PROBABILISTIC DESIGN OF AEROSPACE VEHICLES:
COUPLING GLOBAL AND LOCAL REQUIREMENTS

Natasha Smith1 and Sankaran Mahadevan2
Department of Civil Engineering, Vanderbilt University, Nashville, TN 37235

ABSTRACT

This paper presents a probabilistic optimization methodology for conceptual design of aerospace vehicles that takes into account linkages between global and local design requirements. Multiple disciplinary analyses such as geometry, weights, structures, aerodynamics, trajectory, propulsion, thermal protection, operations and maintainability etc. are involved in the overall conceptual design. The global design considered in this paper optimizes the geometry for minimum weight while satisfying aerodynamic constraints. The local design illustrated here relates to structural sizing of vehicle components, e.g., liquid hydrogen tank. The optimization formulation includes probabilistic constraints, which are evaluated using the limit state-based reliability analysis methodology. The global and local designs are linked through probabilistic data flow relating to vehicle geometry and component weight, and the optimization at both levels is achieved through an iterative process.

INTRODUCTION

Design of an aerospace vehicle is a complex process requiring analysis and optimization across multiple disciplines. In many cases, relatively mature (high and low fidelity) disciplinary analysis tools are available. These disciplinary analyses cannot be taken in isolation since they are coupled to one another through shared input and output. Furthermore, system design objectives and constraints may span several disciplines. Thus, a design process needs to address issues relating to the coupled multidisciplinary system. Integrating disciplinary analyses into a multidisciplinary framework and finding practical ways to solve system optimization problems is a serious challenge.

Aerospace vehicle design optimization to date has been based by and large on a deterministic approach. Input variables are assumed to be non-varying and the system is assumed to behave exactly as an analysis model predicts. In this case, the analysis output will be deterministic, and there will not be any uncertainty-based metric for assessing risk. A probabilistic approach, on the other hand, allows for random variation in the input variables and can also consider the error between model predictions and true system behavior. In this case, the output will also have random variation. This random variation is characterized by a probability distribution function (PDF) defined by its shape and a set of distribution parameters (e.g. mean and standard deviation). Risk objectives can then be defined in terms of this output uncertainty\(^1\). A probabilistic optimization will then characterize uncertain objectives and constraints in terms of probability distribution parameters and statistics (e.g. minimize \(\mu_z\) subject to \(P(x_1 < x < x_2) \leq p_f\), where \(\mu_z\) is the mean of output \(z\) and \(p_f\) is some acceptable probability of failure for \(x\) to be in the interval \([x_1, x_2]\)).

In this paper, probabilistic optimization methods are demonstrated for the design of an aerospace vehicle at two levels: a global geometry design and a component structural sizing application. The global geometry application is inter-disciplinary in that it considers a coupled analysis of geometry, weights, and aerodynamic disciplines. It is also a system-level design in terms of physical architecture, in that the geometry design variables define the global characteristics (length, radius, wing areas, etc.) of a vehicle comprised of many component parts (wings, fuel tanks, engines, etc.). The component sizing application involves a single disciplinary analysis, at the component level (in terms of physical architecture),

---

1 PhD Student, AIAA Student Member
2 Professor, AIAA Senior Member. Phone: 615-322-3040.
Email: sankaran.mahadevan@vanderbilt.edu
Copyright © 2003 by S. Mahadevan. Published by American Institute of Aeronautics and Astronautics

Copyright © 2003 by Sankaran Mahadevan. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission.
analyzed in terms of multiple limit states. As these two design processes are intrinsically linked, this paper also examines the iterative coupling process from the global to local design and vice versa given uncertainties in system parameters.

GLOBAL GEOMETRY DESIGN

As a first effort in the RLV multi-disciplinary design analysis, low fidelity second-order response surface models have been developed for a deterministic sizing analysis of a wing-body, single stage-to-orbit vehicle. For this application, a launch vehicle is sized to deliver 25,000 lb in payload from the Kennedy Space Center to the International Space Station. The vehicle geometry, for illustration purposes, is shown in Fig. 1, and has a slender, round fuselage and a clipped delta wing. Elevons provide aerodynamic and pitch control. Vertical tip fins provide directional control and body flaps provide additional pitch control.

Figure 1: Illustrative Vehicle Geometry Concept

As a first step in the conceptual design, two disciplines (weights/sizing, and aerodynamics) are considered in a constrained optimization problem. A vehicle geometry that minimizes mean dry weight is expected to minimize overall cost, so this is chosen as the objective function. For stability, the pitching moment \( C_m \) for the vehicle should be zero or extremely close to zero. In addition, \( C_m \) should decrease as the angle of attack increases. This is achieved by adjusting the control surfaces trim the vehicle as the angle of attack is increased. Thus the aerodynamic analysis for pitching moment constrains the optimization.

The optimal vehicle design is determined by six design variables: fineness ratio (fuselage length / radius), wing area ratio (wing area / radius\(^2\)), tip fin area ratio (tip fin area / radius\(^2\)), body flap area ratio (body flap area / radius\(^2\)), ballast weight fraction (ballast weight / vehicle weight), and mass ratio (gross lift-off weight / burnout weight). For the aerodynamic part of the analysis, three additional variables are required to describe the adjustment of control surfaces in order to trim the vehicle: angle of attack, elevon deflection, and body flap deflection. The pitching moment constraint must hold during all flight conditions; nine flight scenarios (constructed with three velocity levels and three angles of attack) are used as a representative sample. The representative velocities (Mach 0.3, Mach 2, and Mach 10) were selected as those originally used in Unal, et al. for which response surfaces have already been generated. The deterministic optimization problem may then be written as follows: Minimize vehicle dry weight \( W \) such that the pitching moment coefficient \( C_m \) for each of 9 scenarios is within acceptable bounds \([-0.01, +0.01]\). This is a multidisciplinary problem requiring the synthesis of information from three analysis codes: a geometry-scaling algorithm, a weights and sizing code (e.g., CONSIZ) and an aerodynamic code (e.g., APAS).

The problem is reformulated in probabilistic terms as follows: Minimize mean weight such that the pitching moment coefficient for all 9 scenarios has a low probability (less than 0.1) of failing to be within the acceptable bounds \([-0.01, +0.01]\). Note here that the output parameters (weight \( W \), and pitching moment \( C_m \)) are random variables. They cannot be known exactly since the inputs and analysis model, on which they are based, are subject to uncertainty. Therefore minimizing the mean weight approximates the weight minimization, and the pitching moment constraint is estimated as a probability of failure to be within acceptable bounds. The solution of this revised, probabilistic problem requires characterization of system uncertainties, defining the limit states for failure, probabilistic analysis, and finally optimization.

System uncertainty comes from various sources and may be modeled through probability density functions for input parameters that are treated as random variables. For example, for the design parameters mentioned above, the as-built conditions may not exactly the same as the design specifications made at this early conceptual level. Furthermore, uncertainties in operational performance lead to randomness in the control variables. In addition, a model error variable is introduced to capture the uncertainty in the analysis itself. In this paper, it is considered simply as an additional random variable applied to the predicted pitching moment as a percentage. Thus, the output pitching moment coefficient is

\[
C_m = C_m\text{(pred)}(1+\text{error})
\]

where \( C_m\text{(pred)} \) is the pitching moment predicted from the response surfaces. In this application, each input variable is assumed to have normal distributions, a simplifying assumption to facilitate analysis. However, any other distribution can be easily included within the probabilistic framework used here. Model error
The constraints formulated above in probabilistic terms may be evaluated using limit state-based reliability analysis. A limit state defines the boundary between success and failure in satisfying a constraint. In this problem, each of the 9 scenarios has two limit states, one for the lower bound and one for the upper bound of \( C_m \) (pitching moment coefficient). The lower bound limit state is
\[
g_{\text{lower}} = 0.01 + C_m,
\]
and the upper bound limit state is
\[
g_{\text{upper}} = 0.01 - C_m
\]

The probability of failure is defined as
\[
P_f = P(C_m < -0.01) + P(C_m > 0.01)
\]  
\[
= P(g_{\text{lower}} < 0) + P(g_{\text{upper}} < 0)
\]

The probability of failure for each limit state is the volume integral under the joint probability density function of all the input variables over the failure region (i.e. where \( g < 0 \)) as shown in Figure 2. A first order reliability method or FORM is used to approximate this integral. Finally, a gradient-based non-linear optimizer is used to find the minimal mean weight given the probabilistic pitching moment constraints.

Using the probabilistic optimization process described above, solving the following formulation:

\[
\text{Minimize mean empty weight}
\]
\[
\text{Subject to } P(-0.01 \leq C_m \leq 0.01) \leq 10\%
\]

finds the best acceptable geometry, yielding an optimal mean empty weight for the vehicle (213,300 lb in this case).

**LAUNCH VEHICLE GLOBAL–LOCAL COUPLING**

A launch vehicle is comprised of many components (Fig. 3). Each component must be designed to successfully perform its individual function, but must also integrate or ‘fit’ into the system as a whole. Consider the component design of the liquid hydrogen (LH\(_2\)) fuel tank for the sake of illustration. The design parameters of the tank include the shape, dimensions, and location of the tank as well as the material make-up of the tank walls. The primary operational requirement for the tank is that it must be large enough to hold the fuel necessary to complete the mission of the vehicle. At the same time, the tank must not occupy the same space as other components and must be strong enough to resist loading induced by the entire vehicle. The global system design drives both the fuel requirements (a function of the shape and weight of the vehicle) and the induced loading (a function of the weight distribution of the vehicle).

Of the three elements of tank design (location, shape, and wall material), this analysis focuses on the tank wall material. Trade studies have been conducted addressing optimal shapes and positioning for fuel tanks in the X-33 lifting body configuration. However, stability considerations for the more basic slender body concept dictate that the two fuel tanks (liquid oxygen and LH\(_2\)), which comprise the bulk of the weight of the loaded vehicle, be at either end of the launch vehicle. In addition, the slender body vehicle concept lends itself to cylindrical tank geometry so that only the tank’s length and radius are needed. Furthermore, the volume is obviously dictated by fuel requirements, and the tank radius is limited by the RLV geometry, both of which are products of the global design. For the scope of the following analysis, the LH\(_2\) tank is assumed to be a typical cylindrical tank with given end eccentricity, located at a fixed distance from the end of the vehicle. With tank geometry and location driven by requirements, the final considerations for design are the material and structure of the tank walls. In this respect, the tank should be sized such that it is as light as possible but strong enough to resist stresses induced by inertial loads.
The design goal for the liquid hydrogen tank is to minimize the weight of the tank while meeting the requirements for fuel capacity and structural integrity. The fuel capacity requirement is maintained by choosing the appropriate tank geometry. With tank geometry dictated by the global design, an optimization problem may be formulated to select the best design for the tank wall structure:

\[
\text{Minimize Tank Weight} = f(R)
\]

Subject to

\[
R - S < 0 \quad \text{or} \quad \frac{R}{S} - 1 \leq 0 \quad \{\text{for all failure modes}\}
\]

where \( R \) is the tank resistance and \( S \) is the loading on the tank. The problem is re-formulated to consider uncertainties in \( R \) and \( S \):

\[
\text{Minimize } \mu_{\text{tank weight}} = f(f_R(R)) \approx f(\mu_R)
\]

Subject to

\[
P\left( \bigcup_{\text{all failure modes}} R - S \right) < P_{\text{required}}
\]

This optimization formulation recognizes that the objective (tank weight) and constraints (failure limit states) are random variables. For well-defined optimization, objectives and constraints need to be selected from among the parameters that characterize the random distributions of these variables. In this case, the parameter mean tank weight is selected as the objective, and the probability of system failure is chosen as the constraint.

Thus, evaluating \( P\left( \bigcup_{\text{all failure modes}} R - S \right) \) involves five subtasks: (1) defining an analytical model for the system loading, \( S \), based on information from the global analysis and the mission profile, (2) defining analytical models for various failure modes that incorporate the loading model and resistance, \( R \), in terms of design variables (3) quantifying the uncertainty in the inputs to the failure model, (4) using probabilistic methods to evaluate the failure probability of the individual limit states, and (5) using system probability methods to estimate the union of several limit states.

For the first subtask, system load calculations are based on a simple beam model as depicted in Fig. 4, for the sake of illustration. (The analysis calculates running loads, \( N_x \), \( N_y \), and \( N_{xy} \), nominally stress divided by thickness.) The model accounts for component weights as uniform distributed loads under axial and normal accelerations. In addition, fuel tanks experience ullage and head pressures. Reaction point locations (R1 and R2) represent lift, drag, wheel reactions, and/or the launching structure reactions depending on the mission stage (i.e. lift-off, ascent, landing, etc.).

However, since weight is of paramount importance, an isotropic, thin-walled tank is not the ideal design. Instead more complex concepts such as honeycomb sandwiches and blade-stiffened panels are considered more appropriate. In addition, an efficient wall design would have varying resistance properties according to the loading profile. This requires a more sophisticated approach for estimating tank resistance properties and analyzing failure modes. The material management and structural sizing software, Hypersizer™, is useful for this purpose. There are multiple modes of failure for the tank (i.e. Von Mises interaction failure, isotropic failure, panel buckling), multiple locations along the tank that could fail, and even multiple load cases (at various stages in the vehicle trajectory) that could cause failure. Each of these failure cases may be represented by a corresponding limit state. However, the overall reliability measure for the tank is the system failure probability, which synthesizes all of these modes.
For the second subtask, there are multiple failure criteria (e.g., strength, panel buckling, etc.) that may be used to predict structural failure, and infinite locations on the component that may fail. Each of these cases is associated with a failure limit state. In addition, since loading varies with the flight profile, there are an infinite number of failure limit states associated with flight conditions. System failure occurs if any part of the tank fails at any time during the mission based on any of the failure modes. Thus system failure may be modeled as a series or union of an infinite number of limit states. Obviously, only a finite number of failure limit states can be analyzed; this can be achieved by dividing the continuous spaces (e.g., tank location, flight profile) into finite regions. This concept is demonstrated in the following section through a multi-mode failure model of the system. This model synthesizes three failure modes (i.e. failure according to three different criteria) for a honeycomb sandwich wall tank. The three limit states (corresponding to three failure modes) are chosen for the sake of demonstration. Additional limit states would need to be considered in practice, adding computational expense without changing the basic methodology.

In this example, a segmented tank is considered. It is divided into 10 panels along the length and 4 panels along the circumference (Fig. 5). Since the maximum load varies from panel to panel, they need not be identically designed. Loading on the panels is obtained from the simple beam model. In the analysis that follows, the side panel located close to the rear of the RLV is designed. Hypersizer™ includes a database of properties for a number of isotropic and composite materials, and it is able to consider several panel concepts. This demonstration utilizes a honeycomb sandwich concept consisting of top and bottom plates of Aluminum, AL2024 and Hexcell 1/8”-5052-.0015 for the sandwich material.

Design of the panels must specify the thickness of the plates and sandwich. Given the panel properties and applied loads, Hypersizer™ then evaluates the margins of safety for a number of failure criteria. For the tank walls, the significant failure modes are: isotropic strength in the transverse direction, Von Mises strength, and honeycomb buckling. To facilitate probabilistic optimization, response surfaces for three failure modes (Von Mises, isotropic strength, and honeycomb buckling) were developed from a design of Hypersizer™ experiments. (Equations for Von Mises and isotropic strength failure are known, so that the Hypersizer™ analysis is only needed to verify the effective yield stress for the panel material.)

![Figure 5: Segmented Honeycomb-Wall Tank](image)

The next subtask is to model system uncertainty. As seen in the structural analysis, the system loading is a function of several variables. The system resistance variable(s) constitute the design variable(s). For the segmented tank, the honeycomb thickness ($t_{hc}$) is an additional resistance variable, and plate thickness ($t_{plate}$) is the design variable. All of these have a degree of uncertainty that affect the uncertainty of the calculated running loads. However, variables with an insignificant affect on the output variability are assumed to be deterministic (i.e. constant). The variables are summarized in Table 1 below:

<table>
<thead>
<tr>
<th>Table 1: Input Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$x_1$</td>
</tr>
<tr>
<td>$x_2$</td>
</tr>
<tr>
<td>$x_3$</td>
</tr>
<tr>
<td>$x_4$</td>
</tr>
<tr>
<td>$x_5$</td>
</tr>
<tr>
<td>$x_6$</td>
</tr>
<tr>
<td>$x_7$</td>
</tr>
<tr>
<td>$x_8$</td>
</tr>
<tr>
<td>$x_9$</td>
</tr>
<tr>
<td>$t_{plate}$</td>
</tr>
<tr>
<td>$t_{hc}$</td>
</tr>
<tr>
<td>$c_1$</td>
</tr>
<tr>
<td>$c_2$</td>
</tr>
<tr>
<td>$c_3$</td>
</tr>
<tr>
<td>$c_4$</td>
</tr>
</tbody>
</table>
The first 6 variables in Table 1 are determined by the mission profile for the launch vehicle. They will vary along the flight trajectory. The global variables are obtained from a Monte Carlo analysis of the RLV global design response surfaces as summarized below:

- **x7**: Radius = Fuselage Fineness Ratio * overall length (oal)

Fuselage fineness (fr) is an optimization variable for the global design. Response surfaces for overall length (c1: oal) were available from the weights design of experiments previously.

- **x8**: Fuel Weight = Gross lift-off weight – empty weight – cargo weight

Response surfaces for gross lift-off weight and empty weight come from the weights design of experiments. The cargo weight is 25,000 lb as given by mission requirements. Unfortunately, the response surfaces do not give enough detail on component weights to develop the weight distribution needed for the tank structural analysis. Instead, the parametric estimates below were based on the weight distributions for the structure, wing, and engine of legacy vehicles. As components are designed in detail, this information may be updated to refine the structural analysis.

- c2: Wstructure = 0.5 * empty weight
- c3: Wwing = 0.1 * empty weight
- c4: Wengine = 0.4 * empty weight

For the segmented, honeycomb panel tank, a two part probabilistic analysis accomplishes the fourth subtask in evaluating the probability of structural failure. First, ten thousand Monte Carlo runs were performed (by varying the input variables from Table 1) on the simple beam loading analysis to find the mean and standard deviations of the running loads ($N_x$, $N_y$, and $N_{xy}$). The Kolmogorov-Smirnov test was conducted to assess conformity to a normal distribution type 1. All panel loads followed a normal distribution within a 10% significance level, and most were within 5% significance. FORM was performed (as the second part of the analysis) on the response surfaces of select panels considering $N_x$, $N_y$, and $N_{xy}$ as random variables. The plate thickness and honeycomb thickness were also considered as random design variables.

For the fifth subtask, three failure modes (for a single panel) are considered in series to assess a system failure probability. They are honeycomb buckling, strength failure (Von Mises), and the transverse isotropic strength. (These three limit states are chosen for demonstration purposes, but it should be noted that the true system failure probability is a function of not only multiple failure modes but also multiple failure locations and failure under multiple loading conditions).

Several methods are available for approximating the union or intersection of several events. These include methods developed by Ditlevsen7, Hohenbichler and Rackwitz8, Gollwitzer and Rackwitz9, Madsen et al10 and Xiao and Mahadevan11. In this case, the Hohenbichler approximation9 is used. This method uses the reliability indices, $\beta$, from the individual limit states and a correlation matrix, $\rho$. The correlation matrix is defined as: $\rho_{ij} = \alpha_i \alpha_j \cdot $, where $\alpha_i$ and $\alpha_j$ are the direction cosine vectors such that $\alpha_i = \frac{\nabla g_i}{|\nabla g_i|}$. The Hohenbichler methodology will not be explained in detail here. However, the basic premise is to use an orthogonalization procedure to transform a set of limit states to a set of statistically independent limit states. Then, the probability of failure calculation is simply:

$$P_{fs} = P_{f_1} \ast P_{f_2} \ast P_{f_3} = \Phi(\beta_i) \ast \Phi(\beta_2) \ast \Phi(\beta_3),$$

where $\beta_i$, $\beta_2$, and $\beta_3$ are the reliability indices for the transformed (i.e. statistically independent) limit states.

The problem statement may now be refined as stated below:

Minimize $\mu_{plate}$

Subject to

$$P_{\beta} = P\{g_{VM} < 0 \cup (g_{\text{los}} < 0) \cup (g_{\text{HCB}} < 0)\} \leq P_{\text{acceptable}}$$

(for 3-mode failure of a single segmented honeycomb panel). As with the global design problem, the optimization was performed using a gradient based non-linear optimizer. The optimal plate thickness is plotted against the acceptable system failure probabilities in Figure 6. In addition, the relative contribution of each of the limit states is depicted in Figure 7. It can be seen from this graph that the strength (Von Mises) and isotropic failure modes dominate the design requirements for this panel.
LOCAL-GLOBAL FEEDBACK

Just as the local analysis is affected by the global design, the global analysis is impacted by the refinement of local design. In this case, the local optimization of the liquid hydrogen tank refines or updates the weight estimation relationships for the vehicle. The weight estimates from the low-fidelity global analysis are now replaced by detailed component information after the latter are designed for structural reliability. This new information needs to be re-analyzed in the context of the global multidisciplinary design, which in turn updates information at the local level. This process of passing information between global and local levels continues until acceptable
convergence is reached. This happens with increasing levels of fidelity in effect spiraling between analyses from a conceptual to detailed design.

After the local tank sizing analysis is performed, tank weight may be easily assessed. The original global design was based on the sizing program (CONSIZ) using parametric estimates for various component weights based on historical information. However, the tank weight at the global design point (i.e. the mean values of the global geometry design parameters), gave a predicted tank weight (16,400 lb) that was significantly lower than given by the local tank sizing optimization based on structural reliability (21,300 lb). Therefore the global weight estimating relationship is updated to be compatible with the local design. Given this, a new design of experiments is performed to update the global response surfaces for the vehicle weight and pitching moment. The global optimization is then performed a second time, giving a new optimal vehicle empty weight of 223,100 lb (up from 213,300 lb obtained previously). These changes in the global design will affect the local sizing optimization in the same manner previously described. Of course the global design would also be affected by the local design of other components. Thus the effects of multiple components on each other may be transferred through the global design, and the final design at both global and local levels obtained through convergence at the end of an iterative process.

CONCLUSIONS

This paper has presented and applied a methodology for probabilistic multidisciplinary optimization for aerospace vehicle design. The combination of response surfaces and first order reliability analysis provides a valuable tool for both global conceptual design and low-fidelity component design. In addition, the coupled nature of these analyses causes the flow of probabilistic design information from a global, multidisciplinary analysis to a local, single discipline analysis. Geometric variables (fuselage fineness, overall length), established through the global design and outputs from the global weight response surfaces were used to determine the vehicle loading distribution for the local tank sizing analysis. The tank weight established from the local analysis is then used to update the global weight estimating relationships, allowing the global design to be refined or updated. This process of passing information between global and local levels continues, involving additional component analyses until acceptable convergence is reached and appropriate design confidence is established.

In addition to the global to local coupling, this tank sizing application demonstrates a probabilistic optimization formulation for system failure of multiple limit states. For a fuel tank, structural failure may be caused by many possible failure modes, several locations, and a load history that varies according to the stage of flight. In this analysis, only three modes are considered for demonstration. In reality, a large number of failure limit states are applicable. Then, a technique such as branch and bound enumeration may be appropriate to identify the failure modes of most significance before system reliability methods are applied.

ACKNOWLEDGEMENT

The research reported in this paper was supported by funds from NASA Langley Research Center (LaRC), VA (Cooperative Agreement NCC 1-01031) under the direction of the Multidisciplinary Optimization Branch (Dr. Thomas A. Zang). Roger Lepsch and Jeff Cerro from the NASA LaRC Vehicle Analysis Branch provided deterministic optimization data and code as well as valuable technical discussion. In addition, ICASE‡ provided internship support during the summer of 2002. The support is gratefully acknowledged.

REFERENCES


‡ ICASE is operated at the NASA Langley Research Center (LaRC) by the Universities Space Research Association (USRA) under a contract with the Center


