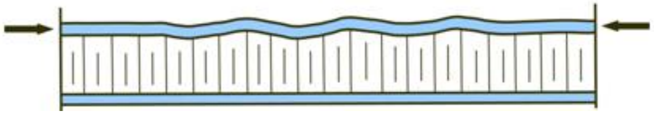
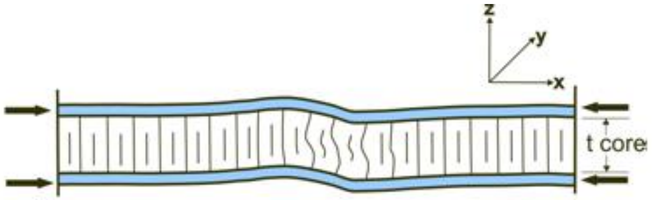
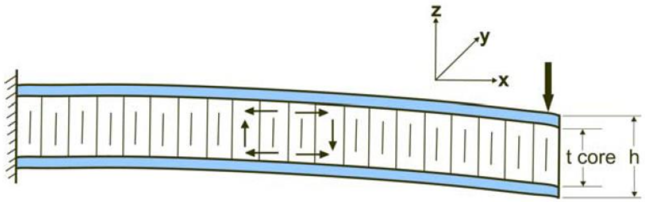
Sandwich Panel Failure Analysis

- **Margins of Safety generated for sandwich panels based on the following analysis:**
  - In plane stress/strain
    - Lamina (Ply by ply analysis)
    - Laminate (Based on Ply percents)
    - Damage tolerance CAI allowables incorporated
  - Facesheet wrinkling
  - Facesheet dimpling
  - Panel shear crimpling
  - Core Shear Failure
  - Flat Wise Tension







Facesheet Wrinkling

- **Face sheet Wrinkling Stress**

- **A pictorial example of face sheet wrinkling is provided in Figure 8.**

FIGURE 8: Face Sheet Wrinkling

$$\sigma_{WR} = k_2 E_f \sqrt{\frac{E_C t_f}{E_f t_C}}$$

Where:

- **E_c = Through-the-thickness elastic modulus of core**
- **E_f = Elastic flexural modulus of face sheet**
- **t_f = Face sheet thickness**
- **t_c = Core thickness**
- **σ_{WR} = Wrinkling stress allowable**
- **k_2 = Symmetric mode wrinkling factor (= 0.82)**

Core Shear Failure

- **Core Transverse Shear Stress**
- **A pictorial example of Core Shear Stress is provided in Figure 9.**

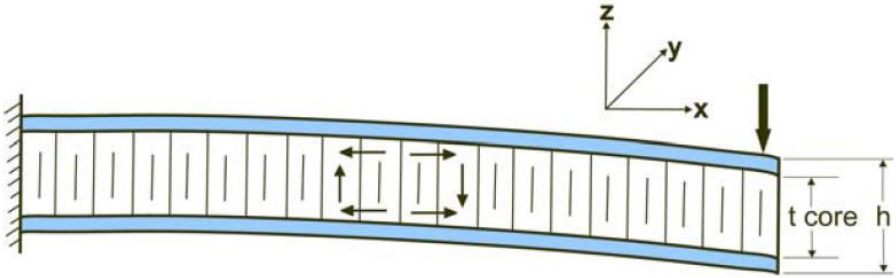


FIGURE 9: Core Shear Stress

$$MS = \frac{R\,K_{sscf}}{\frac{Q}{h_{eff}}} - 1 \qquad h_{eff} = \tfrac{1}{2}t_{upper\ face} + t_{core} + \tfrac{1}{2}t_{lower\ face}$$

Where:

- ***R* =** *Out-of-plane shear strength of core*
- ***K_{sscf}* =** *strength correction factor*
- ***Q* =** *Out-of-plane shear load per unit length*
- ***Q_x* =** *Out-of-plane shear load per unit length in x (ribbon) direction*
- ***Q_y* =** *Out-of-plane shear load per unit length in y (transverse) direction*
- ***h_{eff}* =** *Effective panel height (core + ½ facesheets)*
- ***F_{su}* =** *Out-of-plane ultimate shear strength of core in ribbon direction*
- ***$F_{su\omega}$* =** *Out-of-plane ultimate shear strength of core in transverse direction*
- ***t_{core}* =** *Core thickness*

Flatwise Tension

Sandwich Flatwise Tension

- **Sandwich flatwise tension is a moment-driven failure caused by facesheet pull-off from the honeycomb/foam core**

$$M = 2\left(\frac{N_f H}{2}\right) = N_f H$$

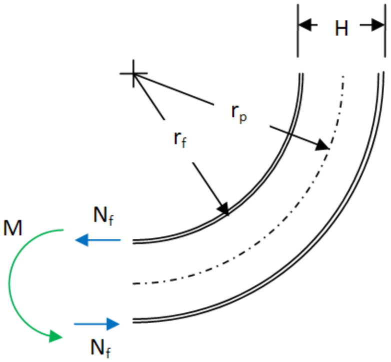


FIGURE 10: Moment Causing Pull-off Stress

$$\sigma_{rr} = \frac{M}{Hr} \qquad MS = \frac{Ftu_{core}}{\sigma_{rr}} - 1$$

Where:

- **M = In-plane Bending moment (Mx or My)**
- **N_f = Force in each facesheet due to imposed bending moment**
- **Ftu_{core} = Through the thickness stress allowable for core**
- **H = Height of Panel**
- **r = Average Radius of curvature $(r_i + r_p)/2$**
- **σ_{rr} = Out-of plane stress (pull-off stress)**

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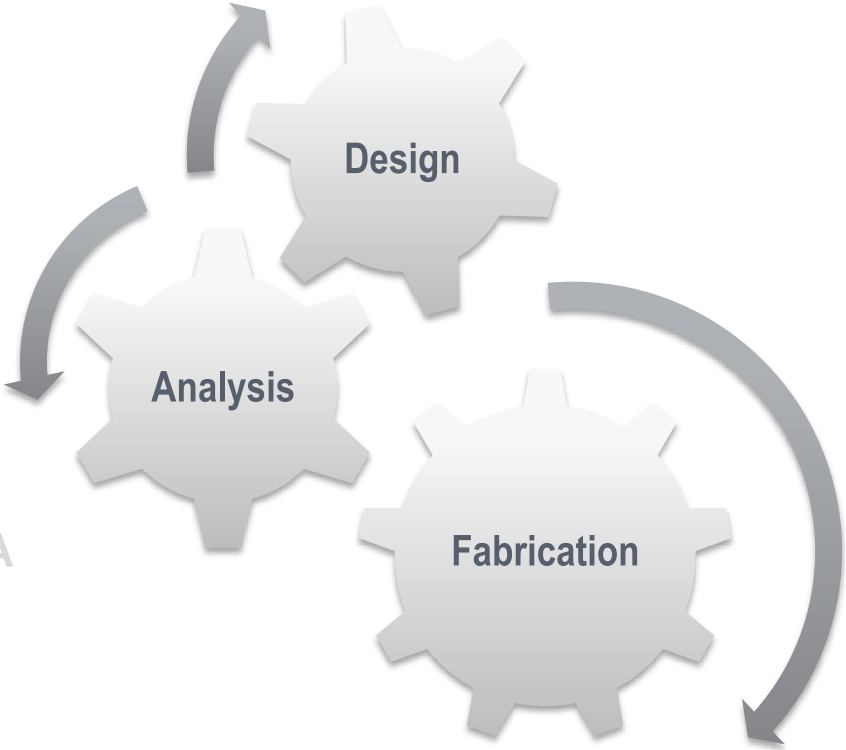
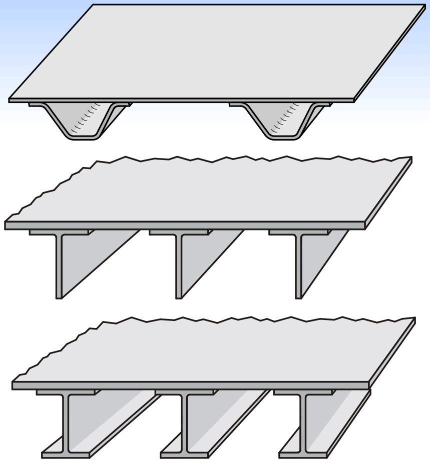
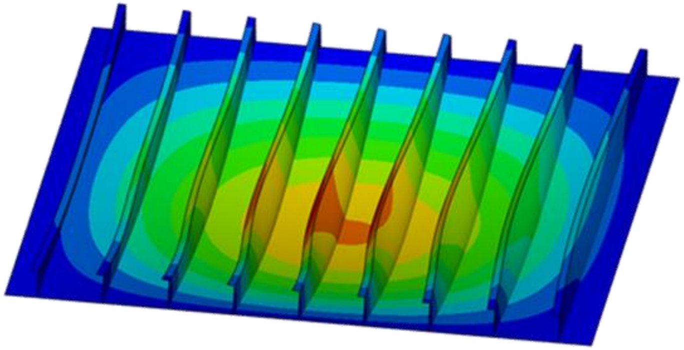
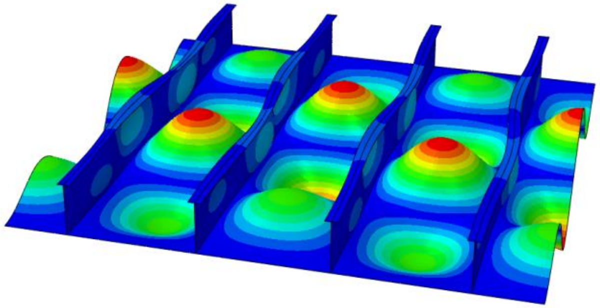


Stiffened Panel Failure Analysis



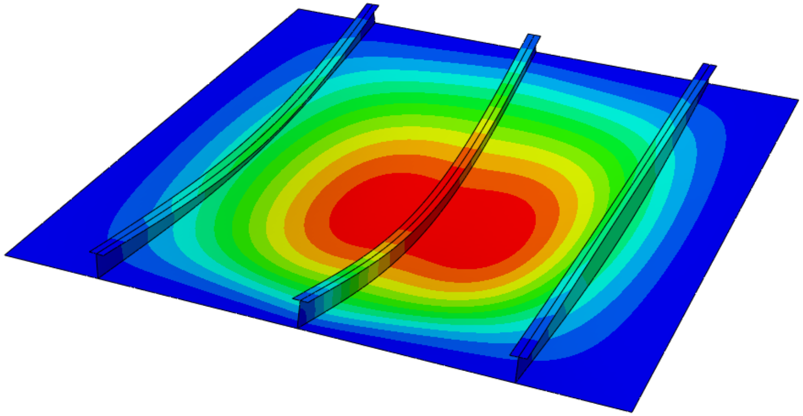
- **Margins of Safety generated for stiffened panels based on the following analysis:**
  - In plane strain
    - Lamina (Ply by ply analysis)
    - Damage tolerance CAI allowables incorporated
  - Stiffener Crippling
  - Stiffener Column Buckling
  - Stiffener Local Buckling
  - Local Post Buckling
  - Torsional Instability – Flexural Torsional Buckling
  - Stiffener delamination
  - Advanced stress analysis techniques
    - Postbuckling (compression and shear)
    - Beam-column





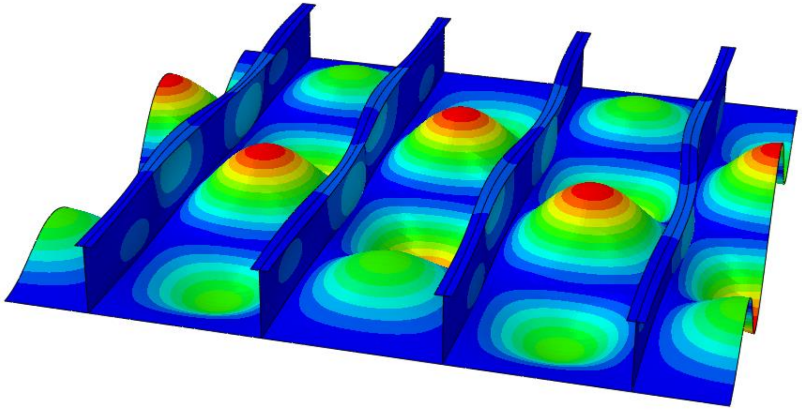
Global Buckling vs. Local Buckling

Global (Panel) Buckling



Global buckling, also referred to panel buckling, typically describes a flexural bifurcation of the entire panel (including stiffeners) due to in-plane compression loads. This bifurcation is typically assumed to be a total collapse.

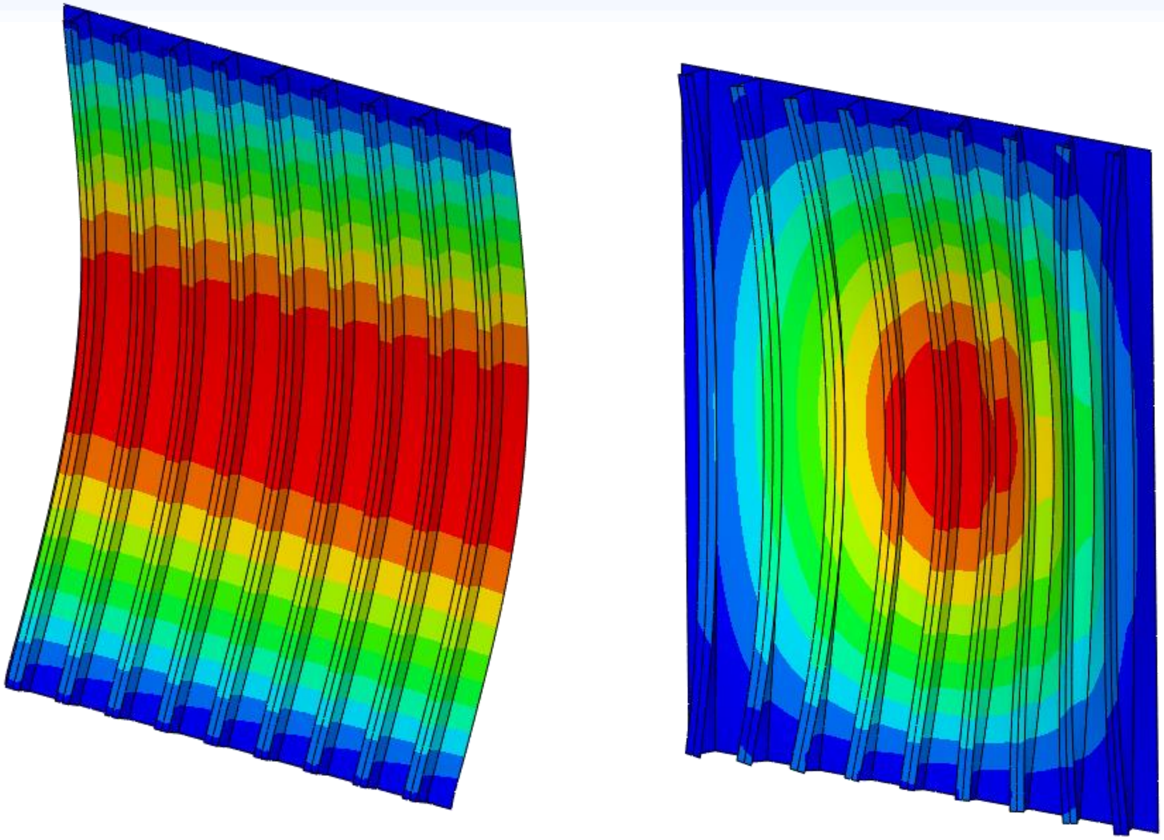
Local Buckling



Local buckling is defined as a buckling mode where the intersecting edges of the cross-section do not deform. The figure above shows the local skin buckling of an I stiffened panel. By default, local buckling is treated as a failure. However in many cases, postbuckling of the skin is permitted at a certain fraction of ultimate load.

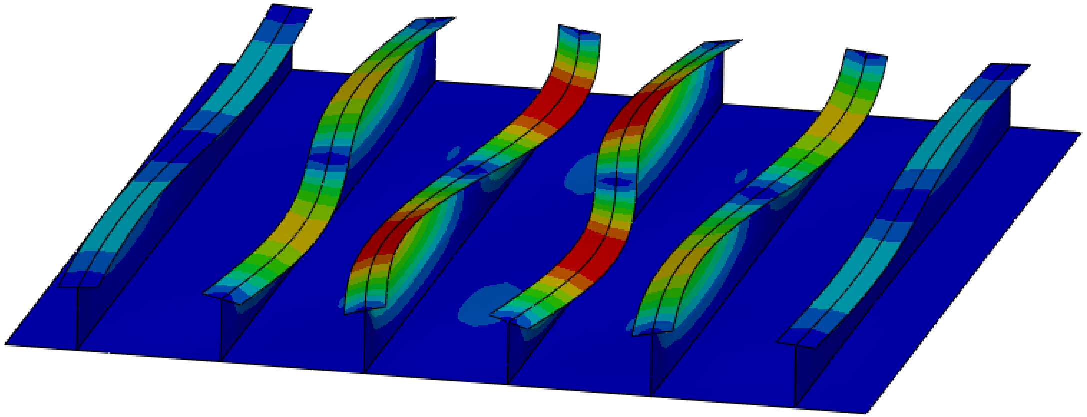
Flexural Buckling

Symmetric
Uncoupled flexure and torsion

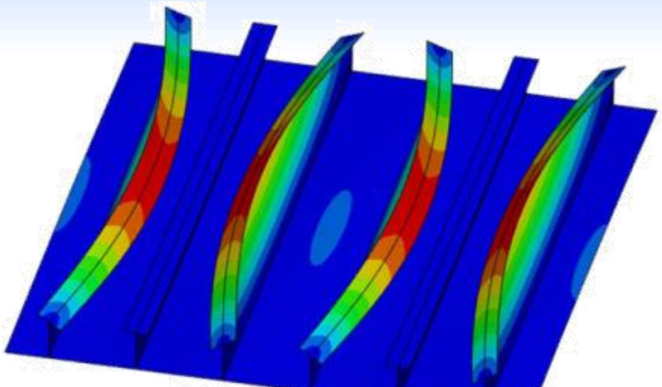


Torsional Buckling

Symmetric
Uncoupled flexure and torsion

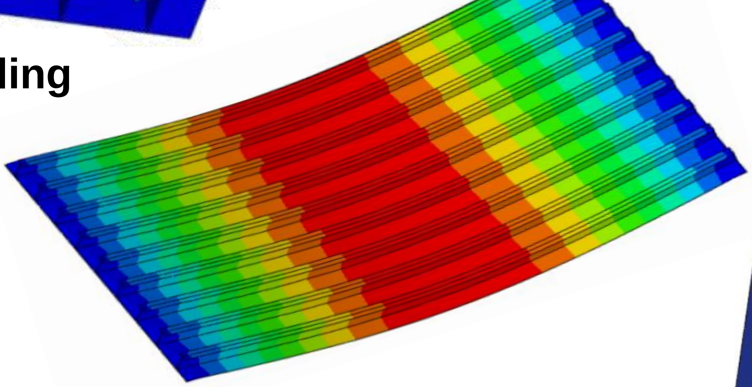


Flexural-Torsional Buckling (FTB)

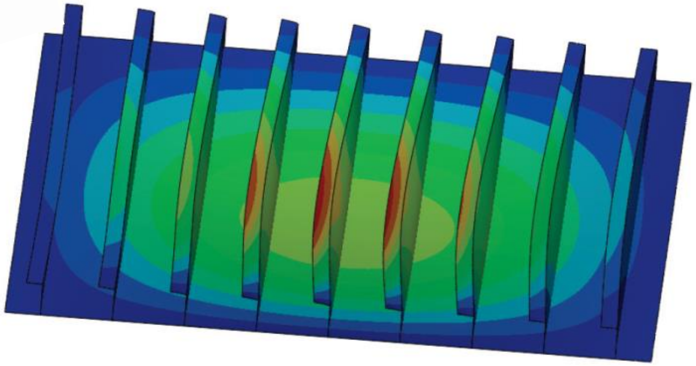


Torsional Buckling

Unsymmetric Stiffener Cross Section

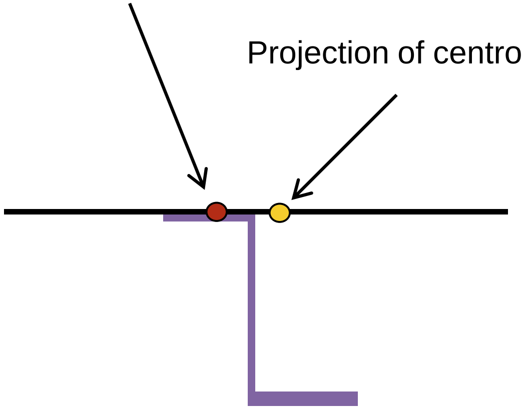


Flexural Buckling



Flexural-Torsional Buckling

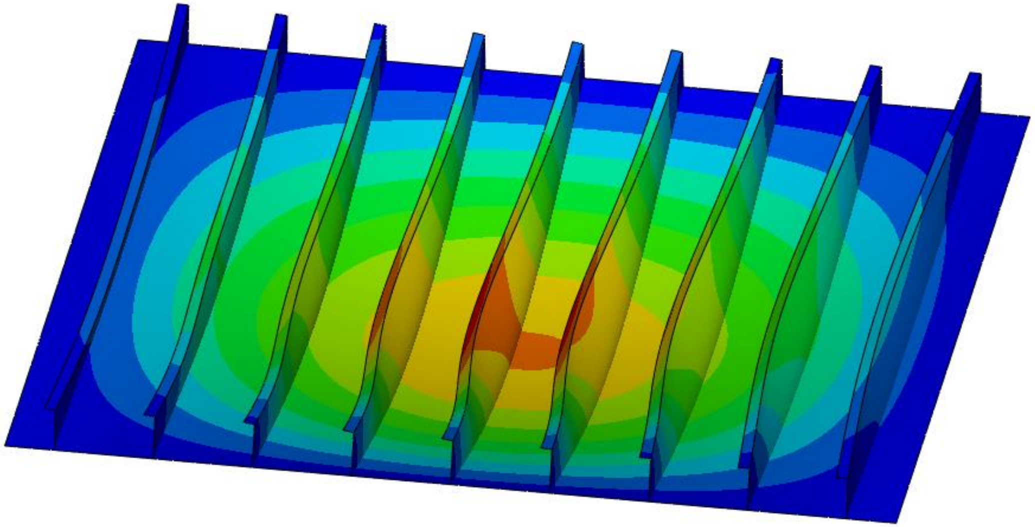
Flexural-Torsional Buckling (FTB)

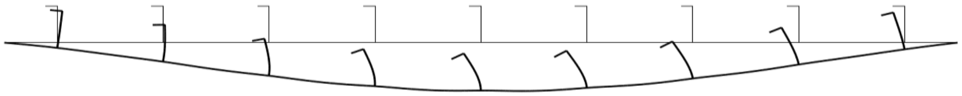


Projection of shear center

Projection of centroid

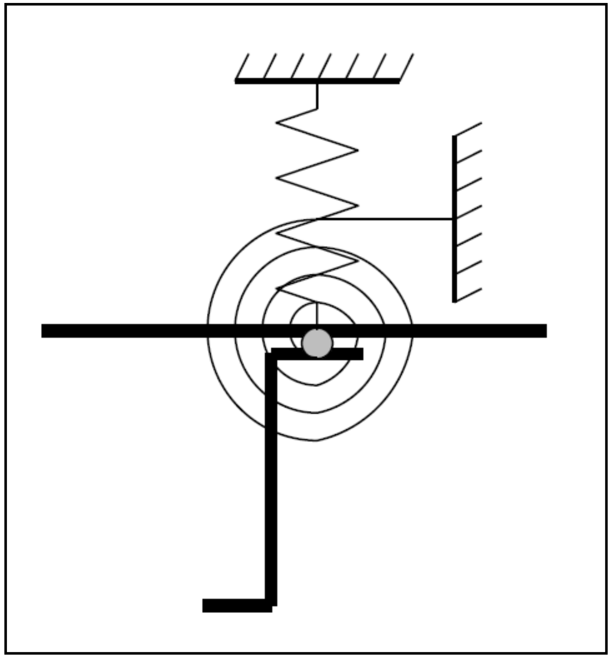
Unsymmetric
Coupled flexure and torsion





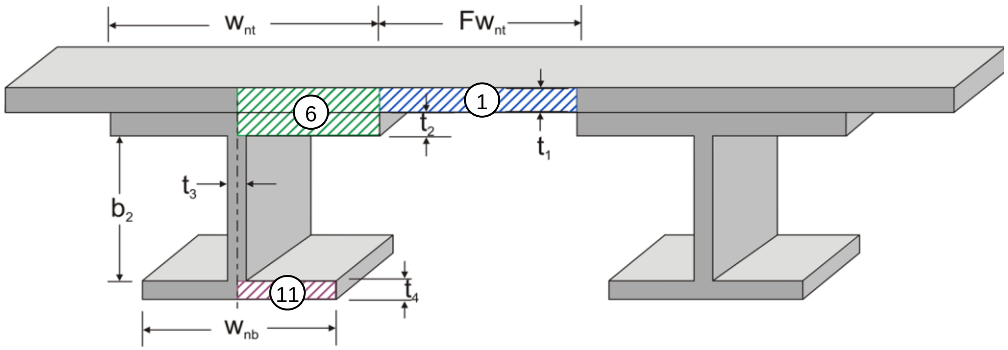
Flexural-Torsional Methods

- Two methods available:
  - Argyris (1954)
  - Levy (1947)
- Skin-stringer section modeled as column
  - Idealized spring striffnesses
    - Skin restraint (posbuckled)
    - Stiffener mode (symmetric vs. antisymmetric)
  - Uniaxial compression only
- Isotropic expressions extended to composites

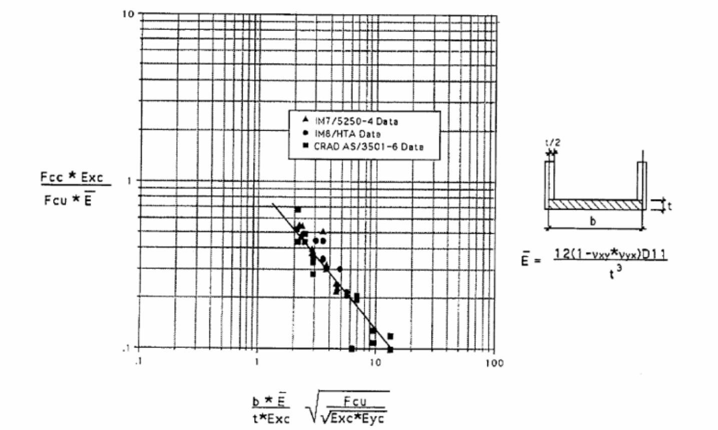


Crippling

- **Mil-Hdbk-17 Crippling method**
  - **Industry standard**



Log-Log Curve, No Edge Free



Log-Log Curve, One Edge Free

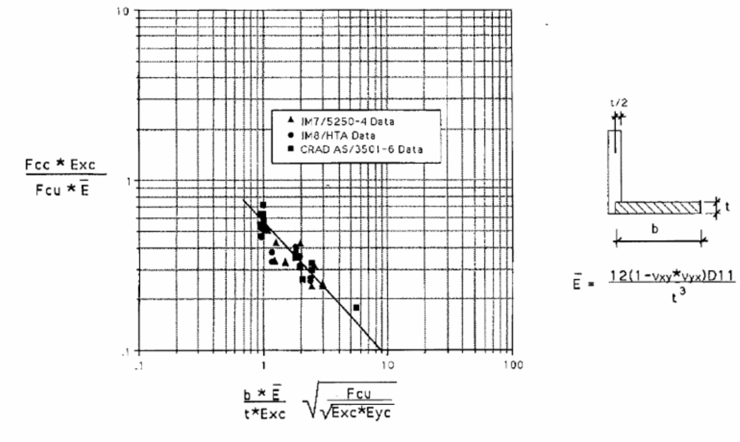


FIGURE 4.7.2.4(a) *One-edge-free crippling test results.*

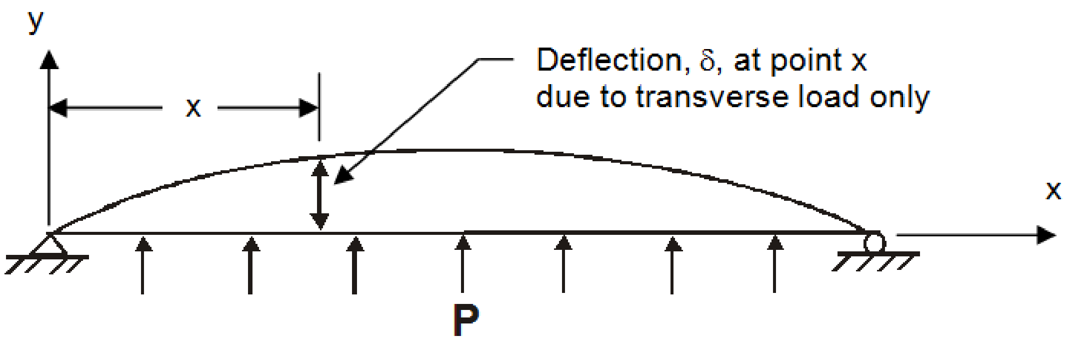
- **Allowable crippling stress for each segment determined from appropriate log-log curve**
- **Perform weighted average to find contribution to total crippling stress of entire section**

$$F_{cc} = \frac{\sum b_n t_n F_{ccn}}{\sum b_n t_n} \qquad \mathrm{P}_{cc} = F_{cc}\ Area$$

Beam-Column Overview

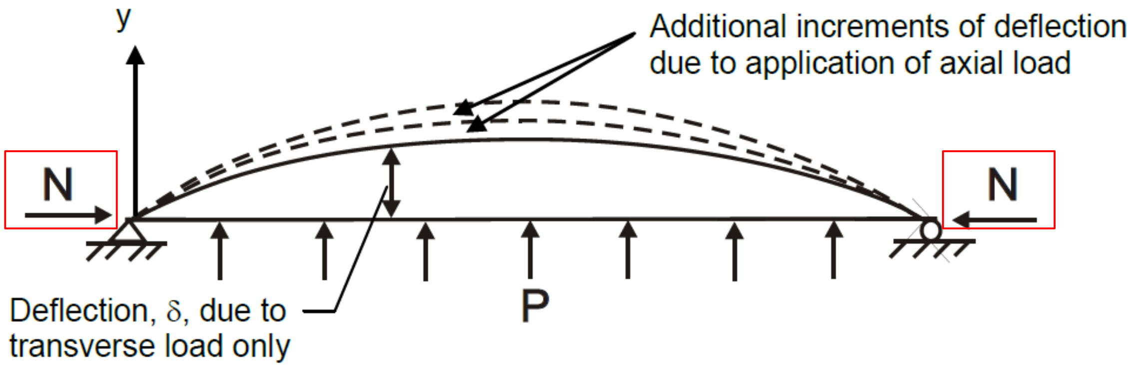
Beam Column analysis is not a failure criteria, it is a stress analysis method that accounts for geometric nonlinear behavior in stiffened panels and beams where the combination of out-of-plane static deformation and in-plane axial compression causes a load eccentricity.

Primary deflection from bending due to application of pressure or initial imperfection



Deflection, δ, at point x due to transverse load only

Secondary, non-linear moment and deflection caused by eccentricity of compression load on deflected shape



Additional increments of deflection due to application of axial load

Deflection, δ, due to transverse load only

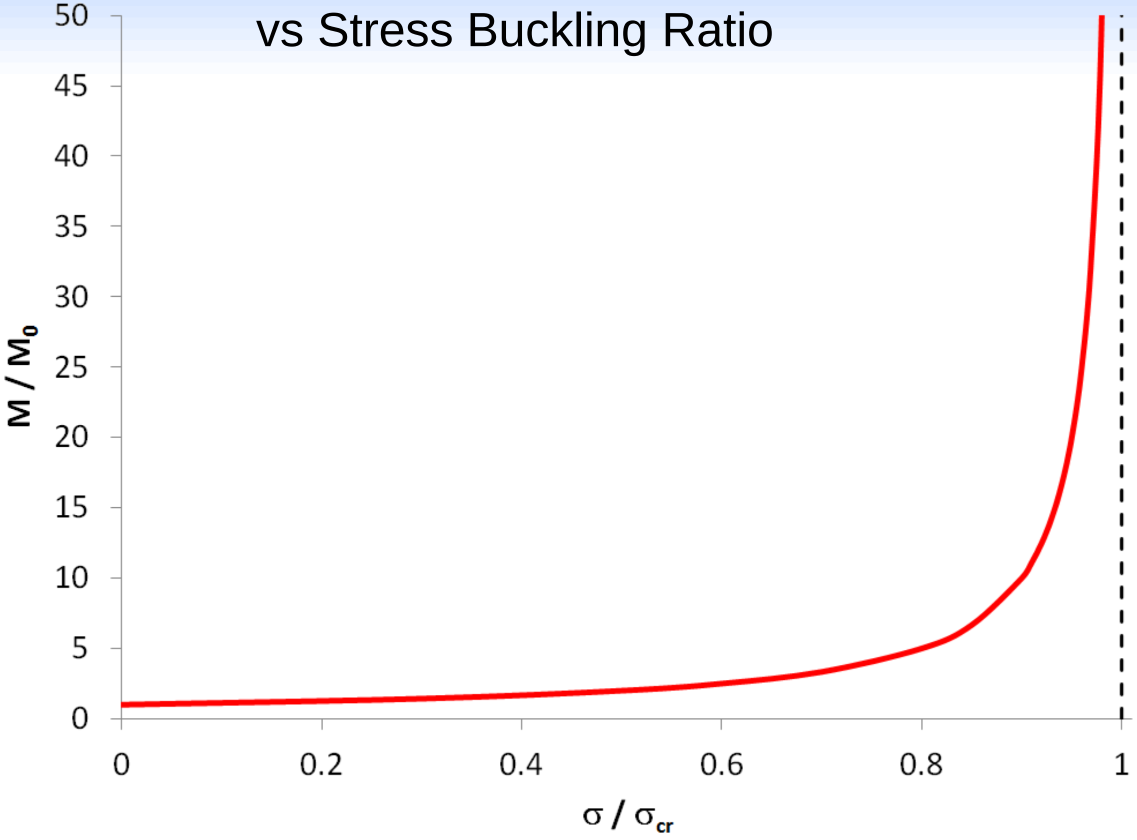
Simple Beam-Column Method

Beam-Column predicts that bending stresses goes to infinity at the panel critical buckling stress

A simple beam-column method is shown where M_{app} is the beam-column moment and M_0 is the moment due to transverse loads only

$$M_{app} = \frac{M_0}{\left(1 - k\frac{\sigma}{\sigma_{cr}}\right)}$$

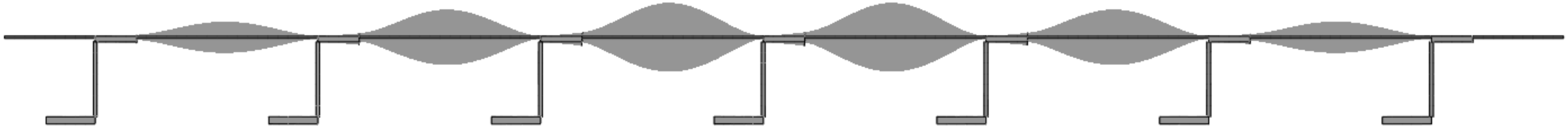
Beam-Column Moment Multiplier vs Stress Buckling Ratio



For low stress values, the beam-column multiplier is negligible. As stress approaches critical buckling stress, bending moment goes to infinity

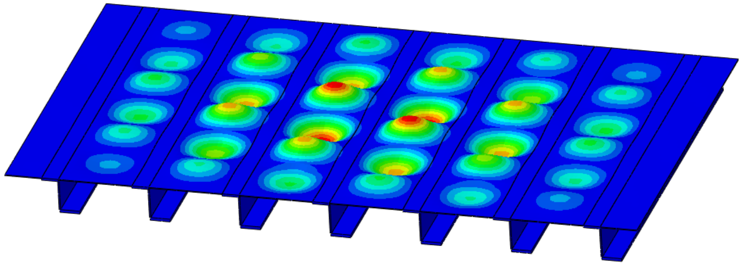
Local Skin Buckling is Not Failure

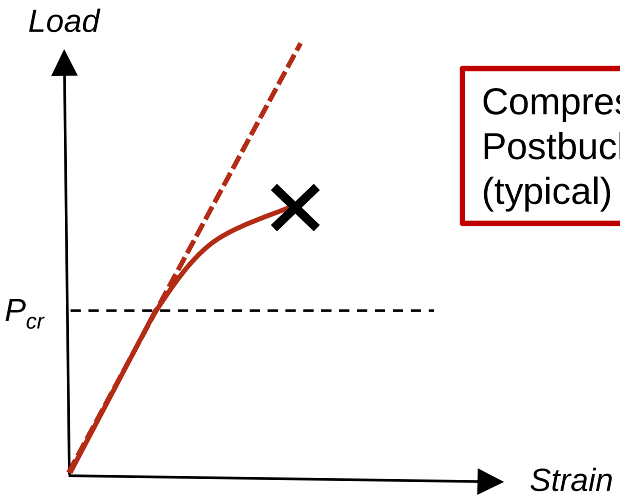
- Compression panels continue to carry load after skin local buckling
  - Plates have stable postbuckling behavior
  - Skin carries small portion of load



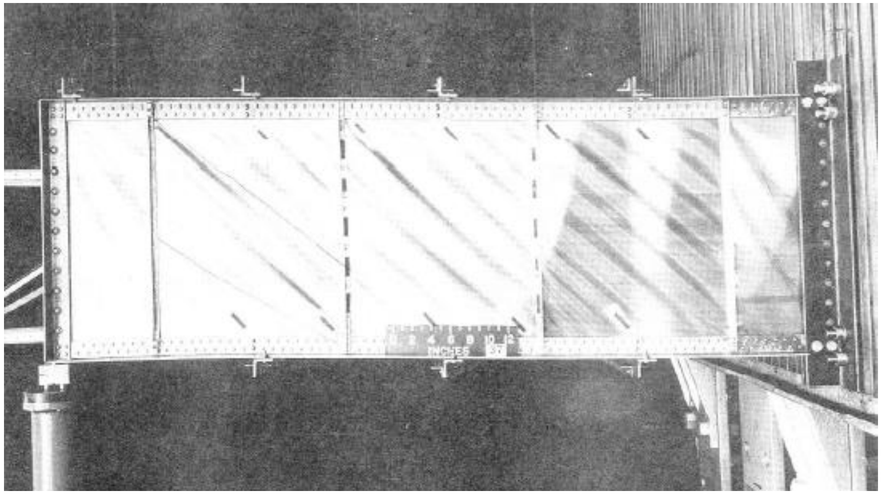
Local Postbuckling

- After skin local buckling, panel continues to carry load
  - Load redistributes
  - Reduced stiffness → effective width
  - Lowered margins (panel buckling, crippling, material strength)





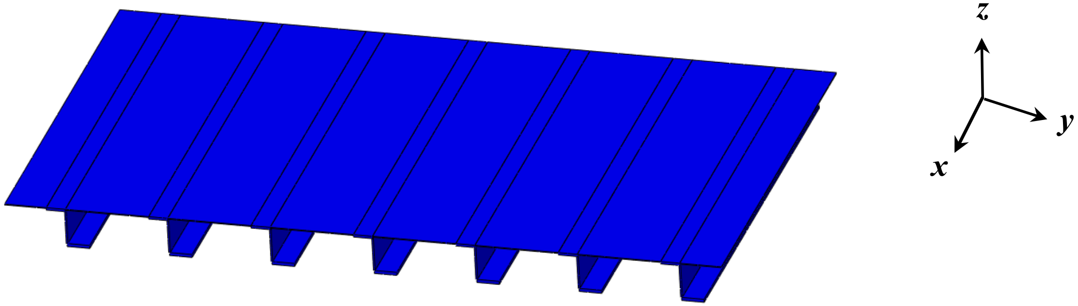
Compression Postbuckling (typical)

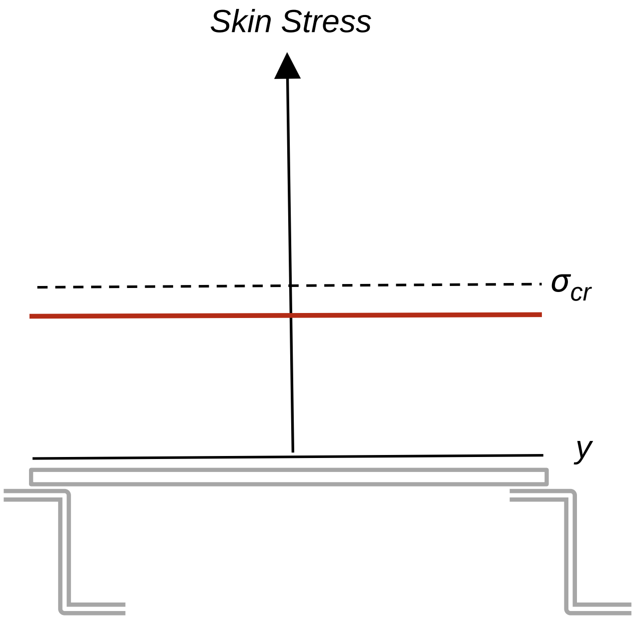


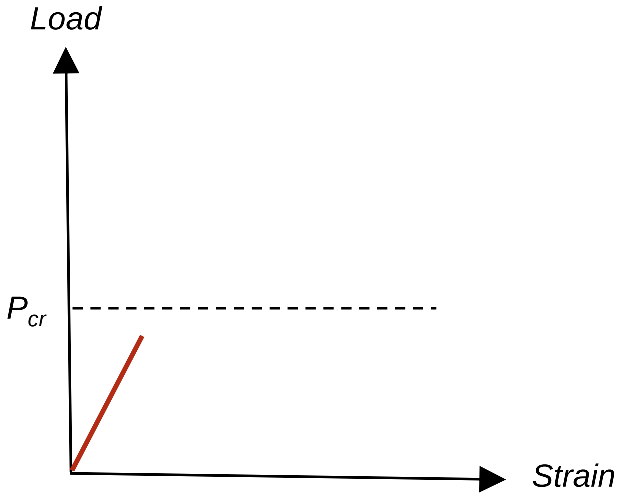
Shear postbuckling – NACA type I-25 test beam (NACA TN 2662, 1952)

Prebuckling: $P < P_{cr,skin}$

- Metallic Zee panel loaded in compression
- Uniform stress

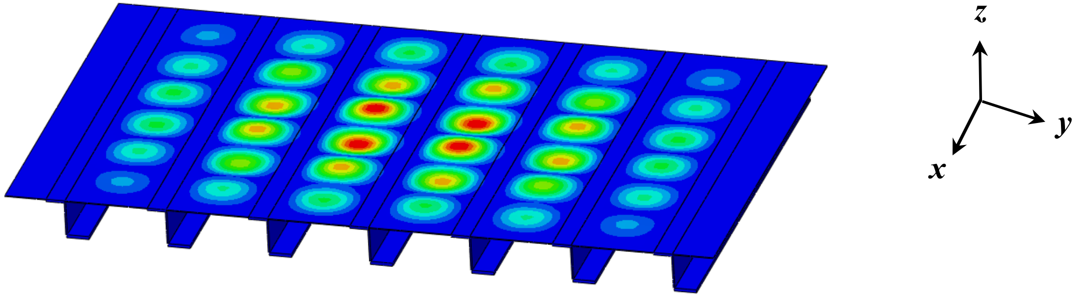


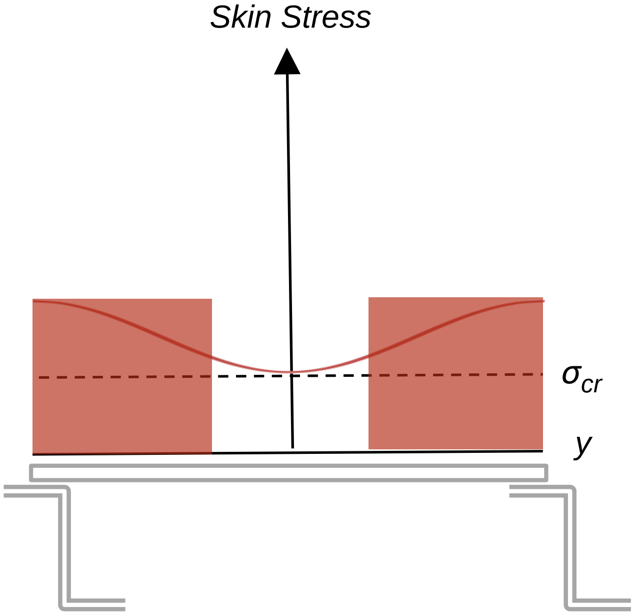


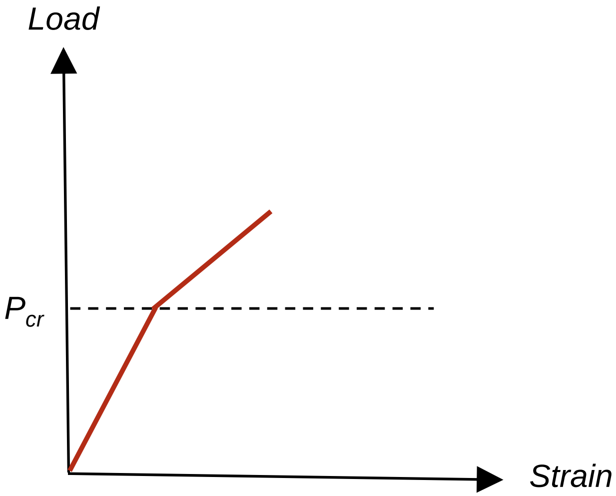


Postbuckling: $P = 2 * P_{cr,skin}$

- Panel stiffness reduced
- Non-uniform stress distribution in skin → effective width

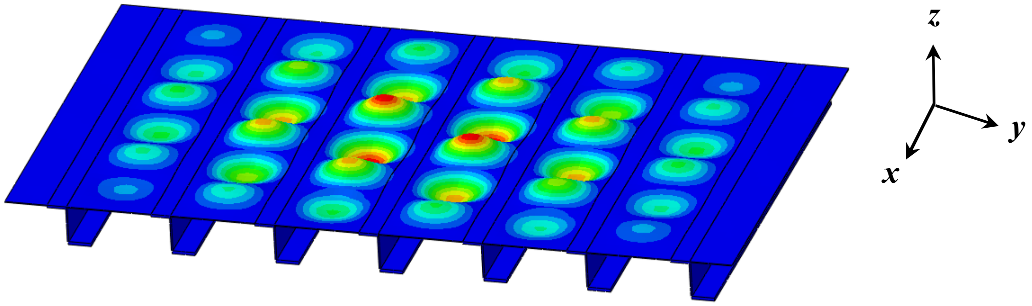


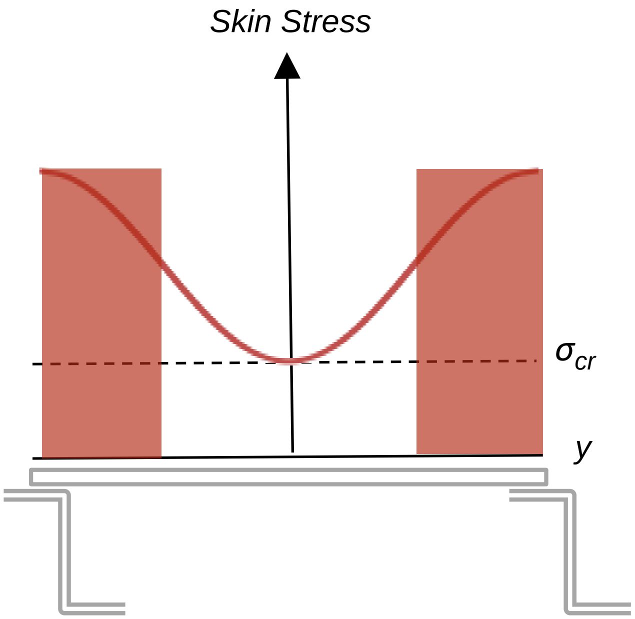


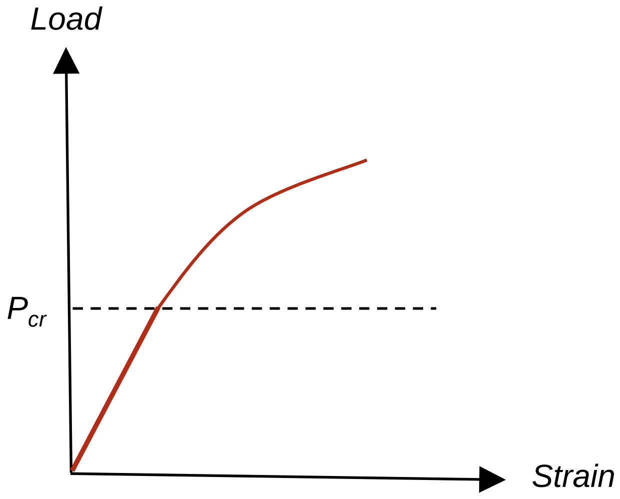


Postbuckling: $P = 3.0*P_{cr,skin}$

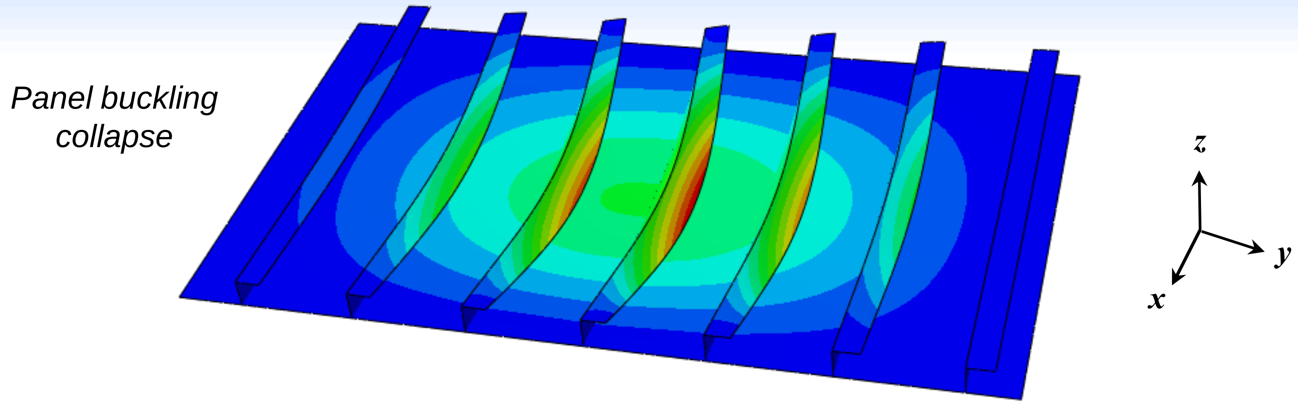
- Additional load shed to edges of skin
- Effective width narrows

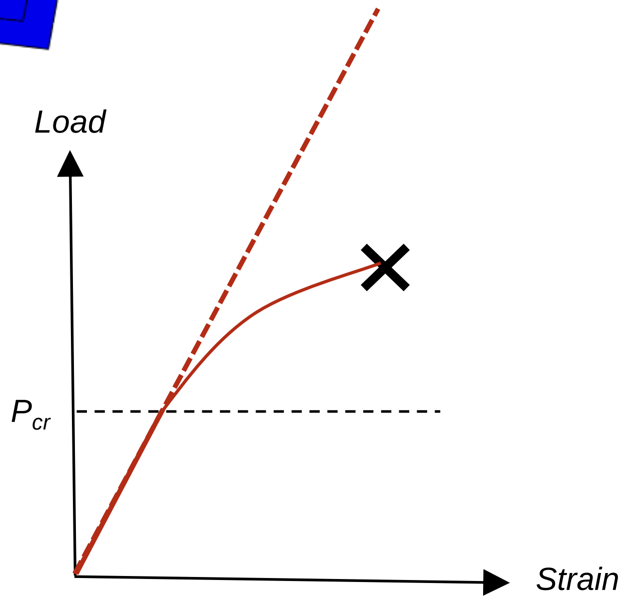






Collapse: $P > 3.0 * P_{cr,skin}$





- **Redistribution of load will lower margins**
  - **Crippling**
  - **Panel buckling**
  - **Strength**

Outline for Presentation

- Composite ply properties
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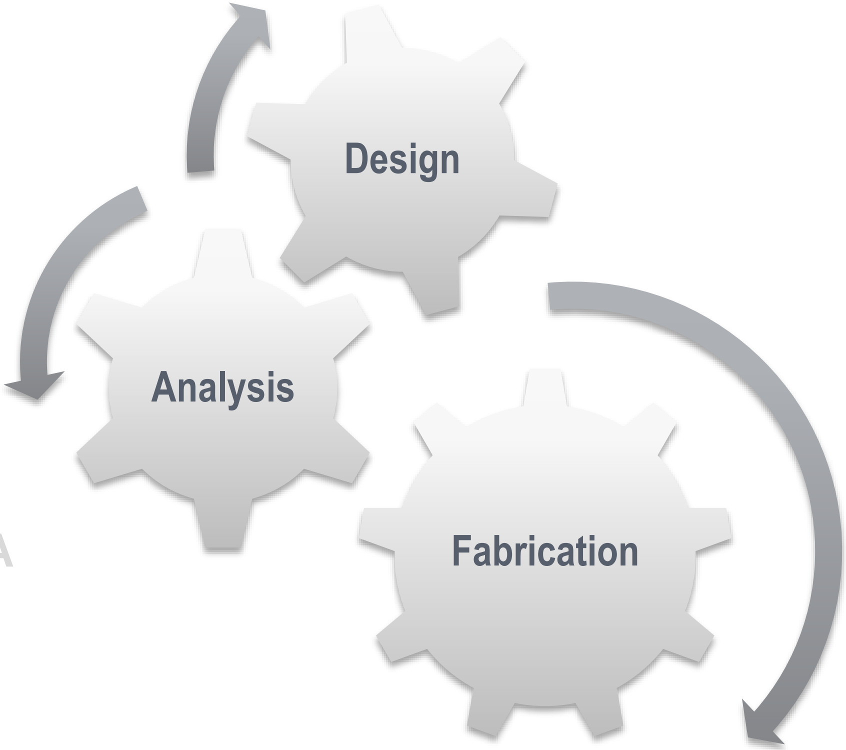
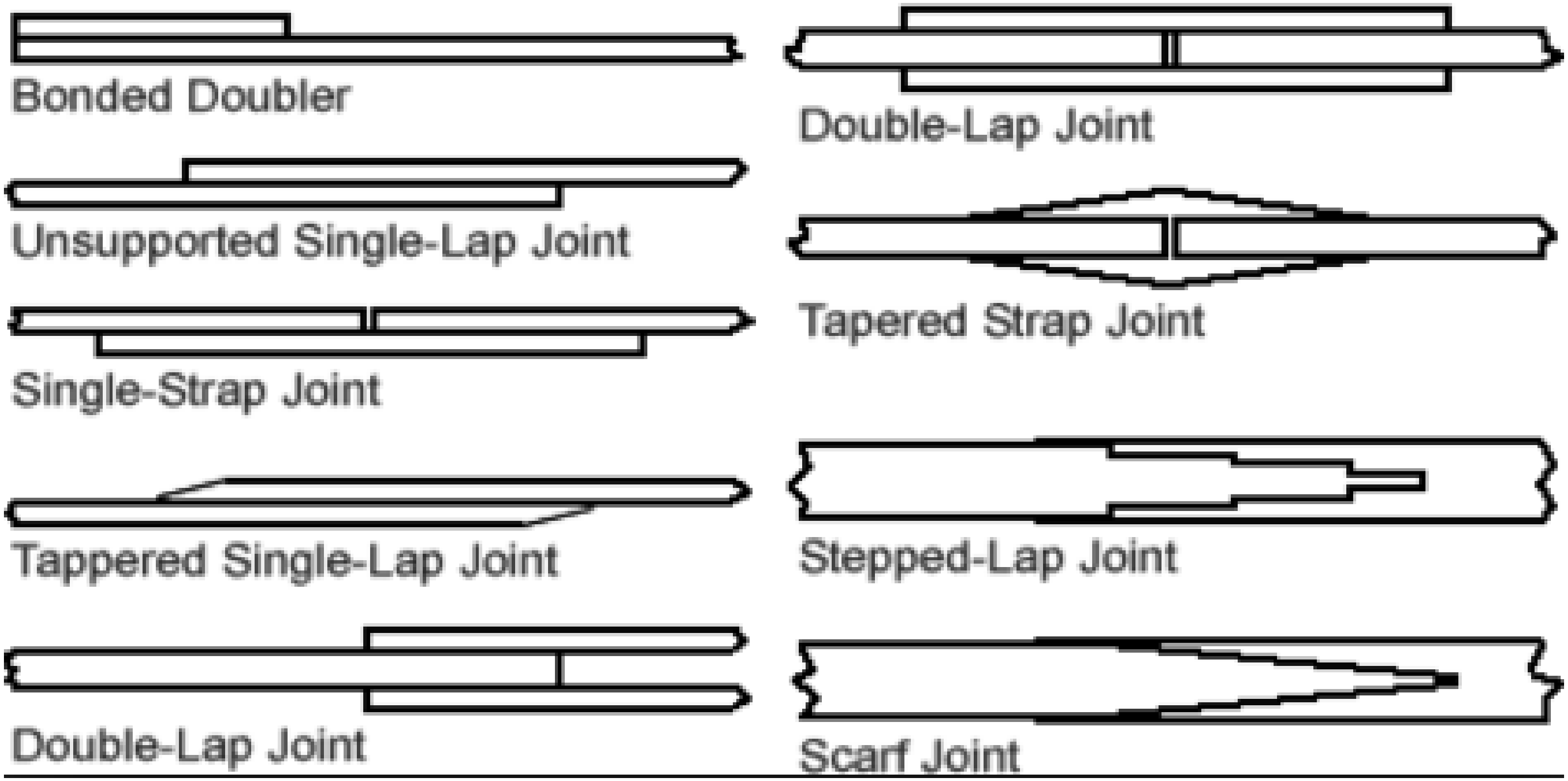
- Extension of CLT to stiffened panels
- Margin of Safety
- Composite strength failure criteria
- Linear buckling
- Honeycomb panel failure
- Stiffened panel failure
- **Composite joints**
- Coupling analytical methods with FEA
- Stiffened panel modeling approaches
- Composite optimization
- Continuous vs. Discrete Sizing
- Designing composites for producibility and repair

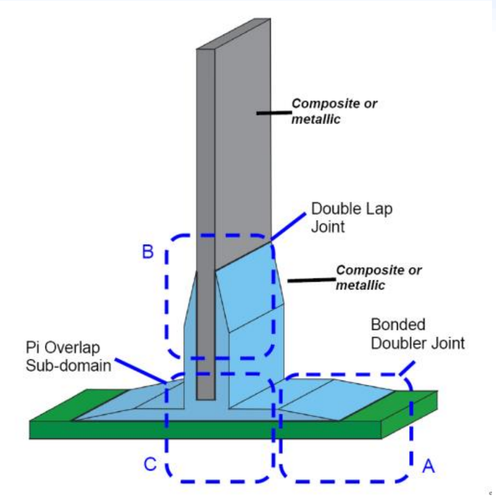


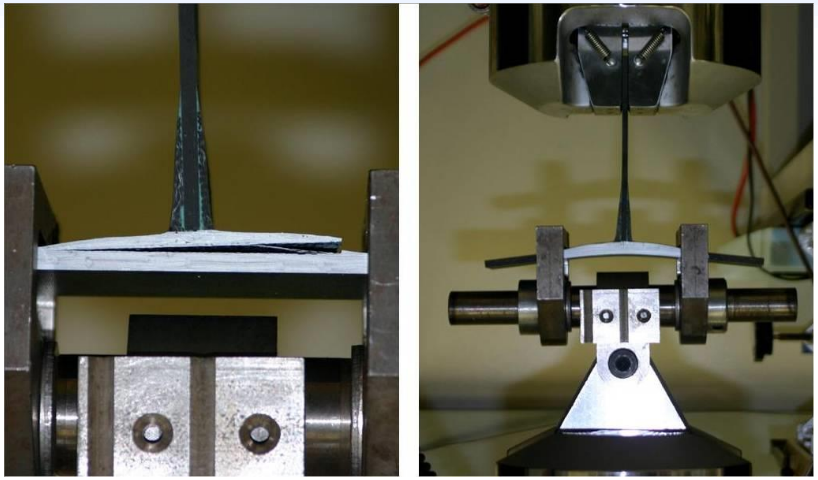
Types of Bonded Joints

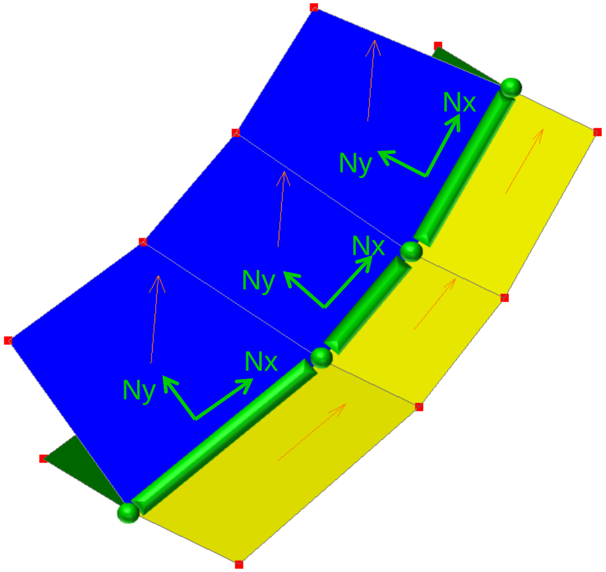


Running Load Analysis

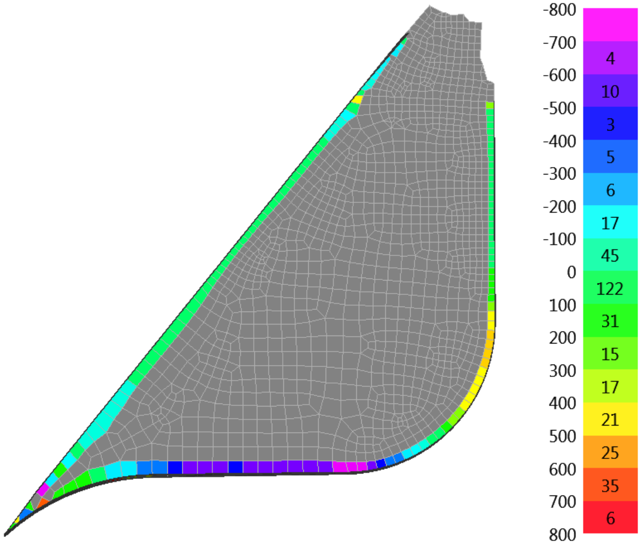
- **Many joints types can be analyzed with this joint configuration. Results based on allowable running loads obtained from testing.**



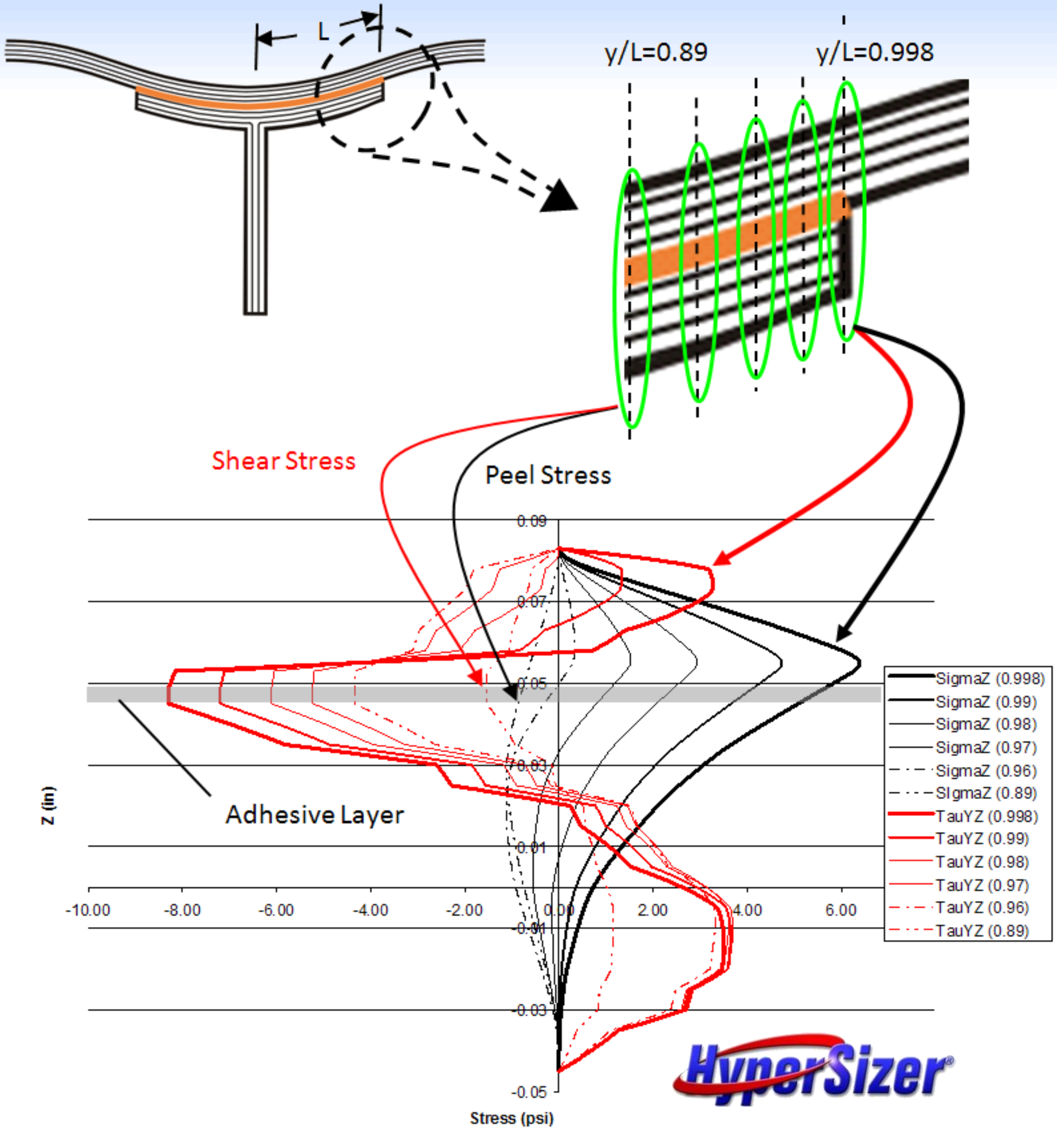


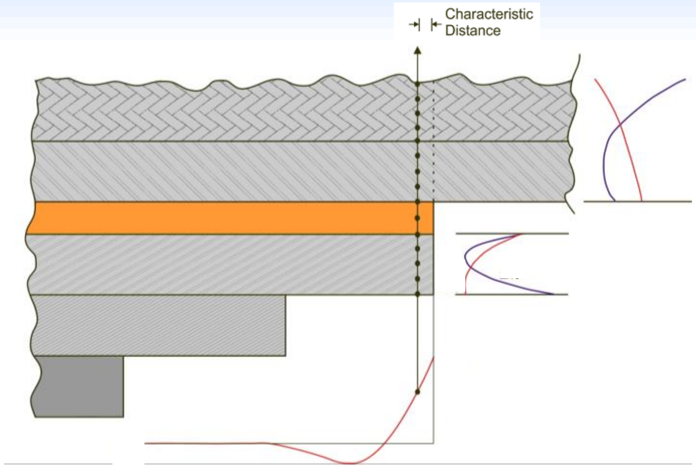


Extract FEA forces and transform grid by grid normal to surface for Pull off and shear loads



Local Analysis – Ply based

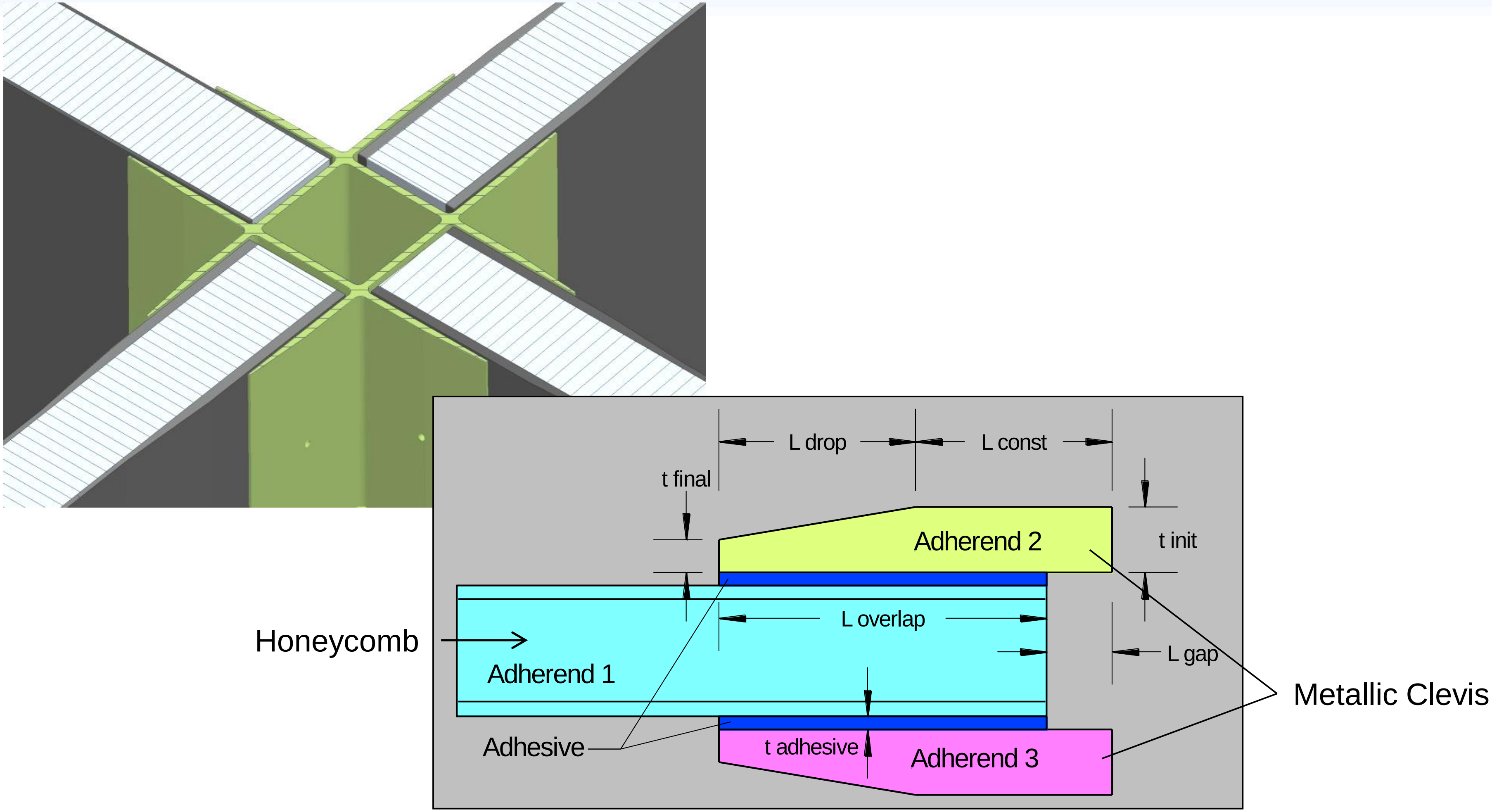




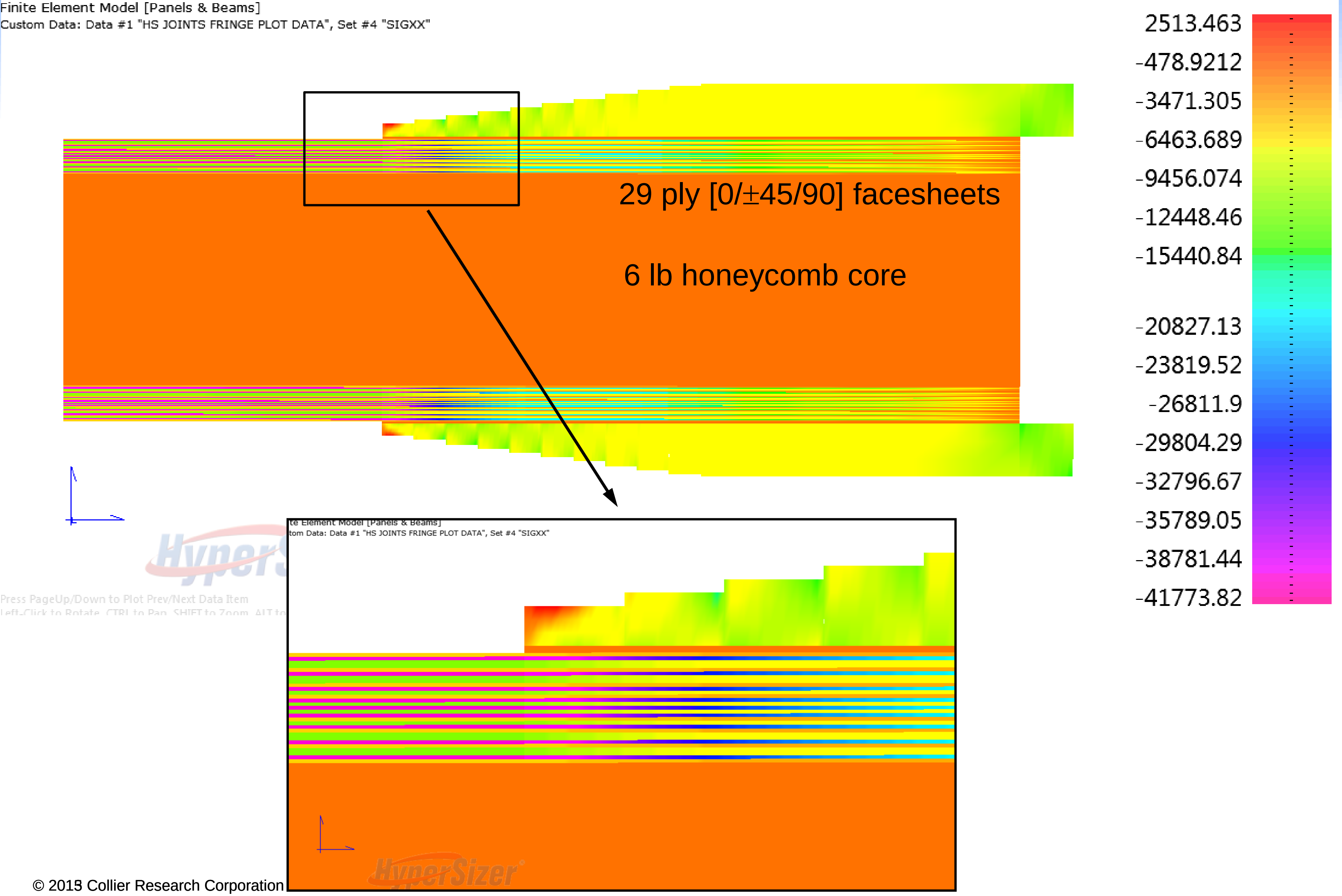
- **Both the peel and interlaminar stresses in the laminates increase dramatically near the flange end**

Bonded Clevis

- **Honeycomb closeout joints can be analyzed with this joint configuration**



Example Ply-By-Ply Fields - σ_{xx} (psi)



Finite Element Model [Panels & Beams]
Custom Data: Data #1 "HS JOINTS FRINGE PLOT DATA", Set #4 "SIGXX"

29 ply [0/±45/90] facesheets

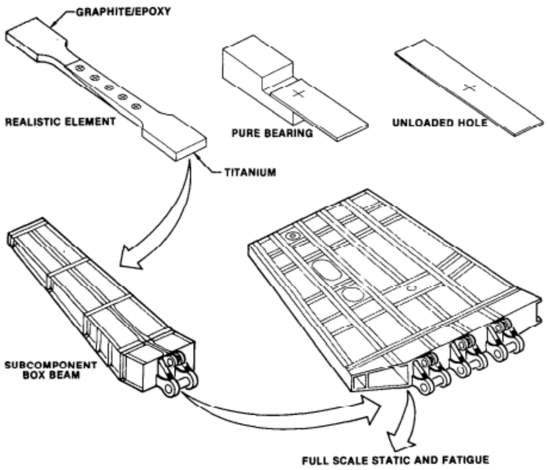
6 lb honeycomb core

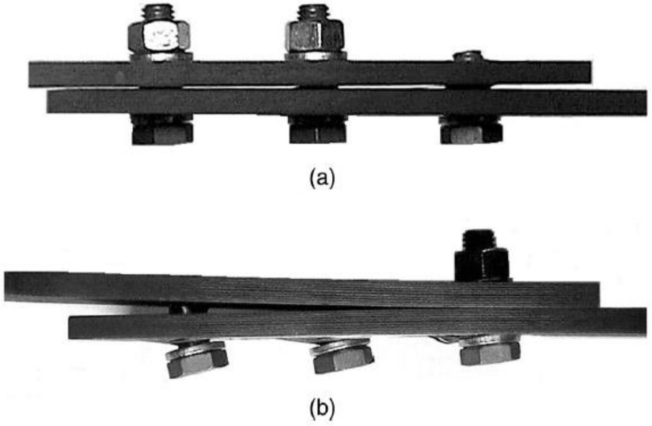
Press PageUp/Down to Plot Prev/Next Data Item

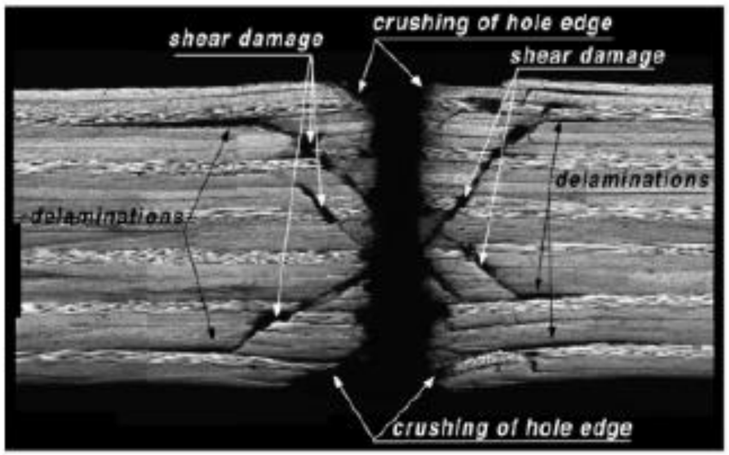
te Element Model [Panels & Beams]
tom Data: Data #1 "HS JOINTS FRINGE PLOT DATA", Set #4 "SIGXX"

Bolted Joint Failure Modes

- Composite bolted joint analysis is challenging
- Bolted joint failures can be catastrophic







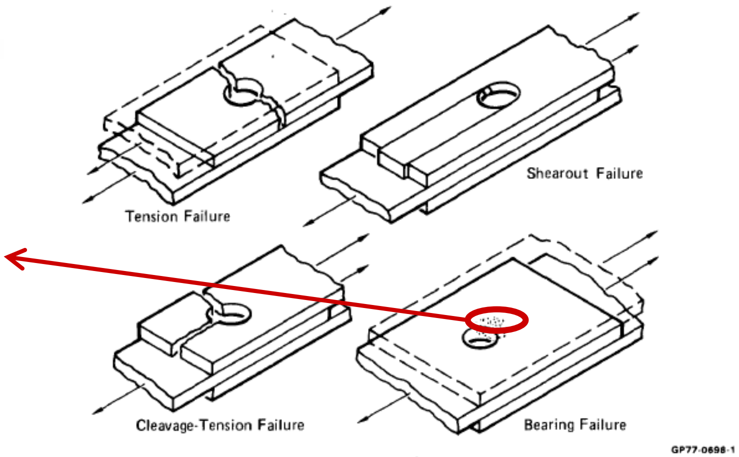
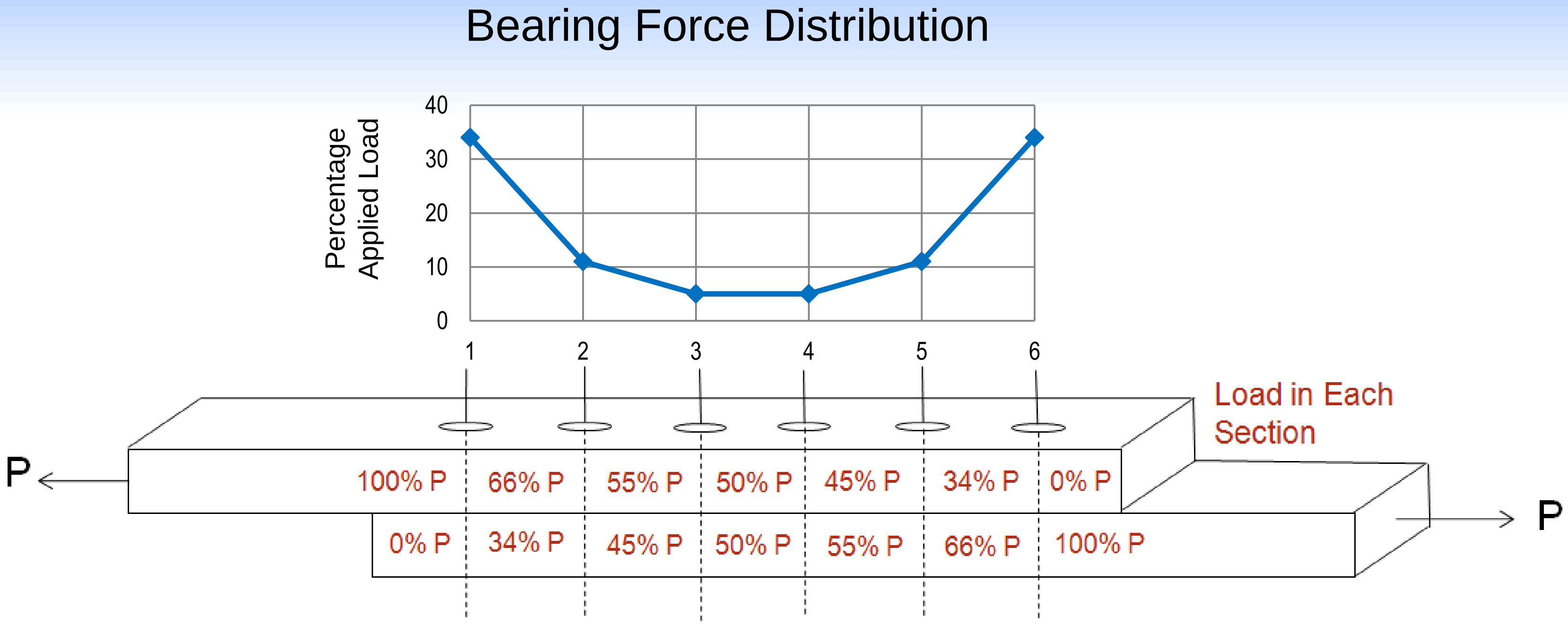


FIGURE 19
MODES OF FAILURE FOR BOLTED COMPOSITE JOINTS

Bearing Force Distribution



- **Composite laminates are stiff and do not yield. So in composite joints, the outer-most fasteners have highest bearing force**
- **Bearing force is dependent on laminate stiffness**

Bearing Analysis Overview

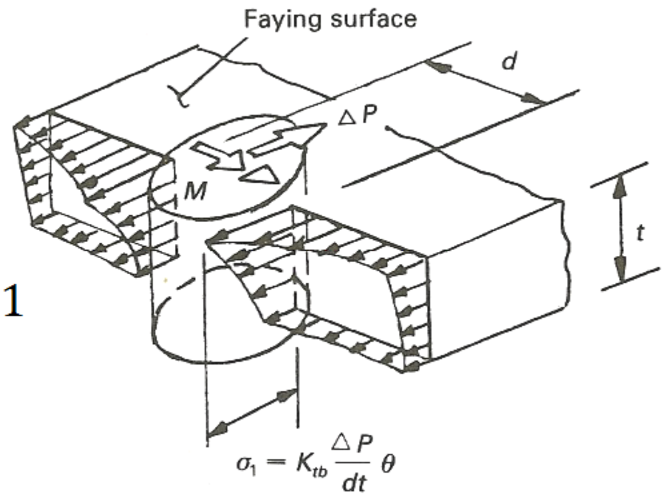
- The bearing analysis requires the fastener geometry, laminate geometry, correction factors, bearing force and bearing stress allowable.

Correction factors used to account for:

- Single shear joints (load eccentricity)
- Hole diameter
- Laminate Thickness
- Fastener fit
- Edge distance
- Fastener spacing
- Liquid and solid shims

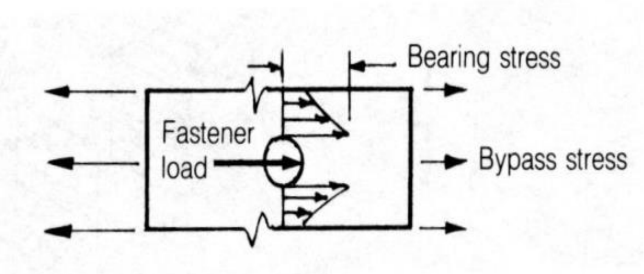
$$[f_{bru}] = \frac{\Delta P}{d\,t}$$

$$MS_{bearing} = \frac{C_f\,[F_{bru}]}{K_f\,[f_{bru}]} - 1$$

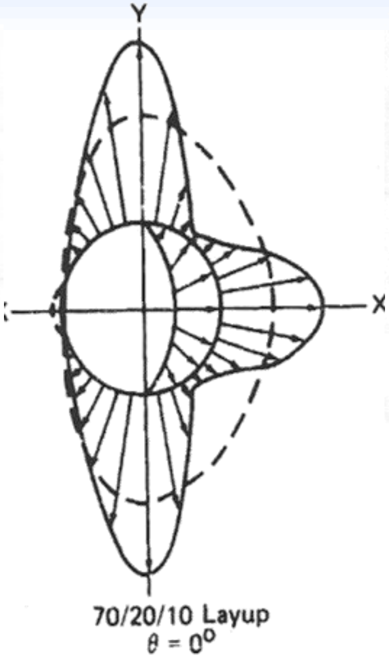


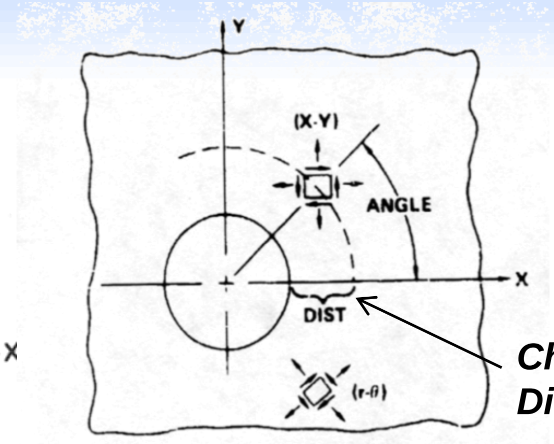
- Advantages:
  - Simple P/A approach to write margins for composite laminates in fastened joints. Easy to include correction factors to impose conservatism for design.
- Disadvantages:
  - Determining bearing stress allowables requires experimental testing.
  - Additional parameters (correction factors) requires additional testing to account for affects not captured in simple bearing analysis.

BJSFM Analysis Overview

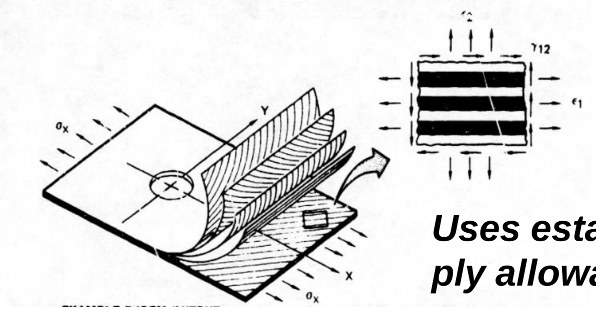


Combined bearing + Bypass stress





Characteristic Distance D_0



Uses established ply allowables

- BJSFM (Bolted Joint Stress Field Modeling) uses closed-form approach to determine the stress field around an open hole. Then measures out a Characteristic Distance from the edge of the hole to determine ply-based failure.

Bearing Force and Load Angle

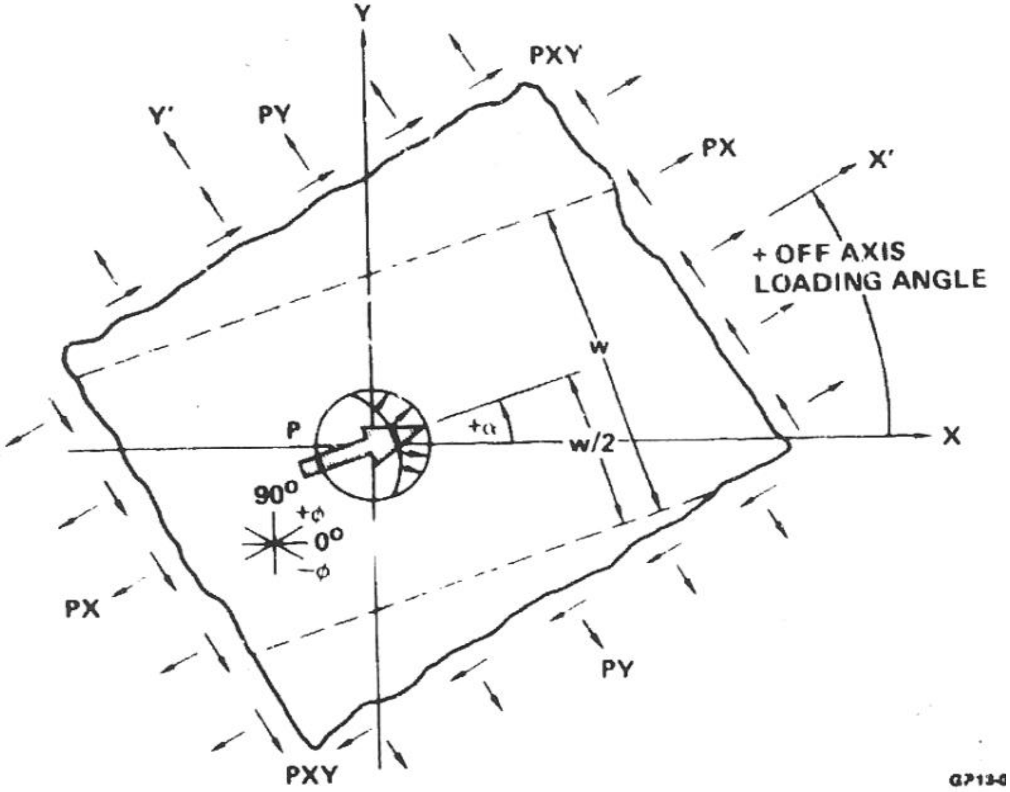
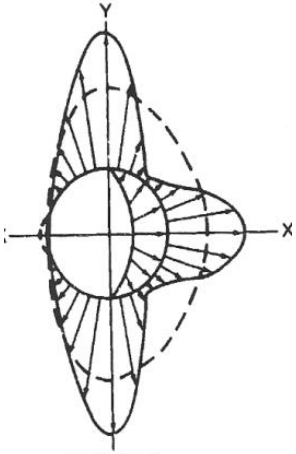
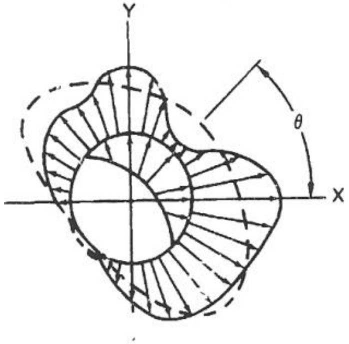


Figure 1. General Load Conditions Analyzed Using BJSFM

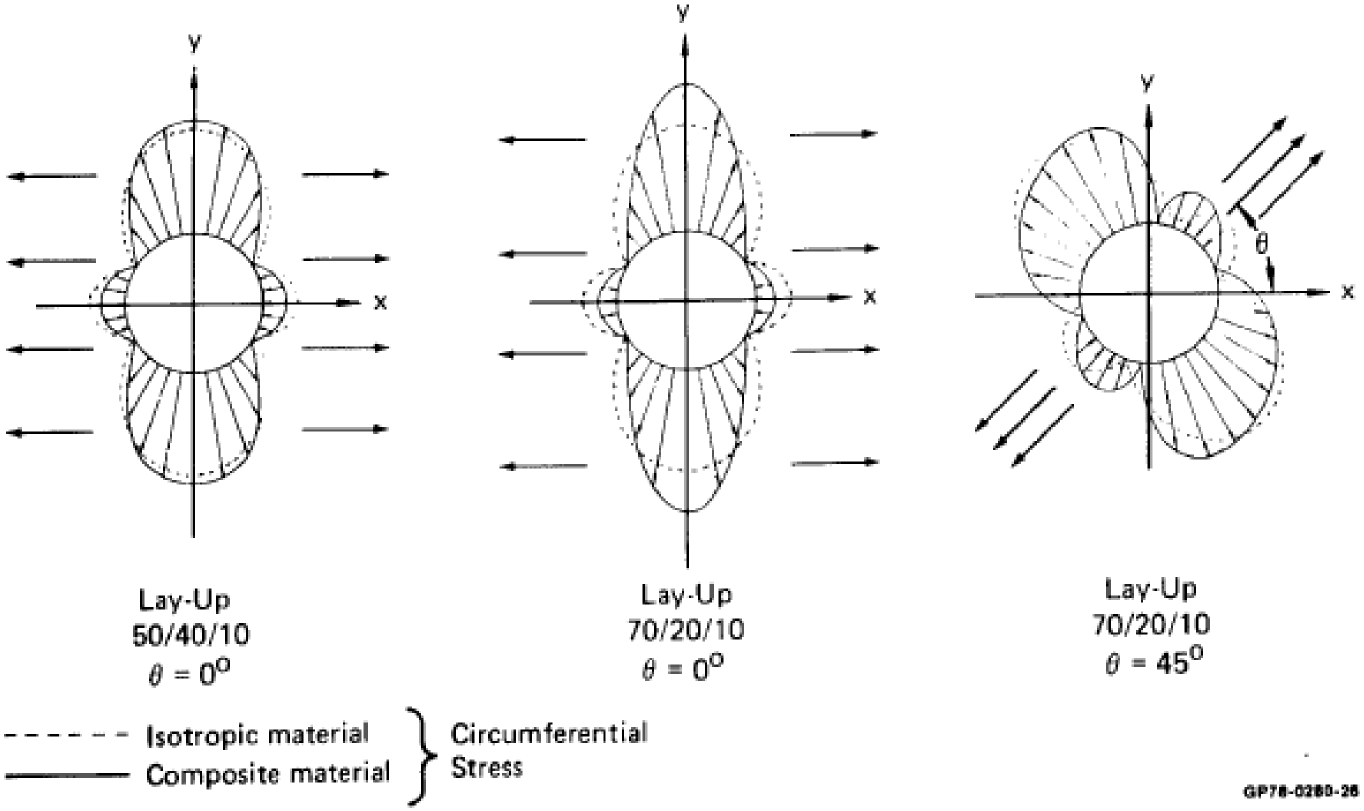


70/20/10 Layup
$\theta = 0^{\circ}$



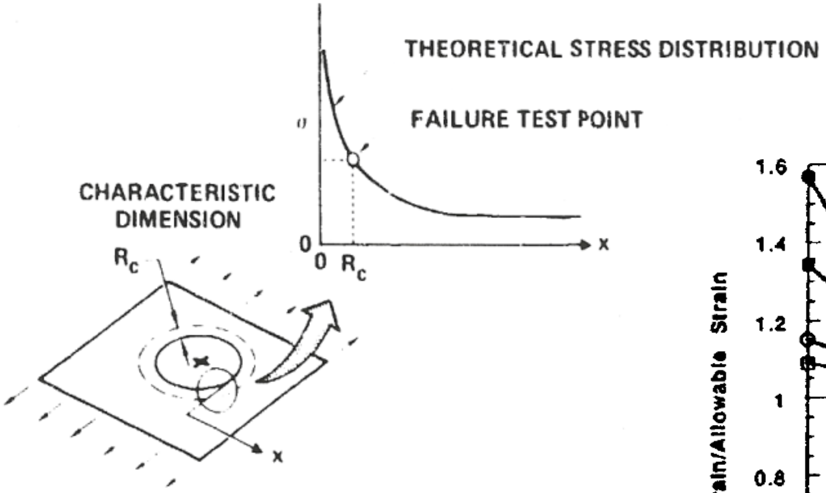
70/20/10 Layup
$\theta = 45^{\circ}$

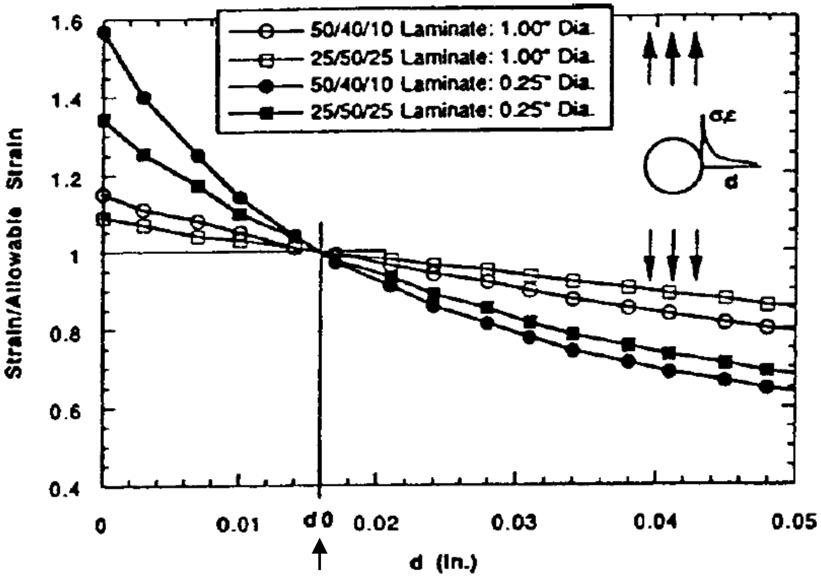
Bypass Load


y
x
Lay-Up
50/40/10
$\theta = 0^o$
y
x
Lay-Up
70/20/10
$\theta = 0^o$
y
x
θ
Lay-Up
70/20/10
$\theta = 45^o$
Isotropic material
Composite material
Circumferential
Stress
GP78-0280-28

MIL HDBK-17-3E, Characteristic Distance

- Characteristic distances are calibrated to damaged (open hole) strain allowables





.016”

Equivalent Margins of Safety

$$MS_{OH} = \frac{\sigma_1}{Ftu} - 1 = MS_{BJSFM}$$

$$MS_{Unnotched} = \frac{\sigma_2}{Ftu} - 1 = MS_{BJSFM}$$

Notes:
Pristine allowables used to determine Ftu
B-basis allowables used for MS_{OH} and $MS_{Unnotched}$

Outline for Presentation

- Composite ply properties
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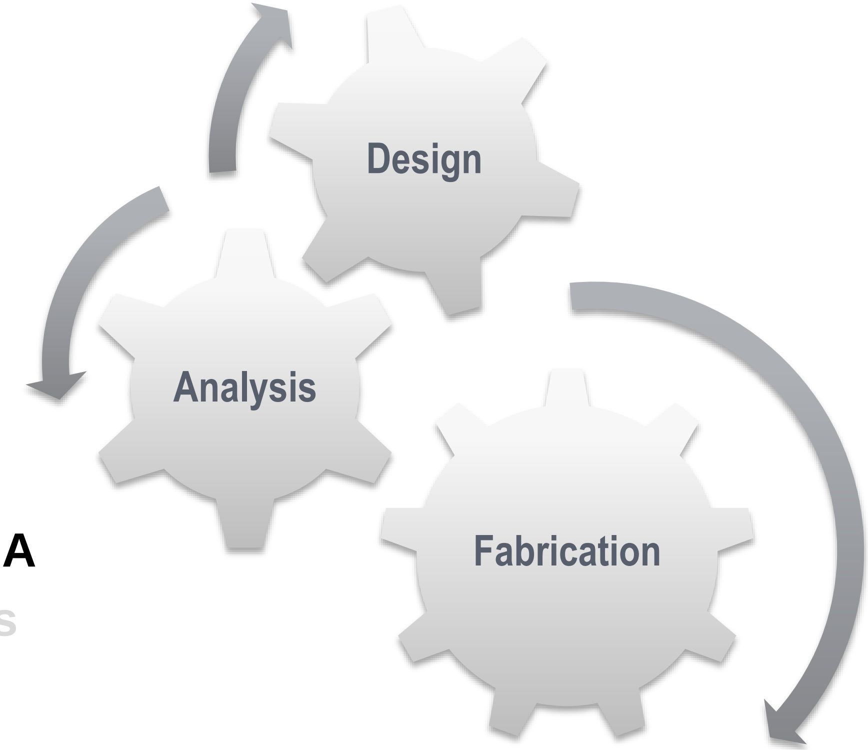
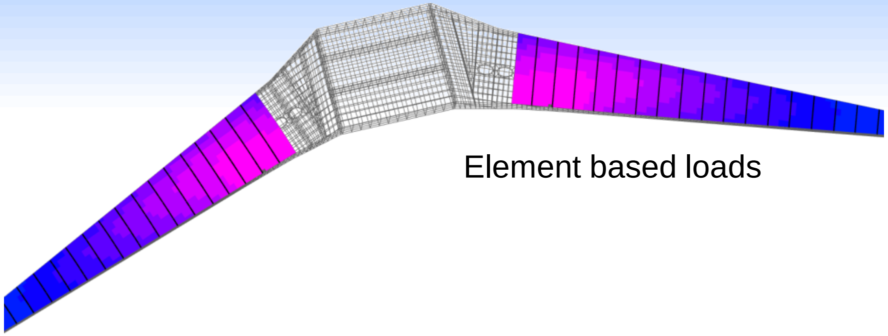
- Stiffened panel failure
- Composite joints
- **Coupling analytical methods with FEA**
- Stiffened panel modeling approaches
- Composite optimization
- Continuous vs. Discrete Sizing
- Designing composites for producibility and repair

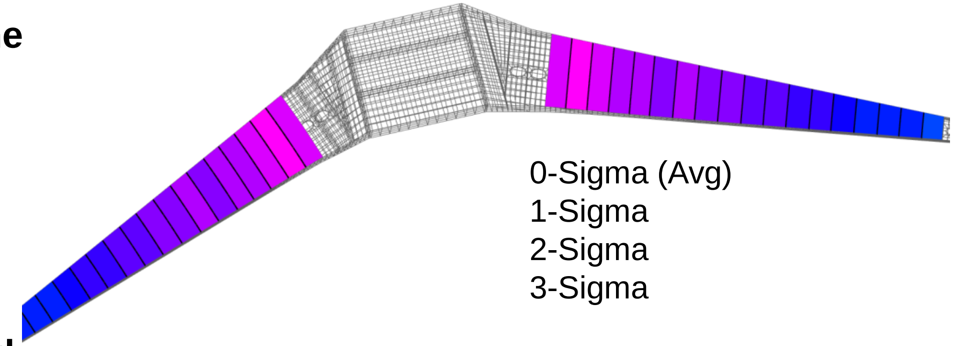


Extracting Element Loads from FEM

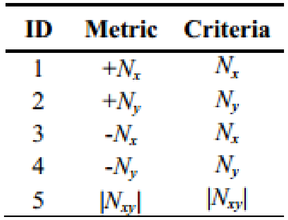
- Element Based
  - Analyzes each element for strength and local stability considering all load cases
  - Returns margins of safety and controlling analysis data for each element
- N-Sigma method
  - Statistically processes loads to determine design-to loads for each component and each load set
  - Analyzes each component for strength and local stability for all load cases
- Element Peak method
  - Determines the critical elements and load cases for a series of metrics
  - Analyzes each component based on peak loads



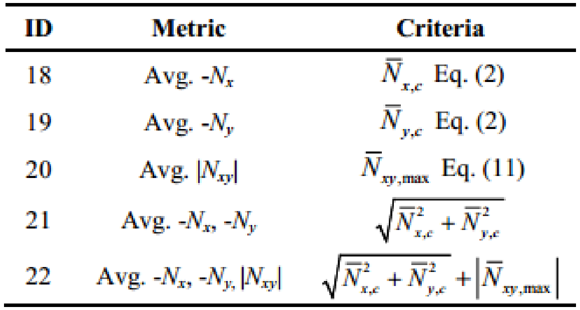
Element based loads



0-Sigma (Avg)
1-Sigma
2-Sigma
3-Sigma



ID	Metric	Criteria
1	$+N_x$	N_x
2	$+N_y$	N_y
3	$-N_x$	N_x
4	$-N_y$	N_y
5	$\lvert N_{xy}\rvert$	$\lvert N_{xy}\rvert$



ID	Metric	Criteria
18	Avg. $-N_x$	$\bar{N}_{x,c}$ Eq. (2)
19	Avg. $-N_y$	$\bar{N}_{y,c}$ Eq. (2)
20	Avg. $\lvert N_{xy}\rvert$	$\bar{N}_{xy,\max}$ Eq. (11)
21	Avg. $-N_x$, $-N_y$	$\sqrt{\bar{N}_{x,c}^2 + \bar{N}_{y,c}^2}$
22	Avg. $-N_x$, $-N_y$, $\lvert N_{xy}\rvert$	$\sqrt{\bar{N}_{x,c}^2 + \bar{N}_{y,c}^2} + \left\lvert \bar{N}_{xy,\max} \right\rvert$

Outline for Presentation

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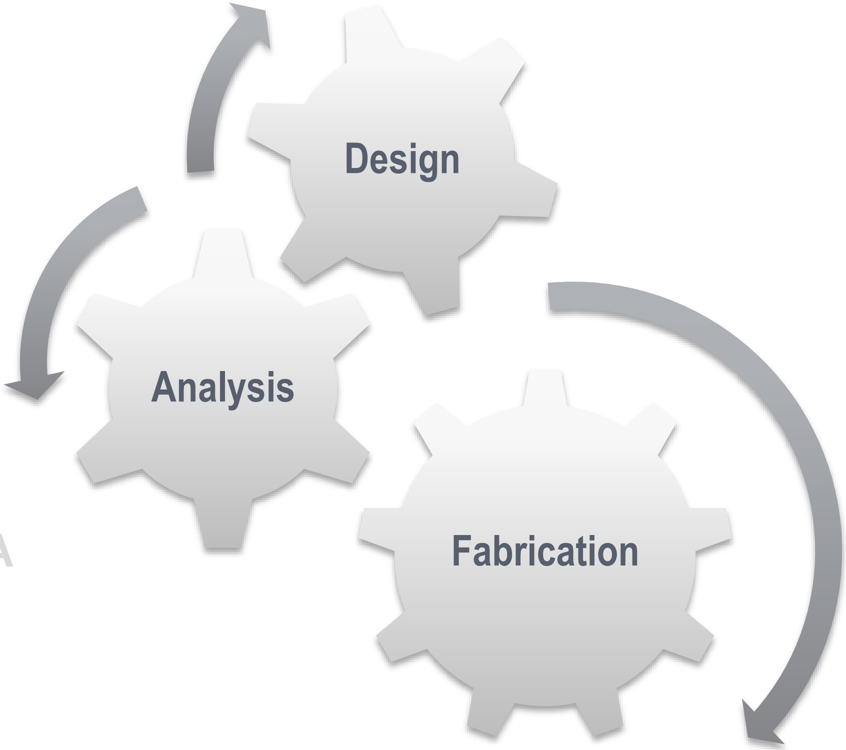
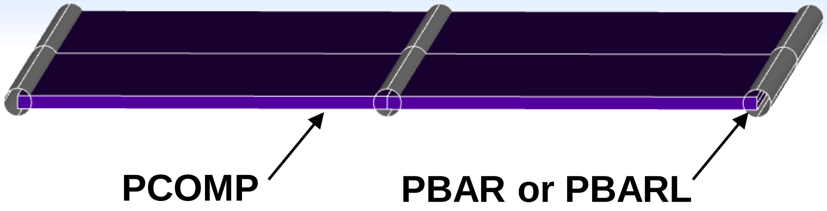


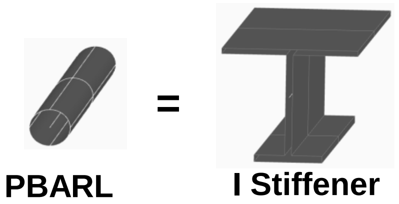
Four Modeling Techniques: Identified

1. Stiffeners Smeared into Shells

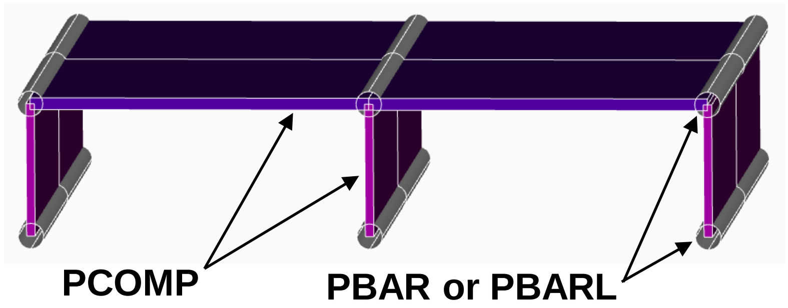
Equivalent Stiffness Matrix, [ABD]

2. Stiffeners Discrete as Beams

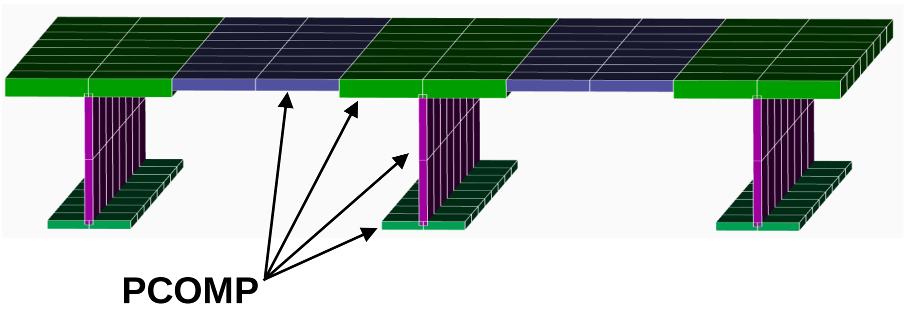




3. Stiffeners Discrete as Beams/Shells



4. Stiffeners Discrete as Shells

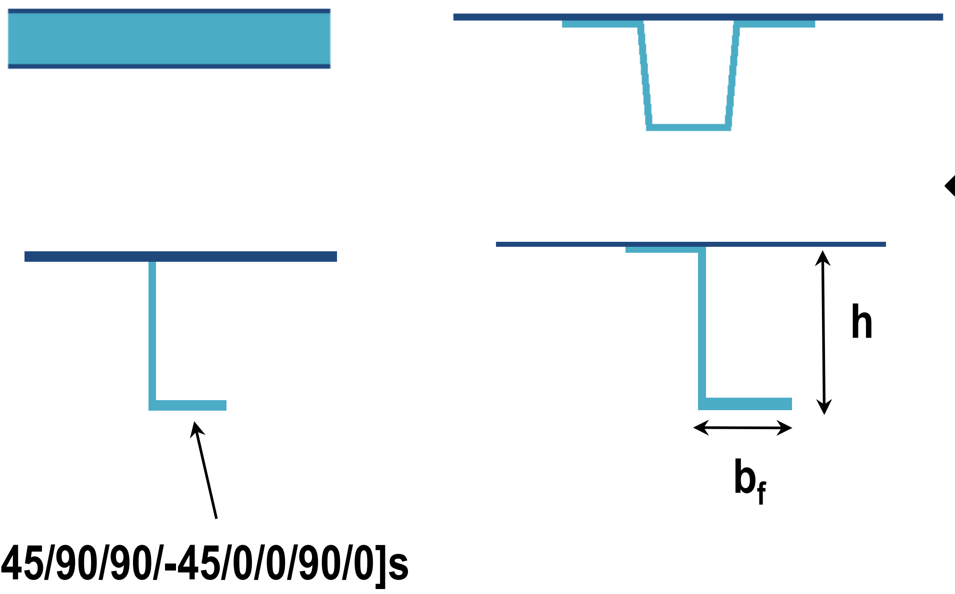


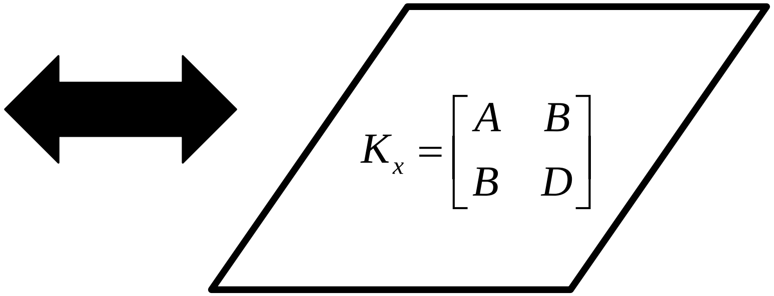
NASTRAN Terminology

Local Stiffness to Global Stiffness

Local Stiffness

Global Stiffness



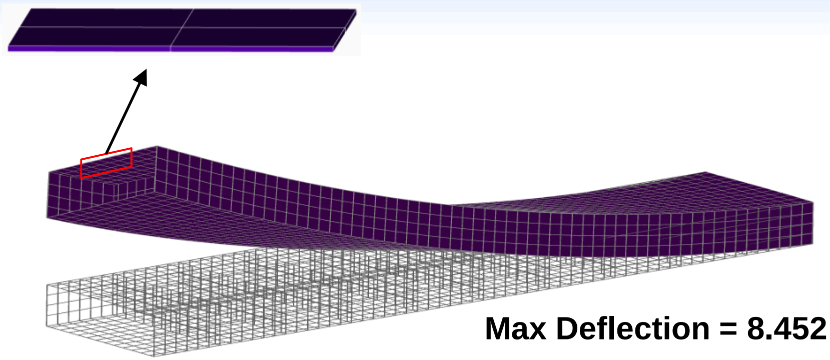


$$K_x = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$$

“smeared stiffness”

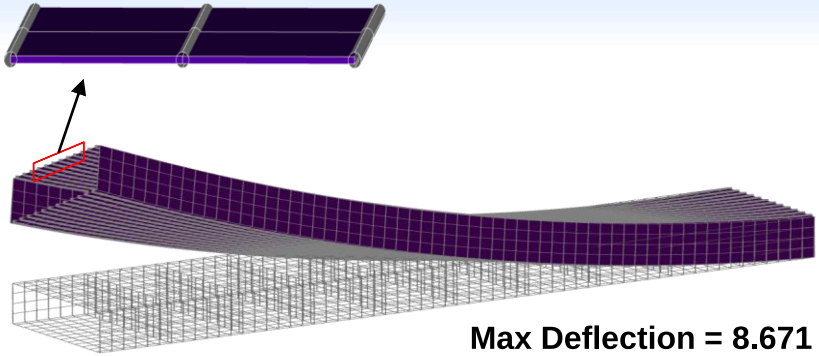
Four Modeling Techniques: Accuracy

1. Stiffeners Smeared into Shells



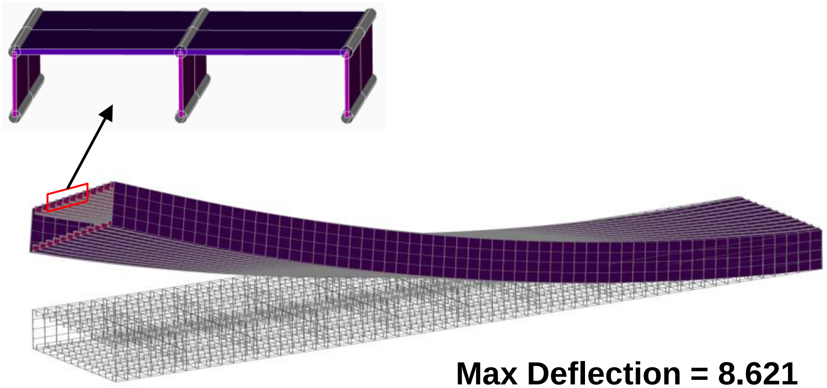
Max Deflection = 8.452

2. Stiffeners Discrete as Beams



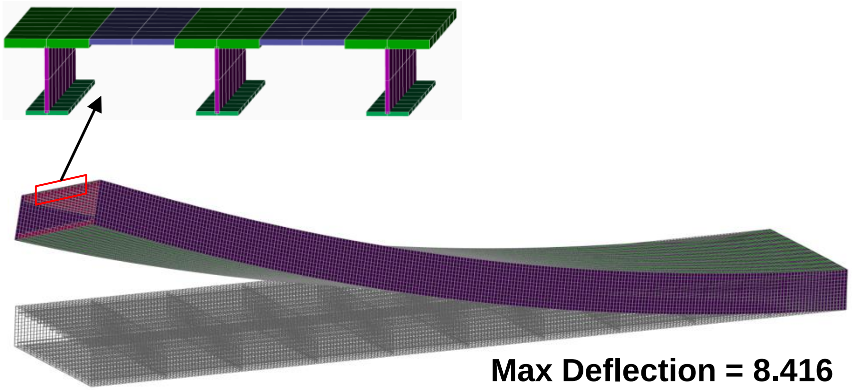
Max Deflection = 8.671

3. Stiffeners Discrete as Beams/Shells



Max Deflection = 8.621

4. Stiffeners Discrete as Shells



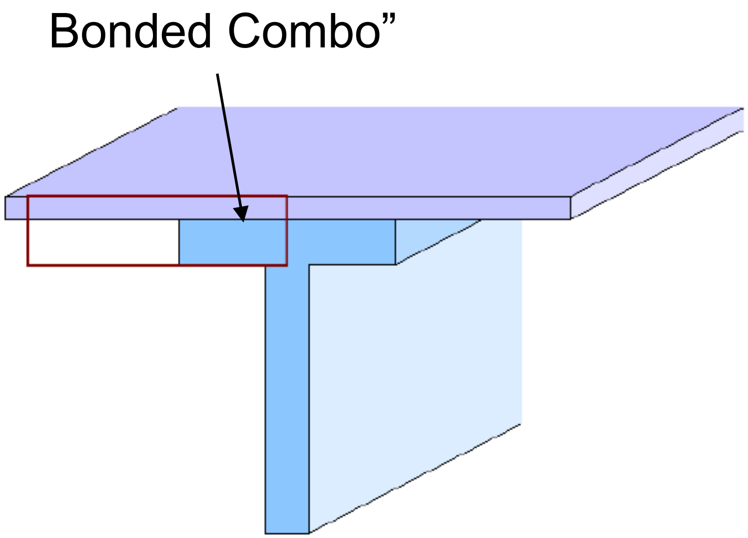
Max Deflection = 8.416

**1.5 Ultimate Load Factor*

** Combined Bending and Torsion Load Cases*

Global Torsional Stiffness

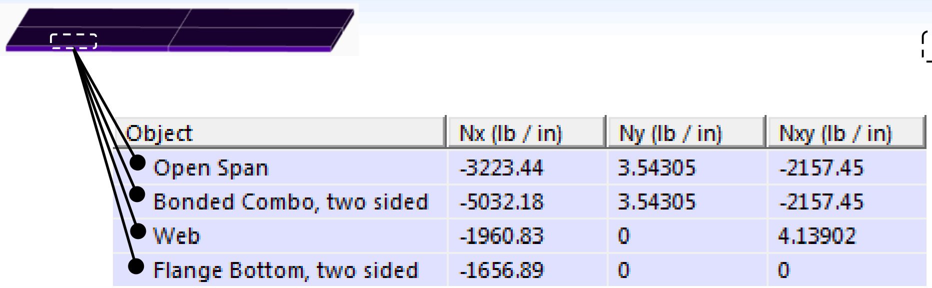
- Torsion Stiffness (GJ) of a closed section is very sensitive to A33 of skin panels around the closed section.
- For modeling techniques 2 and 3 the attached flange is not considered when FEA formulates the A33 stiffness of panel.
- If a smeared stiffness formulation is used it should include the additional shear and transverse stiffness of the bonded combo in equivalent stiffness formulation.



Bonded Stiffened Panel

Uniaxial Modeling Techniques: Accuracy

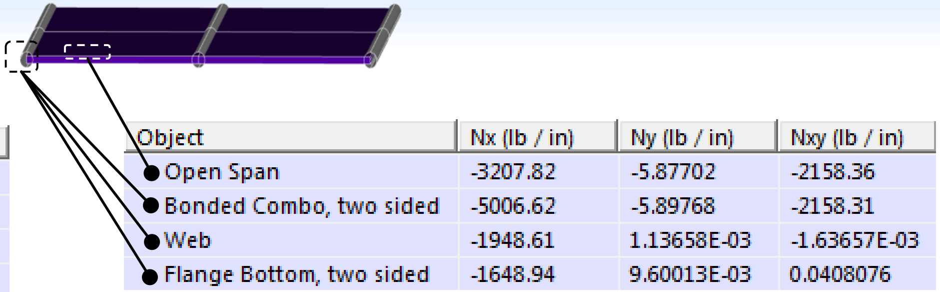
1. Stiffeners Smeared into Shells



Object	Nx (lb / in)	Ny (lb / in)	Nxy (lb / in)
Open Span	-3223.44	3.54305	-2157.45
Bonded Combo, two sided	-5032.18	3.54305	-2157.45
Web	-1960.83	0	4.13902
Flange Bottom, two sided	-1656.89	0	0

Crippling MS = 1.158

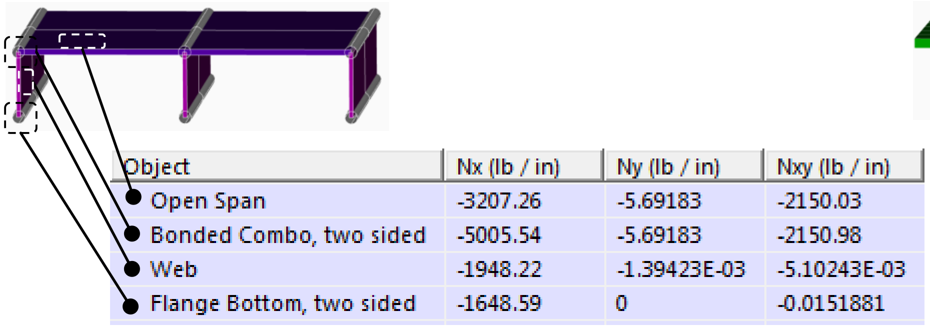
2. Stiffeners Discrete as Beams



Object	Nx (lb / in)	Ny (lb / in)	Nxy (lb / in)
Open Span	-3207.82	-5.87702	-2158.36
Bonded Combo, two sided	-5006.62	-5.89768	-2158.31
Web	-1948.61	1.13658E-03	-1.63657E-03
Flange Bottom, two sided	-1648.94	9.60013E-03	0.0408076

Crippling MS = 1.166

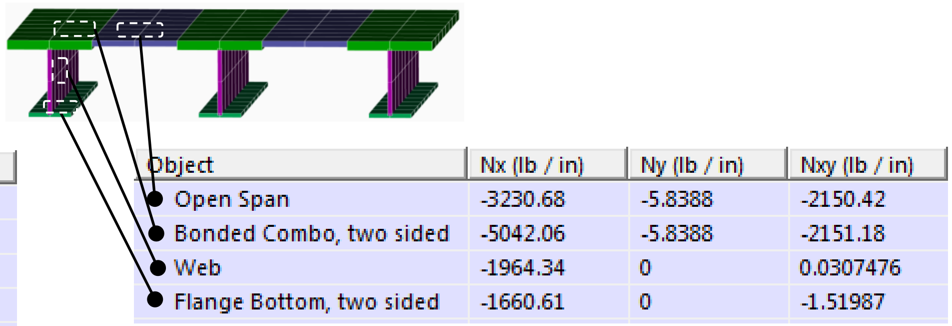
3. Stiffeners Discrete as Beams/Shells



Object	Nx (lb / in)	Ny (lb / in)	Nxy (lb / in)
Open Span	-3207.26	-5.69183	-2150.03
Bonded Combo, two sided	-5005.54	-5.69183	-2150.98
Web	-1948.22	-1.39423E-03	-5.10243E-03
Flange Bottom, two sided	-1648.59	0	-0.0151881

Crippling MS = 1.178

4. Stiffeners Discrete as Shells



Object	Nx (lb / in)	Ny (lb / in)	Nxy (lb / in)
Open Span	-3230.68	-5.8388	-2150.42
Bonded Combo, two sided	-5042.06	-5.8388	-2151.18
Web	-1964.34	0	0.0307476
Flange Bottom, two sided	-1660.61	0	-1.51987

Crippling MS = 1.159

Bending Twisting Load Case
****1.5 Ultimate Load Factor***

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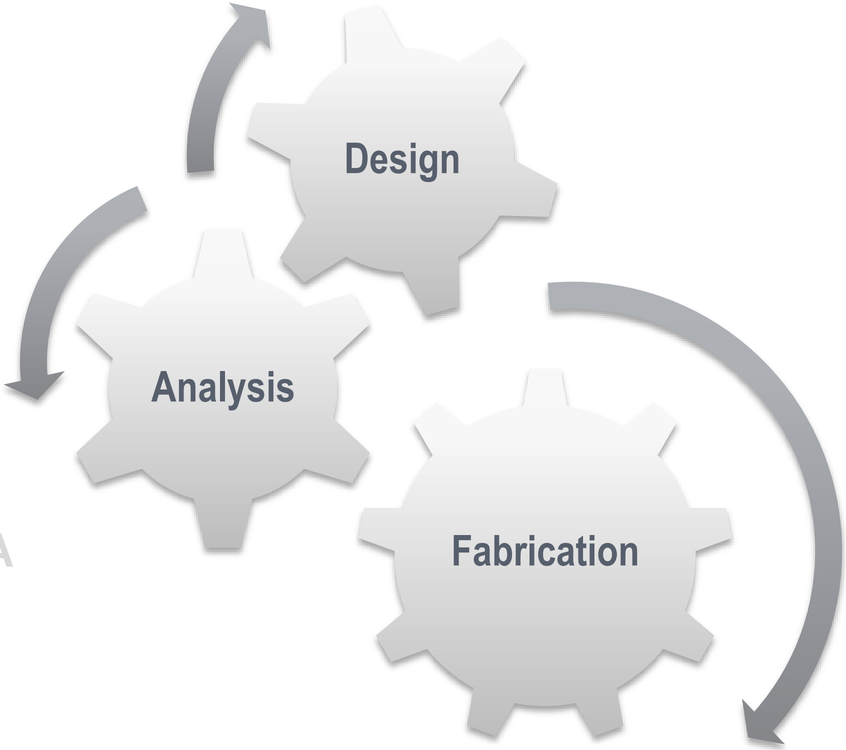
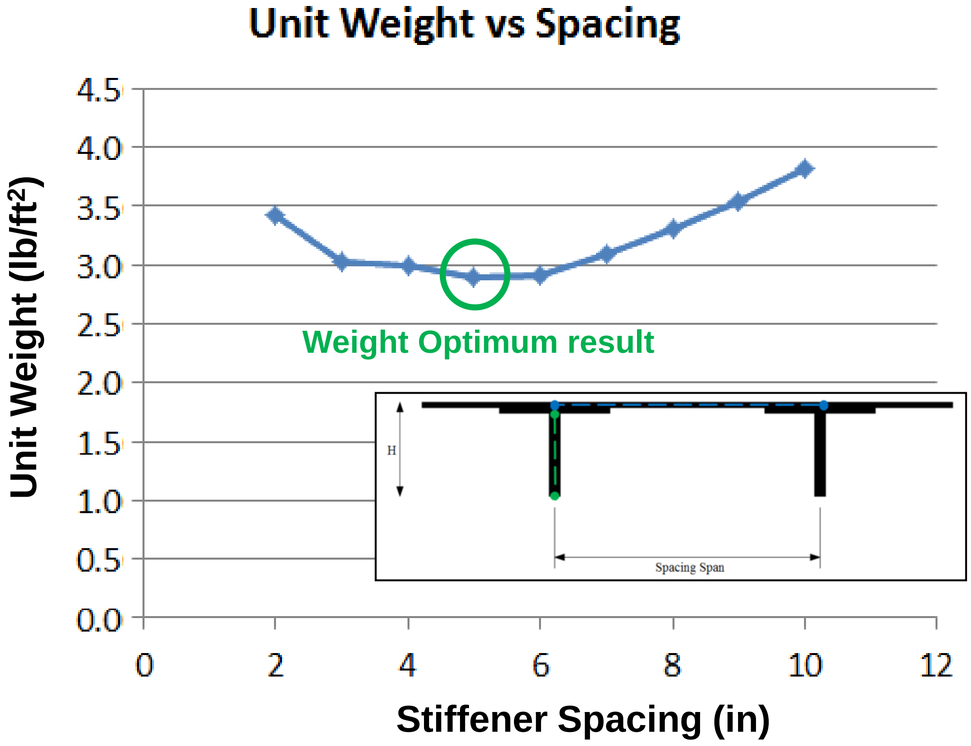
- **Composite optimization**
- Continuous vs. Discrete Sizing
- Designing composites for producibility and repair



What is Optimization?

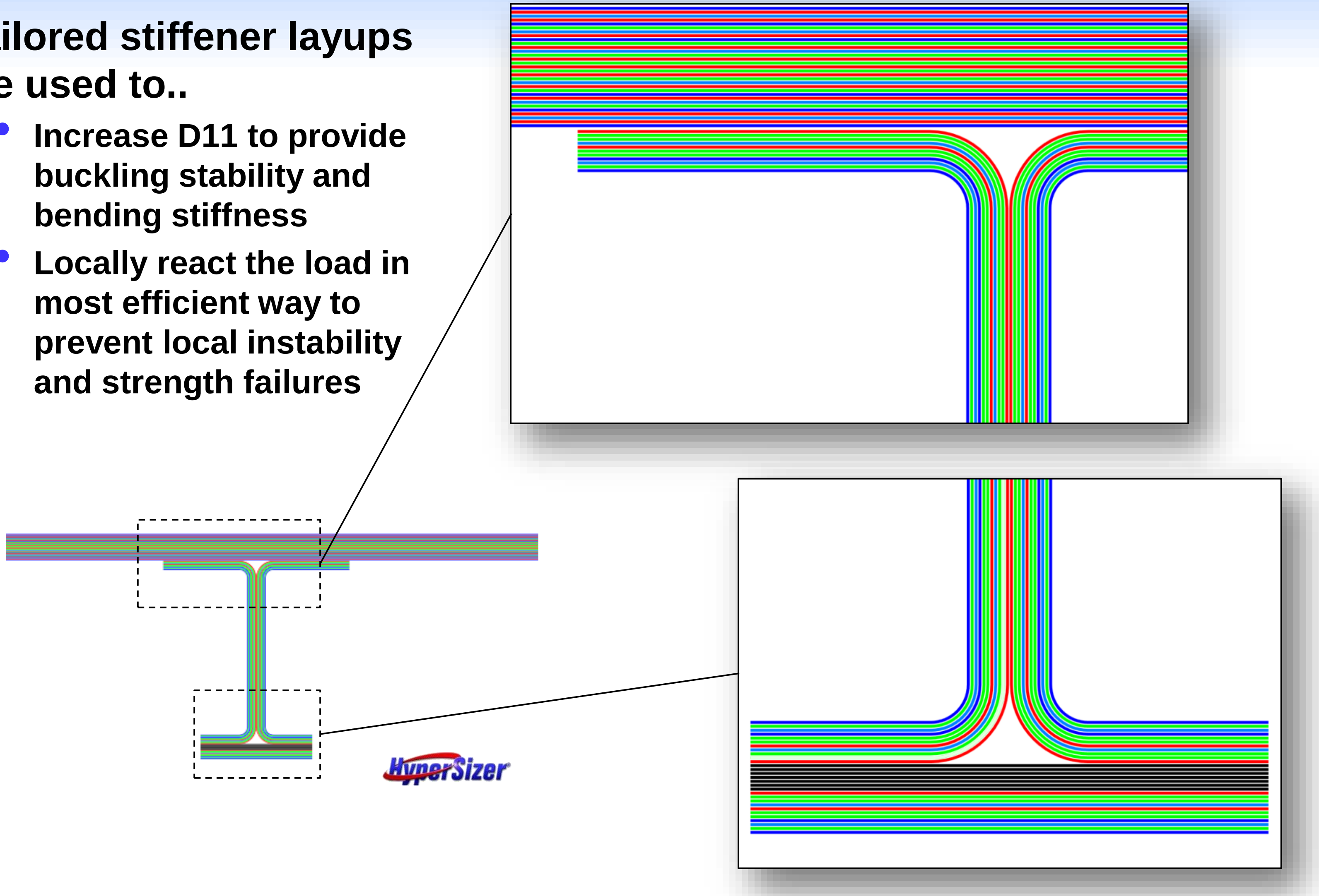
- In mathematical terms, optimization means to find the combination of variables to minimize or maximize some objective (weight, cost, etc.) subject to some constraints.
- In practice, structural optimization approaches are used reduce the weight of a structure by modifying design parameters to better handle the applied loading.
- Composite structures provide more design parameters because the cross sectional shape and material stiffness are variable.



There is no absolute optimum answer but many near optimal answer. Optimization software will find those near optimum answers for the primary purpose to provide information to the engineer to make the right decision based on many considerations.

Composite Optimization

- Tailored stiffener layups are used to..
  - Increase D11 to provide buckling stability and bending stiffness
  - Locally react the load in most efficient way to prevent local instability and strength failures

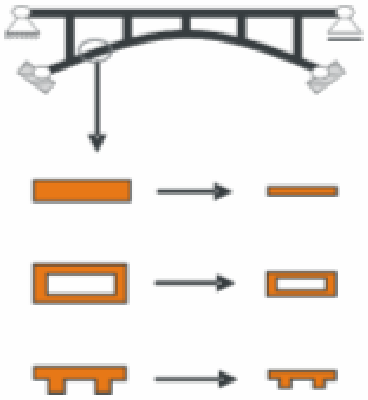


Common Types of Structural Optimization

- Note: Many types of optimization algorithms exist to solve many problems. The types listed below are some common types found in the composites industry.

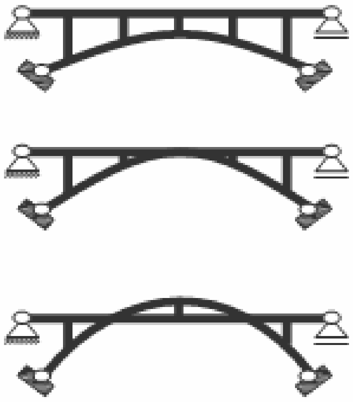
http://carat.st.bv.tum.de/caratuserswiki/index.php/Users:Structural_Optimization/General_Formulation

Sizing



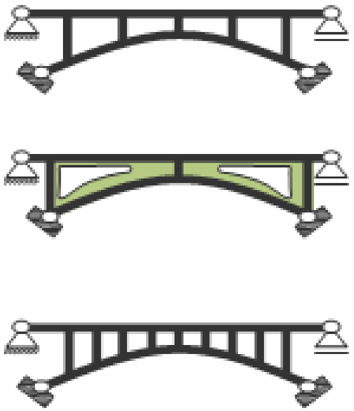
- Finds optimum design variables (thickness, fiber orientation, etc.) while staying within design constraints

Shape Optimization



- Modifies shape of global structure to accomplish objective (moves grids)
- Special forms of shape optimization include
  - Topography
  - Topometry

Topology



- Most flexible approach
- Finds most efficient material distribution in design space (removes elements)
- Special forms of Topology optimization include
  - Full stressed design (FSD)

Mathematical Algorithm

- Computational methods that iterate with an analysis code, like FEA, to converge to a solution.

- Examples:
  - Gradient based
    - Pros – Fast. Relatively few function evaluations needed.
    - Cons – Variables need to be continuous or approximated as continuous. Final solution may not be manufacturable. It is likely it will get stuck in local optimum.
  - Genetic Algorithms
    - Pros – Works with discrete variables. Less likely to get stuck in local optimums
    - Cons – Requires many functions evaluations. Not a good option if the function evaluation involves running FEA.
  - Many more…

Heuristic Algorithm

- **Domain-specific methods that evaluate candidate solutions based on user-defined criteria.**

- **Example: Direct Search Method**
  - **User defines design space by setting bounds and discrete thickness/width intervals. From this information, the candidate solutions generated**
  - **Candidate solutions are sorted by a particular criteria (weight, cost, etc.). Then each candidate solution is evaluated for acceptance based on other criteria (like margin of safety).**

- **Advantages**
  - **Global minimum is guaranteed**
  - **Manufacturable design may be enforced**
  - **May link required properties**
  - **Optimization is independent of margin checks**
- **Disadvantages**
  - **Scaling issues for large design spaces, analysis time**
  - **User required to set the design space boundaries**

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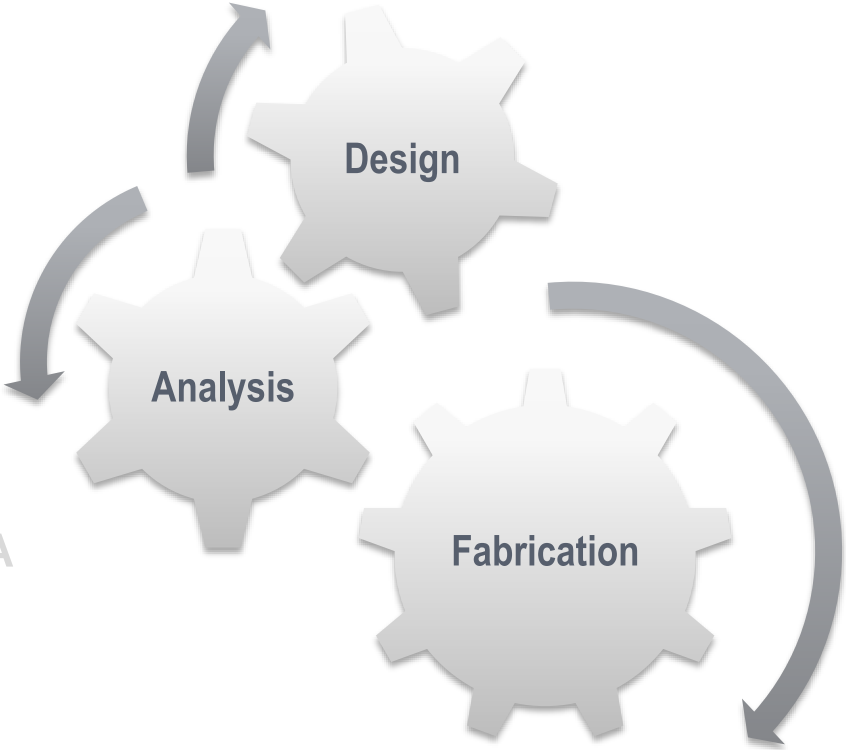
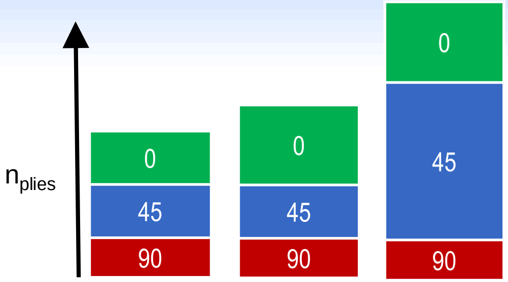
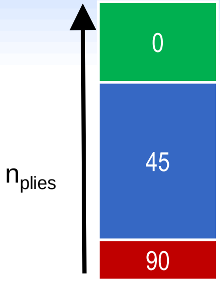


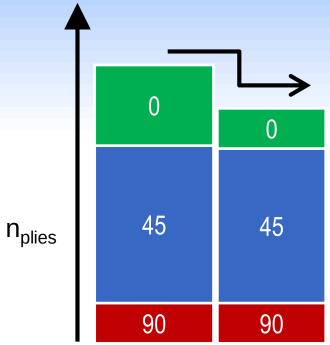
Continuous vs. Discrete Sizing with Composites



Smeared Composite Laminates, Permits Continuous Sizing



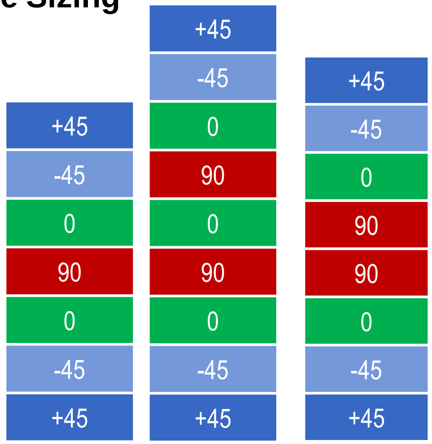
Continuous Sizing Returns Optimal Ply Counts



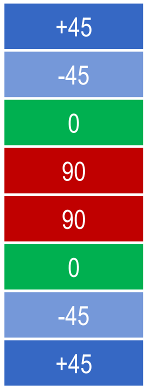
Enforce Compatibility by Matching Counts for Adjacent Sections (Blending)

Continuous Sizing

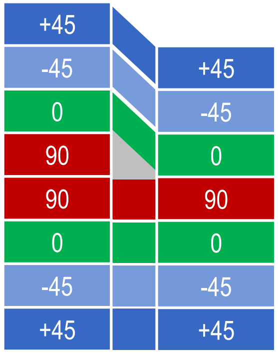
Discrete Sizing



Discrete Laminates, Ply thickness and Orientation Explicitly Defined



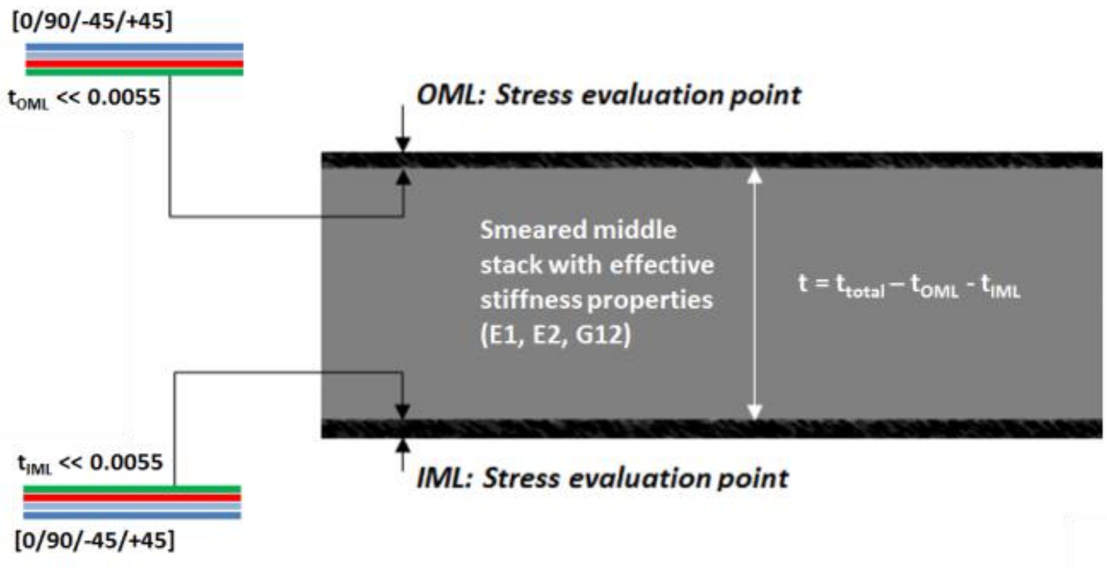
Discrete Sizing Returns Accurate ply-by-ply laminate definition



Enforce Manufacturability by Sequencing the “Global Plies”

Analyzing Smeared Laminates

- Smeared middle stack used to get effective stiffness properties
- Very thin plies defined at IML/OML used to quantify margins of safety.
- Ply allowables used
- Limitation – bending stiffness terms, Dij are approximate



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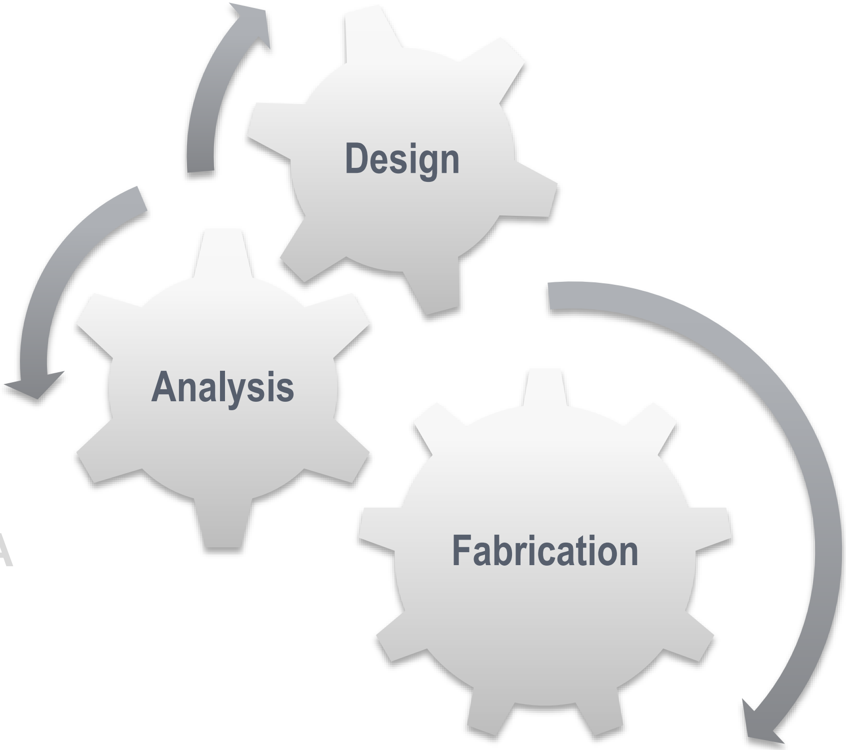
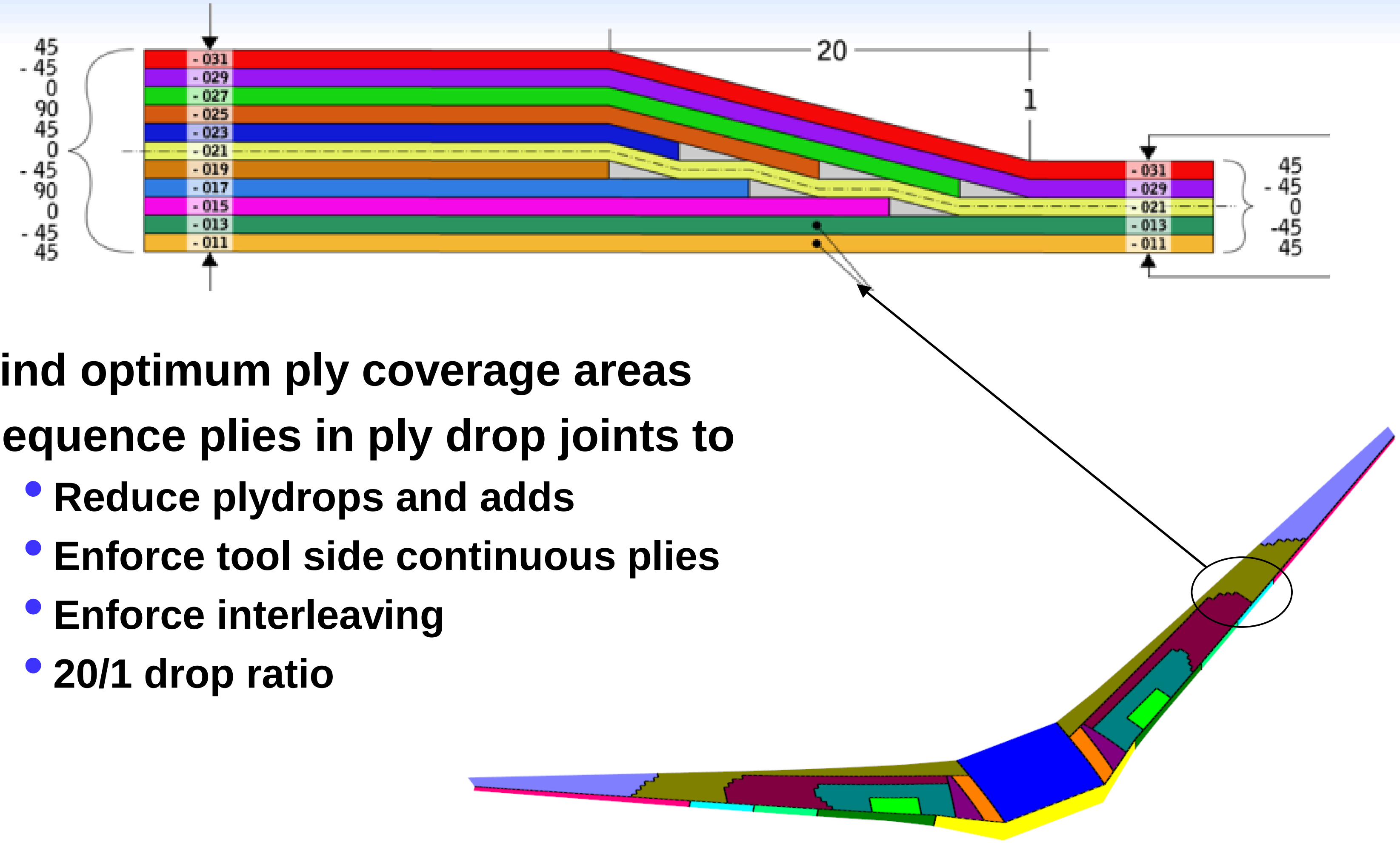


Composite Fabrication Requirements



- **Find optimum ply coverage areas**
- **Sequence plies in ply drop joints to**
  - **Reduce plydrops and adds**
  - **Enforce tool side continuous plies**
  - **Enforce interleaving**
  - **20/1 drop ratio**

Want to Know More?

- Come by our booth: Y112
- Visit our Website: HyperSizer.com

- Contact Information

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  James.Ainsworth@HyperSizer.com