HyperSizer User Manual

Tapered Tube

COLLIER RESEARCH CORPORATION

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> This User Manual is intended for users that are familiar with HyperSizer Basic. Users should be comfortable with setting up running their own workspace analyses.

Introduction

Figure 1 shows a schematic of the tapered tube concept as implemented in HyperSizer. The beam is divided into three sections: joint, taper, and midspan. In general, HyperSizer sizes the taper and midspan sections while the joint section is defined as a constant (not sized).

At the time of this writing, the buckling and beam-column loading of the tapered beam concept is only supported for simple-simple boundary conditions.

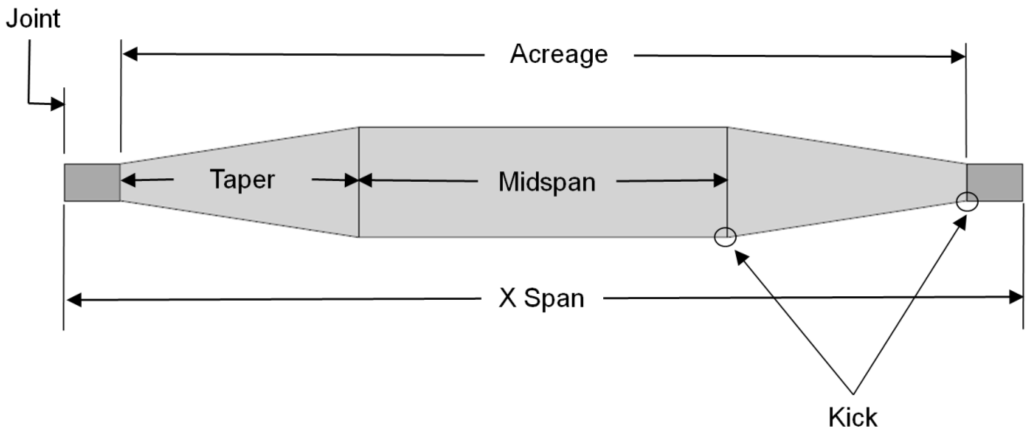


Figure 1 - Schematic of the tapered beam concept.

Midspan Section

The midspan section the is prismatic section of the beam defined in same way as the circular tube concept: thickness, thickness material, and outer diameter.

Tapered Section

Special considerations are used when defining the tapered section. First, the same material is used in both the midspan the tapered section. Additionally, the cross-sectional area throughout the taper *is assumed to be constant and equal to the cross-sectional area of the midspan section*. One implication of this restriction is that the weight per unit length of the midspan is equivalent to that of the tapered section.

The taper angle α is defined with respect to the IML as shown in Figure 2. Taper angle must be greater than zero and less than or equal to 90 degrees. Note that a taper angle of 90 degrees refers to a beam without a tapered section and can be used to model a stepped beam.

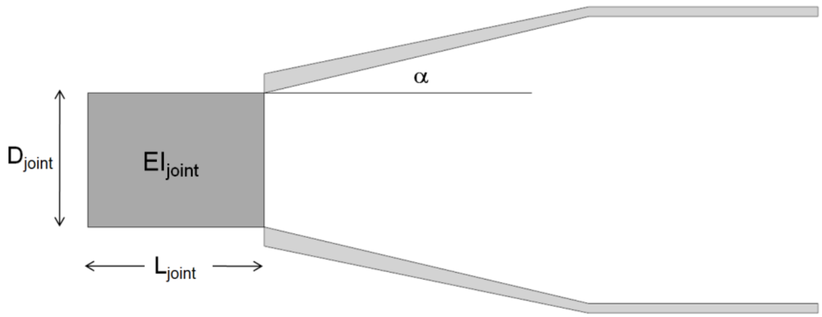


Figure 2 – Close up view of the joint-taper interface. Note that both joint and taper angle α are defined with respect to the IML.

Joint Section

As mentioned before, the joint section is not sized within HyperSizer. Instead, the joint parameters are defined as static group-level constants visible in the Options tab of the Sizing Form. Group-level means that each component in a group will have the same joint parameters.

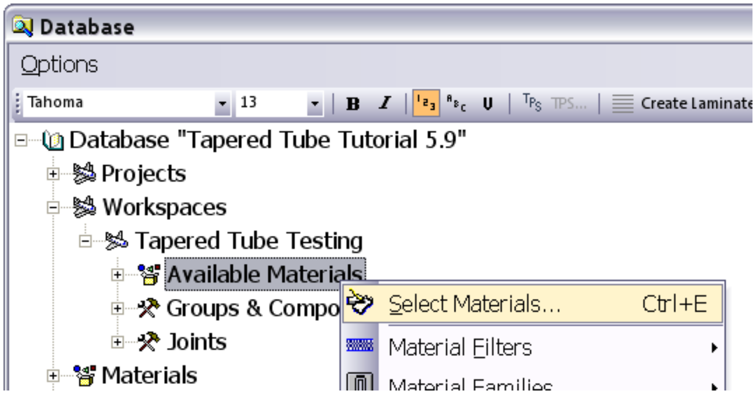
The joint diameter and joint length parameters serve to define the acceptable geometry for a tapered beam. The tapered section is defined as meeting the joint diameter at the IML of the taper.

The diameter must be a positive number. The length of the joint can be zero if no joint section is to be modeled.

Setting up the Workspace

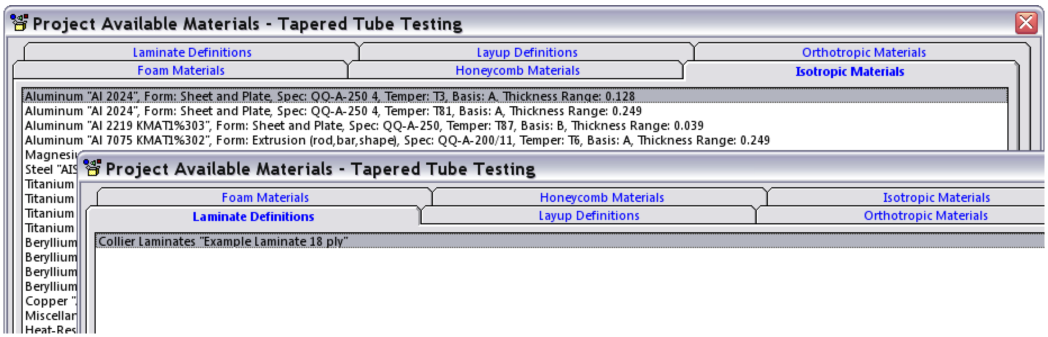
Start by creating a new HyperSizer database of version 5.9 or greater.

Create a new workspace and name it "**Tapered Tube Testing**."



Select materials for the workspace by right-clicking the "Available Materials" entry.

We will be using two materials: "**Al 2024 (T3)**" and "**Example Laminate 18 ply**"

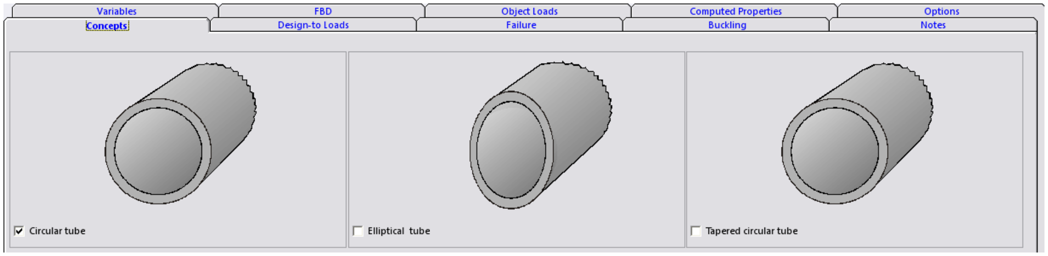


Aluminum Prismatic Tube Example

Create a new group in the **Circular Beam Family.** Number the group **301,** and add component **301** to the group.

Rename both the group and component to **Al Prismatic Tube.** Ignore the error message.

Loop in the Concepts tab and notice the new Tapered Circular Tube concept. For this example we will only use the default **Circular Tube** concept.



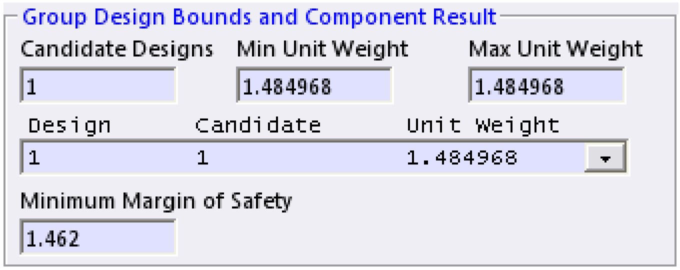
Enter the following group sizing variables. Tube width is ignored because the cross-section is circular. Likewise taper angle is ignored because we are only looking at the circular beam concept.

Parameter	Min	Max	Permutations
Tube - Thickness Material	Al 2024		
Tube - Thickness	0.1	0.1	1
Tube – Height	4.0	4.0	1

Enter the following component variables.

Parameter	Value
Axial Load - Strength	-10,000
Axial Load – Buckling	-10,000
X Span	100
Mechanical Ultimate Factor	1.0

Analyze the component. If everything has been entered properly you should see the following in Component Result Box.



The controlling margin of safety is column-buckling = 1.462. Let's verify this result.

First, calculate the relevant stiffness parameters. Both the bending stiffness and moment of inertia can be verified in the Computed Properties tab.

E_c = 10.7 Msi
$I = \pi (R_{avg})^3 t = \pi * (1.95)^3 * 0.1 = 2.33 \text{ in}^4$
$EI = 2.49 \times 10^7$ lbf in^2

The solution for buckling of a simply-supported prismatic column is defined by the expression.

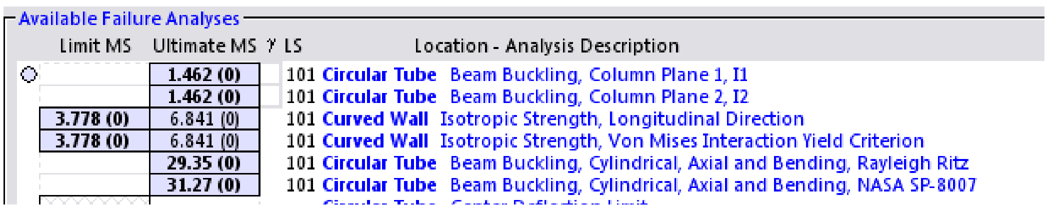
$$P_{critical} = \frac{\pi^2 EI}{L^2}$$

Solving for the critical load by hand:

$P_{critical} = \pi^2 * 2.49 \times 10^7 / (100^2) = 24.6$ kips

MS = $(P_{cr}/P) - 1$ = **1.462**

This matches the HyperSizer result.



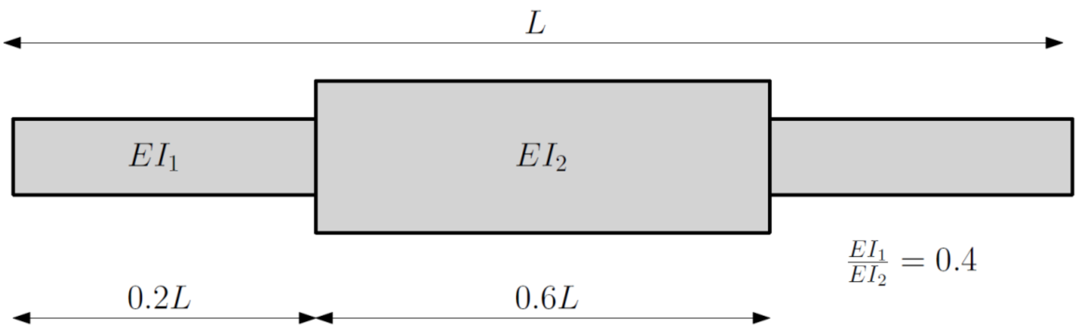
Store both the **Group and Component Designs** for the next steps.

Stepped Aluminum Tube Example

Create a new group in the **Circular Beam Family.** Number the group **302,** and add component **302** to the group.

Rename both the group and component to **Al Stepped Tube.** Ignore the error message.

This component will recreate an example problem found in the classic text, Theory of Elastic Stability (Timoshenko, 1961).



Retrieve the Group and Component Designs from the previous example.

Go to the Concepts tab. Deactivate the Circular Tube concept and activate the **Tapered Circular Tube concept**.

Enter the following group sizing variables. If the Group Design was retrieved properly, you should only need to change the taper angle.

Parameter	Min	Max	Permutations
Tube - Thickness Material	Al 2024	N/a	N/a
Tube - Thickness	0.1	0.1	1
Tube – Height	4.0	4.0	1
Taper - Angle	90.0	90.0	1

Enter the following component variables. They are the exactly the same as the previous example. If the Component Design was retrieved properly, no changes need to be made.

Parameter	Value
Axial Load - Strength	-10,000
Axial Load – Buckling	-10,000
X Span	100
Mechanical Ultimate Factor	1.0

Analyze the component. You should get the same results as the previous example. The taper has no effect because the taper angle is 90°. The joint has no effect because its default length is zero.

Go to the **Options** tab and look at the joint parameters.

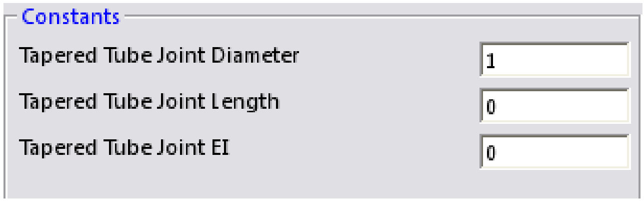
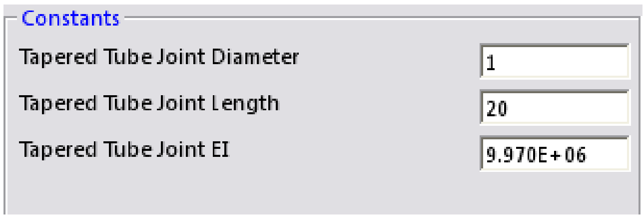


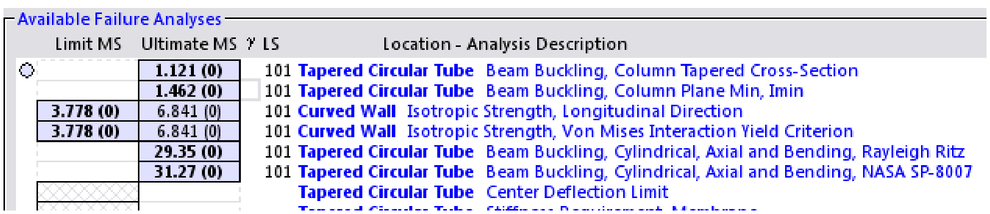
Figure 3 - Default joint parameters. The joint is deactivated if L = 0.

Change the joint parameters to the following values.



Note that the tapered tube joint EI is 0.4 * EI_{mid} and the joint length is 0.2 * L. The diameter is inconsequential because the taper angle is 90°.

Solving the component again we find that the margin of safety for buckling is now 1.121. We can also see the original non-tapered solution of 1.462.



Let's verify that this result matches the Timoshenko result (Article 2.14).

Timoshenko:

$P_{critical}$ = **8.51** EI_{mid} / L^2

HyperSizer:

$P_{critical}$ = (1.121 + 1)*10,000 = 21,210

$P_{critical} * L^2 / EI_{mid}$ = 21,210 * 100^2 / 2.49 x 10^7 = **8.51**

HyperSizer uses the method of successive approximations to solve for buckling loads for columns of varying cross-section as described in Timoshenko Art. 2.15. The same method is also found in Niu Chapter 10.4 (1997).

Store the Group and Component Designs for the next example.

Composite Tapered Tube Example

Now we will try fully tapered tube example using composites.

Create a new group in the **Circular Beam Family.** Number the group **303,** and add component **303** to the group.

Rename both the group and component to **Composite Tapered Tube.** Ignore the error message.

Retrieve the Group and Component Designs from the previous example.

Set the following sizing variables.

Parameter	Min	Max	Permutations
Tube - Thickness Material	Example Laminate 18 ply		
Tube - Thickness	(0.099)	(0.099)	1
Tube – Height	8.0	8.0	1
Taper - Angle	20.0	20.0	1

Set the following component variables.

Parameter	Value
Axial Load - Strength	**-40,000**
Axial Load – Buckling	**-40,000**
X Span	100
Mechanical Ultimate Factor	1.0

Set the following joint parameters (Options tab). Note that these variables are "group-level." Therefore they are stored with the Group Design.

Parameter	Value
Taper Tube Joint Diameter	3.5
Tapered Tube Joint Length	5
Tapered Tuber Joint EI	9.97e6

Analyze the component. You should get the following margins (activate the Sort MS checkbox). The unit weight should be **1.680828**.

Available Failure Analyses

Limit MS	Ultimate MS	LS	Location - Analysis Description
	1.944 (0)	101	Tapered Circular Tube Beam Buckling, Column Tapered Cross-Section
	2.045 (0)	101	Tapered Circular Tube Beam Buckling, Column Plane Min, Imin
	2.713 (0)	101	Curved Wall Composite Strength, Tsai-Wu Interaction
	2.724 (0)	101	Curved Wall Composite Strength, Hoffman Interaction
	2.748 (0)	101	Curved Wall Composite Strength, Max Stress 1 Direction
	2.752 (0)	101	Curved Wall Composite Strength, Tsai-Hill Interaction
	2.756 (0)	101	Curved Wall Composite Strength, Max Strain 1 Direction
	2.821 (0)	101	Curved Wall Composite Strength, Tsai-Hahn Interaction
	3.3 (0)	101	Tapered Circular Tube Beam Buckling, Cylindrical, Axial and Bending, Rayleigh Ritz
	3.497 (0)	101	Tapered Circular Tube Beam Buckling, Cylindrical, Axial and Bending, NASA SP-8007
	3.744 (0)	101	Curved Wall Composite Strength, Max Strain 2 Direction
	4.157 (0)	101	Curved Wall Composite Strength, Max Stress 2 Direction
	4.157 (0)	101	Curved Wall Composite Strength, LaRC03 Matrix Cracking
	6.745 (0)	101	Curved Wall Composite Strength, Max Stress 12 Direction
	6.745 (0)	101	Curved Wall Composite Strength, Max Strain 12 Direction
	15.16 (0)	101	Curved Wall Composite Strength, LaRC03 Fiber Failure
			Tapered Circular Tube Center Deflection Limit

Beam-Column Loading

Beam-column is a term for a long, slender structural member that is loaded both axially (column) and transversely (beam). When axial loads act in isolation only axial displacements occur. However, when lateral loads are applied in conjunction with axial loads, the lateral deflections due to transverse loads cause the axial loads to apply moments of the form M=Py where y is the lateral deflection of the beam. This creates a non-linear loading effect i.e. deflections are no longer directly proportional to the applied loading.

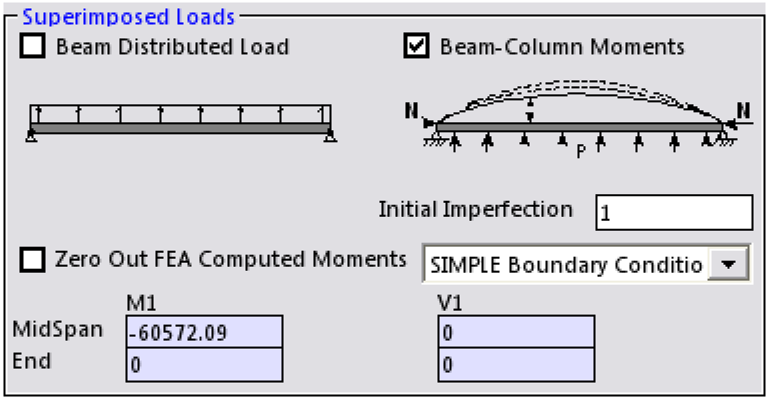
In HyperSizer, beam-column loading is applied in terms of an initial lateral deflection δ_o or initial imperfection. For the special case of the tapered tube a well-known approximation is used to calculate the beam-column moment at the midspan of a simply supported member.

$$M_{midspan} = P\frac{\delta_o}{1 - P/P_{critical}}$$

$P_{critical}$ refers to the Euler buckling load. Here we can see that as P approaches the critical load, the beam-column moments approach infinity for *any* initial deflection. Simple beam theory (small deflections) are used in deriving this expression so caution must be used when calculating results for large deflections.

Keep in mind that if a structure is assumed to behave as a beam-column, traditional Euler buckling will not occur. Buckling by definition involves the rapid transition from an unstable equilibrium state to stable one - bifurcation. If the beam-column has initial curvature (either directly assumed or due to transverse loads) no “Euler” type bifurcation occurs.

Apply a beam-column imperfection of L/100 = 1" using the FBD tab of the Sizing Form. First **click the Beam-Column Moments checkbox**, and then type in an initial imperfection of **1**. Set the boundary conditions to **SIMPLE**.



Analyzing the component again, you will now see the moments have appear in the design-to-loads boxes. Although the moments are reported as being on the end, they are actually accounted for as midspan moments. Let's verify this result.

Free Body Diagram Output (Controlling Factored Loadcase)

Controlling Analysis Load: STRENGTH	M1,κ1 (End A)	M2,κ2 (End A)	M1,κ1 (End B)	M2,κ2 (End B)	V1	V2	Axial,εx	Torque,φ
Virtual Loads								
Design-to Loads	-60572.1	0	-60572.1	0	0	0	-40000	0
Design-to Deformation	-4.907683E-04	0	-4.907683E-04	0			-2.529335E-03	0

First we back out the critical load from the HyperSizer computed margin. This critical load is independent of an beam-column effect because there strictly is no Euler buckling for beam-columns.

$P_{critical}$ = (MS + 1)*P = (1.944 + 1)*40,000 = **117,760 lbf**

$1/(1-P/P_{cr})$ = 1 / (1 – 40,000/117,760) = **1.5144** *(amplification factor)*

M = P * δ_o * 1.51 = 40,000 * 1 * 1.51 = **60,576 lbf in**

Notice that when beam-column moments are activated, strength margins decrease due to the effect of the additional compressive stress due to the applied moment.

Local (Wall) Buckling

Figure 4 shows an example of local buckling in a cylindrical panel. This type of failure can become critical in thin-walled tubes. HyperSizer has two methods of calculating margins for wall buckling: NASA SP-8007 and Rayleigh-Ritz.

NASA SP-8007 is the published method of solving for the cylindrical type buckling. It solves separate margins for buckling due to axial and moment loads. A combined margin is calculated using an interaction equation.

The Rayleigh-Ritz method is a numerical solution that accounts for the directly for the combined effects of axial and moment loads. This method is a good deal slower than SP-8007 so it is usually a good practice to disable Rayleigh-Ritz for initial sizings and turn it back on for detailed sizings.

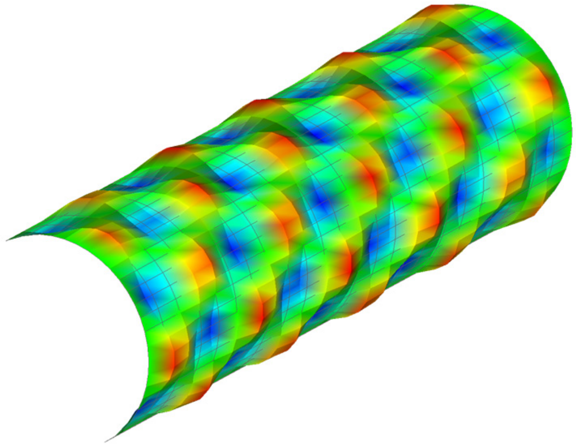


Figure 4 - HyperSizer mode shape of local buckling in a cylindrical panel

With the beam-column analysis enabled, the buckling margins should be the following. Isolate the margins to only show the "Buckling, Beam" category. Note the SP-8007 and Rayleigh Ritz solutions are fairly close. Also note that the column buckling solutions do not change when beam-column effects are introduced.

Available Failure Analyses

Limit MS	Ultimate MS	LS	Location - Analysis Description	
	1.733 (0)	101	Tapered Circular Tube	Beam Buckling, Cylindrical, Axial and Bending, NASA SP-8007
	1.781 (0)	101	Tapered Circular Tube	Beam Buckling, Cylindrical, Axial and Bending, Rayleigh Ritz
	1.944 (0)	101	Tapered Circular Tube	Beam Buckling, Column Tapered Cross-Section
	2.045 (0)	101	Tapered Circular Tube	Beam Buckling, Column Plane Min, Imin

It is well known that cylindrical structures are imperfection sensitive. Many analytical buckling solutions do not account for this sensitivity and thus are very unconservative compared with actual test results. Knockdown factors are derived to reduce to this unconservatism.

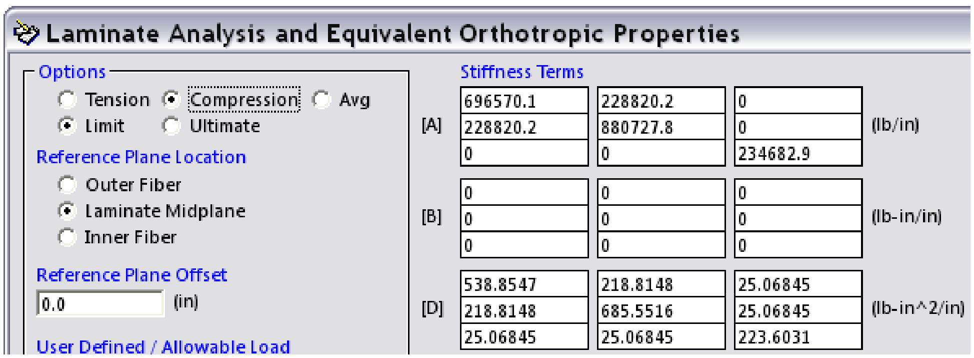
HyperSizer has the option to include SP-8007 derived knockdown factors (denoted as γ in the original paper) based on geometric and stiffness parameters. **By default for wall buckling of tubes, the SP-8007 knockdown factors are turned on.**

Let's verify that the knockdown factors are enabled by looking at the Buckling Detail file. In the sizing form go to **Options – Buckling Detail**. You should see the following text.

```
---------- SP-8007 Knockdown Factors for wall buckling ------------

t_effective               =      .0965
phi                       =      .3999
gamma_axial               =      .7030
gamma_bending             =      .7591
Effective Knockdown       =      .7250
```

Let's verify these calculations. To do so, we need to look at the compressive ABD matrix of the laminate.



For an orthotropic material:

$$t_{eff} = \sqrt{12}\sqrt[4]{\frac{D_{11}D_{22}}{A_{11}A_{22}}}$$

t_{eff} = 3.464 * (538.9 * 218.8 / 696570 / 880728)$^{1/4}$ = **0.0965 in**

φ is defined as:

$$\phi = \frac{1}{16}\left(\frac{r_{average}}{t_{eff}}\right)^{1/2}$$

φ = (1/16) * (3.9505/0.0965)$^{1/2}$ = **0.4**

The gamma factors are defined as:

$$\gamma_{axial} = 1 - 0.901\left(1 - e^{-\phi}\right)$$

$$\gamma_{bending} = 1 - 0.731\left(1 - e^{-\phi}\right)$$

γ_{axial} = 1 – 0.901 * (1 – e$^{-0.4}$) = **0.703**

$\gamma_{bending}$ = 1 – 0.731 * (1 – e$^{-0.4}$) = **0.759**

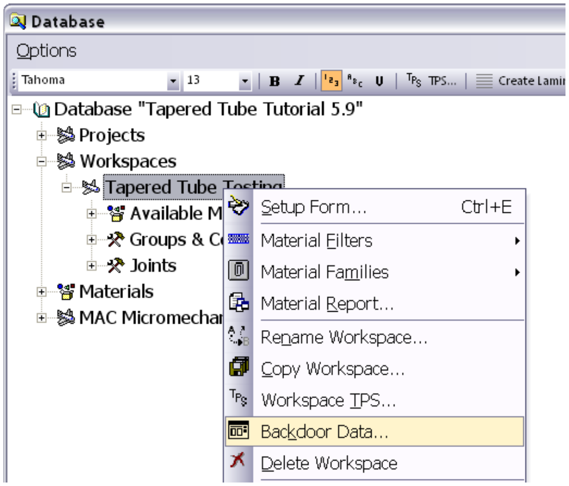
These gamma factors are applied to the SP-8007 solution in the traditional manner. For the Rayleigh Ritz solution these gamma factors are not directly applicable because this method handles axial and bending loads simultaneously. We therefore derive an "effective knockdown" to apply the Rayleigh Ritz solution.

The effective knockdown is derived using the SP-8007 results *with and without* the gamma knockdowns applied.

$$\text{Effective Knockdown} = \frac{FS_{\text{SP-8007 with Gamma}}}{FS_{\text{SP-8007 without Gamma}}}$$

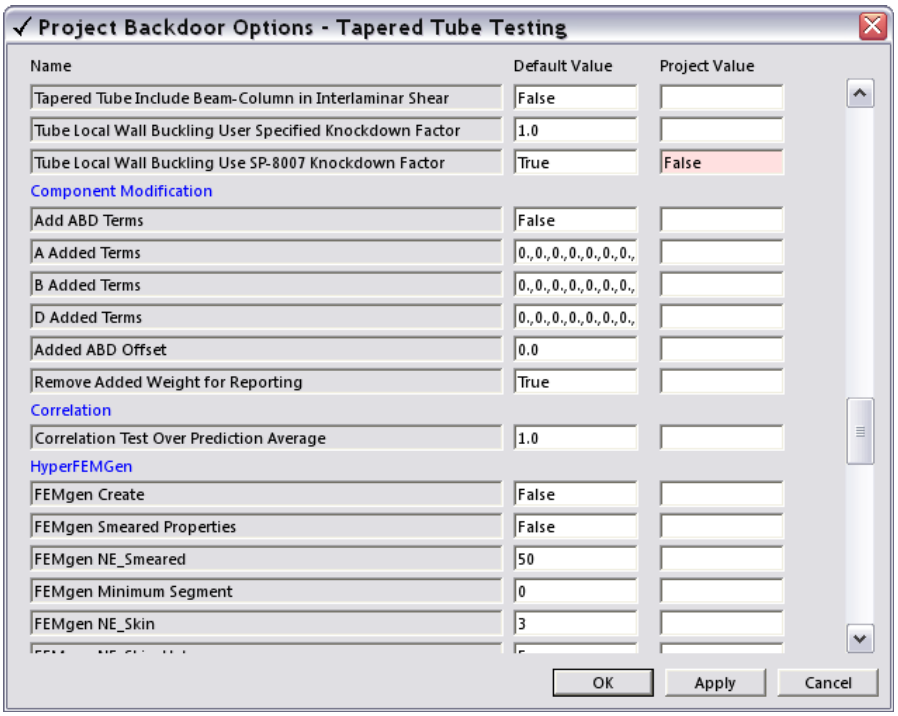
To verify the effective knockdown calculation, we need to know the margin (factor of safety) of the SP-8007 method when no gamma factors are applied. We can do this by disabling the gamma factors and reanalyzing the component.

To deactivate the SP-8007 knockdown factors, we need to go into the backdoor data file. **Right-click the workspace** in the database browser and select **Backdoor Data**.



In the Project Backdoor Options window scroll down until you find the "**Tube Local Wall Buckling Use SP-8007 Knockdown Factor**". It is located a little more than halfway down the window.

Type in "**False**" for the **Project Value**.



Press **Apply** to save the changes.

Backdoor data is applied on the project-level. This means that these values are set for all groups and components within a given project. The **Project Value** field sets the value for the selected project only. The **Default Value** will set the value for all projects within the database.

Notice that there are two other backdoor options of interest. "*Tube Local Wall Buckling User Specified Knockdown Factor*" will apply a constant knockdown factor to both the SP-8007 and Rayleigh Ritz margins. "*Tapered Tube Include Beam-Column in Interlaminar Shear*" refers to the ILS shear kick analysis discussed in the next section. When this value is False, no moments are included when calculating this margin.

Return to the sizing form and **reanalyze the component**.

The MS for SP-8007 without the SP-8007 knockdown factors is **2.769**. The buckling detail file no longer contains the data about the knockdown factors.

Turn the knockdown factors back on. The MS of SP-8007 with the gamma factors is **1.733**.

The derived effective knockdown factor is then:

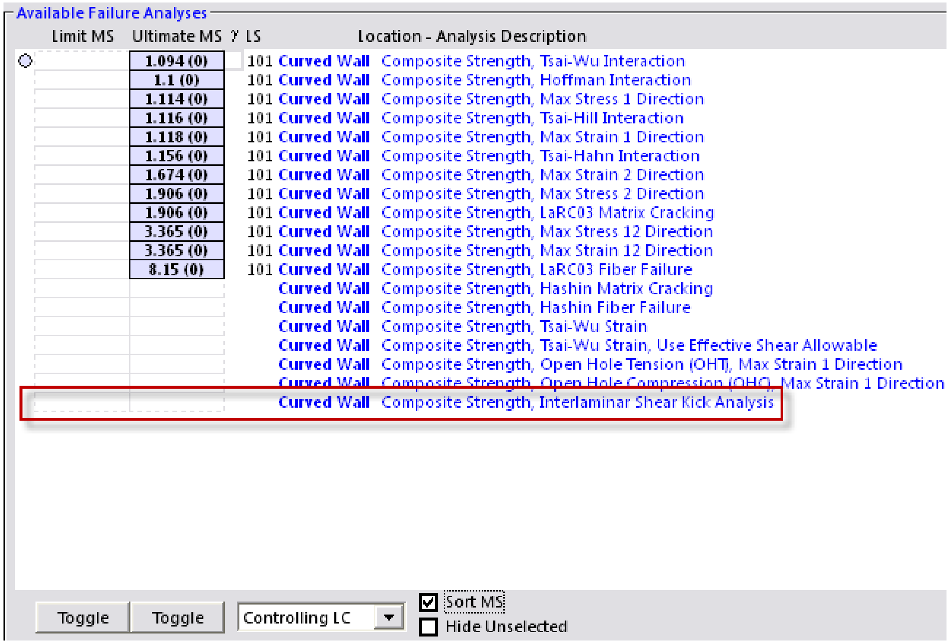
Effective Knockdown = (1 + 1.733)/(1 + 2.769) = **0.725**

This value matches the value found in the buckling detail file. The "effective knockdown" is used applied to the margin the Rayleigh-Ritz margin.

Interlaminar Shear Kick Analysis

The kick refers to the regions when the taper interfaces with the midspan and joint sections. The geometric discontinuity will create a region of high interlaminar shear (ILS).

Go to the Failure tab and isolate the view to the **Material Strength, Composite, Ply** category.



You'll notice that the shear kick analysis is deactivated. **Activate the shear kick analysis**.

Analyze the component. The margin should be **-0.4036.**

The following expression is used to calculate the ILS at the kick. Note that the 3/2 factor is an approximation of the shear distribution of the thickness of the laminate.

$$f_{ILS} = \frac{3}{2} \frac{P \tan \alpha}{\left(\pi D_{mean}\right) t}$$

Let's verify this margin.

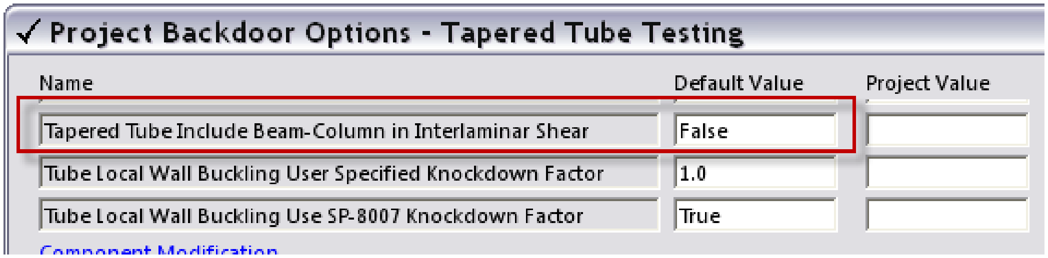
f_{ILS} = 1.5 * 40,000 * tan(20*π/180) / (π * 7.901 * 0.99) = **8887 psi**

The interlaminar shear stress allowable used is the minimum of Fsu13 and Fsu23.

F_{sux3} = min(8120, 5300) = **5300 psi**

MS_{kick} = (5300/8887) – 1 = **-0.403**

Note that although beam-column moments are present, their effects are not included due to an option in the backdoor data file. Turning this option to True will use the point of maximum axial stress in the ILS calculation.



We now have a negative margin for this component. To get the margin positive we can decrease the taper angle (at the cost of stiffness) or increase the diameter (at the cost of local stability and weight).

Bibliography

NASA. (1965). *NASA SP 8007: Buckling of Thin-Walled Circular Cylinders.* Washington D.C.: NASA.

Niu, M. C. (1997). *Airframe Stress Analysis and Sizing.* Conmilit Press Ltd.

Timoshenko, S. (1961). *Theory of Elastic Stability.* McGraw Hill.