HyperSizer® Stress Report

4 March 2008

Project - Training - AP1 i02 - Detailed Sizing2

Report Summary

Contents:	Project Report
Number of Active Load Cases:	6
Controlling Load Case:	Mechanical Load Set #105 (Run Deck #1)
Controlling Component:	9:Longerons, over the wing
Controlling Failure Mode:	Beam Buckling- Column Plane 2- I2
Lowest Margin-of-Safety (MS):	-0.8519

Table of Contents

1 Overall Summary

HyperSizer Project "Training - AP1 i02 - Detailed Sizing2"
Design & Stress Analysis Report
Tuesday, March 4, 2008 at 2:41:33 PM
Entire Project

2008-01-20: Craig modified significantly to excercise almost all panel concepts and failure analyses for Stress Reports: (see notes tab)

1.1 General Information

General Information	
Database	C:\HyperSizer Data\Projects\Collier\Stress Reports\[Database]\HyperSizer Database CCM 2008-02-12 5.3.17.hdb
Project Name	Training - AP1 i02 - Detailed Sizing2
Owner	HyperSizer Admin
Creation Date	Tuesday, December 17, 2002 at 1:20:12 PM
Modification Date	Friday, February 29, 2008 at 2:06:21 PM
Project Notes	26Aug05, GP23, Cp 36, is this doing the Tee panel, bonded joint optz, that shows weight increase when optz with joint failure? If so, will need to turn on joint analysis is the backdoor table, and then make sure no other components have it turned on. See the Assembly called "composite panels" 2008-01-20 trying to get the interlaminar shear analysis to work for laminates. Gives application error when turning on for a stiffened panel. so only two componens with interlaminar shear are: solid laminate: gp6, cp30, combined with the BJSFM example, high Qx and Qy user defined load. sandwich: gp1 cp41 (not reasonable cause the core will fail way before this high Qx and Qy user defined load. suspected bug when putting in allowables for equiv ortho. Hs failure tab says MS=very high, always. Also most likely a Hs bug: .... Group 5 (about 10 components) all select the same equiv ortho material. not believable cause the biaxial loads vary so much.

1.2 Summary of Last Run

Report Summary	
Contents	Project Report
Number of Active Load Cases	6
Controlling load case	Mechanical Load Set #105 (Run Deck #1)
Controlling component	9:Longerons, over the wing
Controlling failure mode	Beam Buckling- Column Plane 2- I2
Lowest margin-of-safety (MS)	-0.8519

Last Run Summary	
Run Time (hh:mm:ss)	00:13:16
Weight Total (lb)	2718
Beam, Unit Weight (lb / ft)	2.371

Last Run Summary	
Beam, Total Length (ft)	536.2
Beam, Total Weight (lb)	1271
Panel, Unit Weight (lb / ft^2)	1.664
Panel, Total Area (ft^2)	869.3
Panel, Total Weight (lb)	1447
Controlling Failure Mode, Strength (lb)	1130
Controlling Failure Mode, Buckling (lb)	77.71
Controlling Failure Mode, Local Buckling (lb)	720.9
Controlling Failure Mode, Minimum Gage (lb)	111
Controlling Failure Mode, Negative Margin Structure (lb)	678

Report Options	
Report Detail	Summary of Controlling Load Case Only
Composite Layup Detail	Not Included
Margin-of-Safety Sorting	Sort by Load Case
Sort by Object	No
Element-Based Margins and Loads	Included
Material Reports	Not Included
Material Report Units	US

1.3 Summary of Assemblies

Assembly Weight Summary						
Assembly	**Area (ft^2)**	**UW Area (lb / ft^2)**	**Length (in)**	**UW Length (lb / ft)**	**Weight (lb)**	**% Total Weight**
2 "Composite Panels"	289.4	1.028	0	0	297.5	15.13%
1 "Forward Fuselage Barell"	238.1	1.343	270.7	2.76	1067	54.25%
3 "Sandwich and Solid Laminates"	450.3	1.337	0	0	601.8	30.61%
Totals	**977.9**	**1.247**	**270.7**	**2.76**	**1966**	

**components may belong to more than one assembly*

1.4 Project Setup

Finite Element Model Filenames		
Run Deck		
1	FEM Filename	S:\HyperSizer Data\Projects\Collier_Training UM\AP1\FEA\Ap1_i02.dat
	FEA Force Filename	S:\HyperSizer Data\Projects\Collier_Training UM\AP1\FEA\AP1_i02.f06
	PM Filename	C:\HyperSizer Data\Projects\Collier_Training UM\AP1\FEA\AP1_i03.PM1

FEM Conversion Units	
Temperature	Fahrenheit (°F)
Length (Area)	Inches (in) to Inches: 1
Mass	Pounds (lb) to Pounds Mass: 1
Force	Pounds (lb) to Pounds Force: 1

**Used during FEM import and FEM Stiffness Update*

Project Units	
Temperature	°F
Weight	lb
Length	in
Unit Weight (Panel, Area)	lb / ft^2
Unit Weight (Beam, Length)	lb / ft
Force	lb

**For display in reports and the HyperSizer interface*

2 Loads

2.1 Mechanical and Thermal Load Sets

Available Load Sets					
Load Set	**Run Deck**	**Type**	**Limit Factor**	**Ultimate Factor**	**Description**
101	1	Mechanical	1	1	Wing Pressure
102	1	Mechanical	1	1	Internal Tank Pressure
103	1	Mechanical	1	1	Fuselage Bending
104	1	Mechanical	1	1	Wing and Tank Pressure
105	1	Mechanical	1	1	Tank Press & Fuse Bending
106	1	Mechanical	1	1	All Mechanical
201	1	Thermal	1	1	Aero Heating

2.2 Load Cases

Available Load Cases					
Load Case	**MLS ID**	**TLS ID**	**Description**	**Weight (lb)**	**% Total**
1*	101		Mechanical Load Set #101 (Run Deck #1) "Wing Pressure", Thermal None.	83.81	3.1%
2*	101	201	Mechanical Load Set #101 (Run Deck #1) "Wing Pressure", Thermal Load Set #201 (Run Deck #1) "Aero Heating"	279.6	10.3%
3	102		Mechanical Load Set #102 (Run Deck #1) "Internal Tank Pressure", Thermal None.		
4*	103		Mechanical Load Set #103 (Run Deck #1) "Fuselage Bending", Thermal None.	708.4	26.1%
5*	104		Mechanical Load Set #104 (Run Deck #1) "Wing and Tank Pressure", Thermal None.	237.5	8.7%
6*	105		Mechanical Load Set #105 (Run Deck #1) "Tank Press & Fuse Bending", Thermal None.	739.5	27.2%
7*	106		Mechanical Load Set #106 (Run Deck #1) "All Mechanical", Thermal None.	669.2	24.6%

** - indicates active load case*

3 Margins-of-Safety for Load Cases

Load Case Margin of Safety Summary						
Load Case	**MS**	**Component**	**Lim/Ult**	**Object ID**	**AID**	**Analysis**
1	.09	36	Ultimate	6	180	Joint- Bonded- Adhesive- Longitudinal & Transverse Shear Strain
2	-.61	9	Ultimate		27	Beam Buckling- Column Plane 2- I2
4	-.85	9	Ultimate		27	Beam Buckling- Column Plane 2- I2
5	-.58	73	Ultimate	6	162	Joint- Bonded- Fracture- Principal Transverse
6	-.85	9	Ultimate		27	Beam Buckling- Column Plane 2- I2
7	-.85	9	Ultimate		27	Beam Buckling- Column Plane 2- I2

4 Margins-of-Safety for Analyses

Lowest Margin of Safety for Each Analysis					
MS	Component	Load Case	Lim/Ult	AID	Analysis
.51	39	2	Ultimate	2	Panel Buckling- Flat- Simple BC- Uniaxial or Biaxial
6.97	39	2	Ultimate	3	Panel Buckling- Flat- Simple BC- Shear
.46	39	2	Ultimate	5	Panel Buckling- Flat- Simple BC- Uniaxial or Biaxial w/Shear Interaction
.47	39	2	Ultimate	7	Panel Buckling- Flat- Simple BC- Uniaxial or Biaxial w/TSF
4.31	57	4	Ultimate	8	Panel Buckling- Flat- Simple BC- Shear w/TSF (Transverse Shear Flexibility)
.41	39	2	Ultimate	9	Panel Buckling- Flat- Simple BC- Uniaxial or Biaxial w/TSF & Shear Interaction
.03	44	4	Ultimate	11	Panel Buckling- Curved or Flat- All BC
-.66	9	6	Ultimate	25	Beam Buckling- Column Plane 1- I1
-.66	9	6	Ultimate	26	Beam Buckling- Column Plane 1 w/TSF- I1
-.85	9	6	Ultimate	27	Beam Buckling- Column Plane 2- I2
-.85	9	6	Ultimate	28	Beam Buckling- Column Plane Min- Imin
.07	3	4	Ultimate	33	Beam Buckling- Cylindrical- Axial and Bending- Rayleigh Ritz
.05	3	4	Ultimate	34	Beam Buckling- Cylindrical- Axial and Bending- NASA SP-8007
-.61	9	4	Limit	40	Local Buckling- Longitudinal Direction
.07	48	2	Limit	41	Local Buckling- Transverse Direction
1.18	49	6	Limit	42	Local Buckling- Shear Direction
-.61	9	4	Limit	43	Local Buckling- Interaction
-.15	17	7	Ultimate	50	Crippling- Isotropic- method Niu- formed and extruded sections
-.15	9	6	Ultimate	51	Crippling- Isotropic- method LTV- formed and extruded sections
-.19	17	7	Ultimate	53	Crippling - Buckling interaction- Johnson-Euler
.30	33	7	Ultimate	90	Wrinkling- Eqn 1- Isotropic or Honeycomb Core- X- Y & Interaction
.07	57	4	Ultimate	91	Wrinkling- Eqn 2- Honeycomb Core- X- Y & Interaction
6.01	39	2	Ultimate	94	Intracell Dimpling- X- Y & Interaction
625.51	39	7	Ultimate	101	Crushing- Flexural Bending Load
.59	38	6	Ultimate	102	Crushing- Joint Support Load
1.66	34	7	Ultimate	103	Sandwich Face/Core Flatwise Tension
.55	57	4	Ultimate	104	Shear Crimping- X- Y & Interaction {Hexcel}
.41	57	7	Ultimate	105	Shear Strength- X (Longitudinal) direction {Hexcel}
.16	39	2	Ultimate	106	Shear Strength- Y (Transverse) direction {Hexcel}
-.42	9	6	Ultimate	110	Isotropic Strength- Longitudinal Direction
.00	53	2	Limit	111	Isotropic Strength- Transverse Direction
1.22	54	6	Ultimate	112	Isotropic Strength- Shear Direction
-.43	9	6	Ultimate	113	Isotropic Strength- Von Mises Interaction Yield Criterion
2.58	37	5	Ultimate	114	Isotropic Strength- Max Shear Criterion
2.58	37	5	Ultimate	115	Isotropic Strength- Max Principal Stress Criterion
.01	78	5	Ultimate	135	Composite Strength- Max Strain 1 Direction
3.87	30	6	Ultimate	136	Composite Strength- Max Strain 2 Direction
5.99	37	5	Ultimate	137	Composite Strength- Max Strain 12 Direction
2.20	37	5	Ultimate	138	Composite Strength- Max Stress 1 Direction
4.25	30	6	Ultimate	139	Composite Strength- Max Stress 2 Direction
5.99	37	5	Ultimate	140	Composite Strength- Max Stress 12 Direction
1.56	30	6	Ultimate	141	Composite Strength- Tsai-Hill Interaction
.21	41	6	Ultimate	142	Composite Strength- Tsai-Wu Interaction
1.45	30	6	Ultimate	143	Composite Strength- Tsai-Hahn Interaction
.01	39	2	Ultimate	144	Composite Strength- Hoffman Interaction
4.25	30	6	Ultimate	147	Composite Strength- LaRC03 Matrix Cracking
2.49	30	6	Ultimate	148	Composite Strength- LaRC03 Fiber Failure
.23	74	6	Ultimate	151	Composite Strength- Open Hole Tension (OHT)
.13	74	5	Ultimate	152	Composite Strength- Open Hole Compression (OHC) after impact

Lowest Margin of Safety for Each Analysis					
MS	Component	Load Case	Lim/Ult	AID	Analysis
5.14	42	7	Ultimate	153	Composite Strength- Laminate- Longitudinal Strain Tension
.51	42	2	Ultimate	154	Composite Strength- Laminate- Longitudinal Strain Compression
5.05	42	2	Ultimate	155	Composite Strength- Laminate- Shear Strain
7.85	68	5	Ultimate	156	Composite Strength- Laminate- Open Hole Tension (OHT)
2.89	68	5	Ultimate	157	Composite Strength- Laminate- Open Hole Compression (OHC) after impact
-.78	73	7	Ultimate	162	Joint- Bonded- Fracture- Principal Transverse
2.58	36	1	Ultimate	163	Joint- Bonded- Fracture- Max Stress or Strain 1 direction
3.07	36	1	Ultimate	164	Joint- Bonded- Delamination- Peel Dominated
2.25	36	1	Ultimate	165	Joint- Bonded- Delamination- Peel and Transverse Shear 1
1.87	36	1	Ultimate	166	Joint- Bonded- Delamination- Peel and Transverse Shear 2
2.05	36	2	Ultimate	167	Joint- Bonded- Delamination- Tong- Peel- Transverse Shear & Axial- 1
1.67	36	2	Ultimate	168	Joint- Bonded- Delamination- Tong- Peel- Transverse Shear & Axial- 2
1.95	36	2	Ultimate	169	Joint- Bonded- Delamination- Tong- Peel- Transverse Shear & Axial- 3
1.57	36	2	Ultimate	170	Joint- Bonded- Delamination- Tong- Peel- Transverse Shear & Axial- 4
1.17	36	1	Ultimate	171	Joint- Bonded- Delamination- Tong- Peel- Transverse Shear & Axial- 5
.95	36	1	Ultimate	172	Joint- Bonded- Delamination- Tong- Peel- Transverse Shear & Axial- 6
1.74	36	1	Ultimate	173	Joint- Bonded- Delamination- Peel- Longitudinal & Transverse Shear
.85	36	1	Ultimate	174	Joint- Bonded- Delamination- Peel- Longitudinal & Transverse Shear- Axial and Tr
3.38	36	1	Ultimate	175	Joint- Bonded- Adhesive- Peel Dominated
.82	36	1	Ultimate	176	Joint- Bonded- Adhesive- Von Mises Strain
1.19	53	2	Ultimate	177	Joint- Bonded- Adhesive- Maximum Principal Stress
1.26	36	1	Ultimate	178	Joint- Bonded- Adhesive- Peel- Longitudinal & Transverse Shear
1.64	36	1	Ultimate	179	Joint- Bonded- Adhesive- Longitudinal & Transverse Shear Stress
.09	36	1	Ultimate	180	Joint- Bonded- Adhesive- Longitudinal & Transverse Shear Strain
.01	30	6	Ultimate	190	Joint- Bolted- Single Hole- BJSFM- loaded and far field
4.63	41	6	Ultimate	203	Composite Strength- Interlaminar Shear

5 Margins-of-Safety for Components

Component Margin of Safety Summary					
Component	**MS**	**Load Case**	**Lim/Ult**	**AID**	**Analysis**
2	.05	7	Ultimate	110	Isotropic Strength- Longitudinal Direction
3	.05	4	Ultimate	34	Beam Buckling- Cylindrical- Axial and Bending- NASA SP-8007
7	-.83	4	Ultimate	27	Beam Buckling- Column Plane 2- I2
8	-.63	4	Ultimate	27	Beam Buckling- Column Plane 2- I2
9	-.85	6	Ultimate	27	Beam Buckling- Column Plane 2- I2
11	.07	2	Limit	40	Local Buckling- Longitudinal Direction
12	.02	7	Ultimate	110	Isotropic Strength- Longitudinal Direction
13	.01	7	Ultimate	110	Isotropic Strength- Longitudinal Direction
14	.15	4	Limit	43	Local Buckling- Interaction
15	.03	5	Ultimate	50	Crippling- Isotropic- method Niu- formed and extruded sections
17	-.53	4	Limit	43	Local Buckling- Interaction
18	.01	6	Limit	40	Local Buckling- Longitudinal Direction
19	.06	7	Ultimate	53	Crippling - Buckling interaction- Johnson-Euler
20	-.42	7	Limit	43	Local Buckling- Interaction
21	.23	4	Limit	43	Local Buckling- Interaction
22	.48	4	Limit	43	Local Buckling- Interaction
23	.48	6	Limit	43	Local Buckling- Interaction
24	.19	7	Limit	43	Local Buckling- Interaction
30	.01	6	Ultimate	190	Joint- Bolted- Single Hole- BJSFM- loaded and far field
33	.30	7	Ultimate	90	Wrinkling- Eqn 1- Isotropic or Honeycomb Core- X- Y & Interaction
34	1.66	7	Ultimate	103	Sandwich Face/Core Flatwise Tension
35	.01	6	Ultimate	53	Crippling - Buckling interaction- Johnson-Euler
36	.09	1	Ultimate	180	Joint- Bonded- Adhesive- Longitudinal & Transverse Shear Strain
37	2.20	5	Ultimate	138	Composite Strength- Max Stress 1 Direction
38	.24	6	Ultimate	106	Shear Strength- Y (Transverse) direction {Hexcel}
39	.01	2	Ultimate	144	Composite Strength- Hoffman Interaction
40	.13	7	Ultimate	144	Composite Strength- Hoffman Interaction
41	.21	6	Ultimate	142	Composite Strength- Tsai-Wu Interaction
42	.51	2	Ultimate	154	Composite Strength- Laminate- Longitudinal Strain Compression
43	.20	7	Ultimate	110	Isotropic Strength- Longitudinal Direction
44	.03	4	Ultimate	11	Panel Buckling- Curved or Flat- All BC
45	.06	4	Ultimate	11	Panel Buckling- Curved or Flat- All BC
46	.05	7	Ultimate	113	Isotropic Strength- Von Mises Interaction Yield Criterion
47	.18	2	Limit	43	Local Buckling- Interaction
48	.06	6	Ultimate	113	Isotropic Strength- Von Mises Interaction Yield Criterion
49	.04	6	Ultimate	113	Isotropic Strength- Von Mises Interaction Yield Criterion
50	.03	4	Ultimate	11	Panel Buckling- Curved or Flat- All BC
51	.06	4	Limit	43	Local Buckling- Interaction
52	.21	2	Limit	43	Local Buckling- Interaction
53	.00	2	Limit	111	Isotropic Strength- Transverse Direction
54	.01	2	Limit	111	Isotropic Strength- Transverse Direction
55	.05	4	Ultimate	11	Panel Buckling- Curved or Flat- All BC
56	.00	6	Limit	40	Local Buckling- Longitudinal Direction
57	.07	4	Ultimate	91	Wrinkling- Eqn 2- Honeycomb Core- X- Y & Interaction
58	.46	6	Ultimate	135	Composite Strength- Max Strain 1 Direction
59	.05	2	Ultimate	135	Composite Strength- Max Strain 1 Direction
60	.04	7	Ultimate	135	Composite Strength- Max Strain 1 Direction
61	.25	4	Ultimate	135	Composite Strength- Max Strain 1 Direction
62	.42	6	Ultimate	135	Composite Strength- Max Strain 1 Direction

Component Margin of Safety Summary					
Component	MS	Load Case	Lim/Ult	AID	Analysis
63	.09	7	Ultimate	135	Composite Strength- Max Strain 1 Direction
64	.08	7	Ultimate	135	Composite Strength- Max Strain 1 Direction
65	.46	6	Ultimate	135	Composite Strength- Max Strain 1 Direction
66	.63	7	Ultimate	135	Composite Strength- Max Strain 1 Direction
67	.02	6	Ultimate	135	Composite Strength- Max Strain 1 Direction
68	2.89	5	Ultimate	157	Composite Strength- Laminate- Open Hole Compression (OHC) after impact
69	.10	6	Ultimate	135	Composite Strength- Max Strain 1 Direction
70	.08	4	Ultimate	135	Composite Strength- Max Strain 1 Direction
71	.47	5	Ultimate	135	Composite Strength- Max Strain 1 Direction
72	.68	7	Ultimate	135	Composite Strength- Max Strain 1 Direction
73	-.78	7	Ultimate	162	Joint- Bonded- Fracture- Principal Transverse
74	.13	5	Ultimate	152	Composite Strength- Open Hole Compression (OHC) after impact
75	1.62	4	Ultimate	135	Composite Strength- Max Strain 1 Direction
76	.12	7	Ultimate	135	Composite Strength- Max Strain 1 Direction
77	.04	6	Ultimate	135	Composite Strength- Max Strain 1 Direction
78	.01	5	Ultimate	135	Composite Strength- Max Strain 1 Direction
80	.15	6	Ultimate	113	Isotropic Strength- Von Mises Interaction Yield Criterion
81	.09	6	Ultimate	113	Isotropic Strength- Von Mises Interaction Yield Criterion

6 Margins-of-Safety for Finite Elements

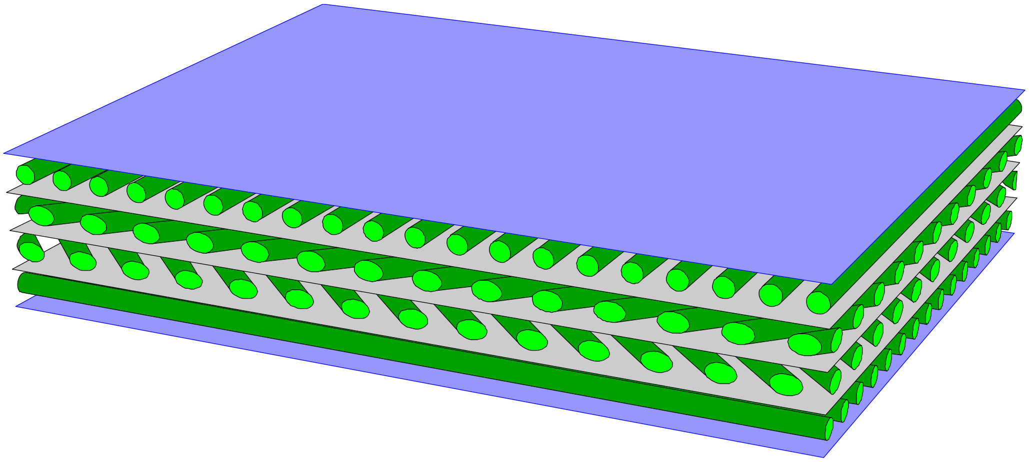
This method reports a separate margin for each element that belongs to a structural component using individual FEA element loads. This is in contrast to the statistical loads processing approach which statistically determines the appropriate design-to load from all of the elements collectively that comprise the component.

6.1 Element Summary

Element Summary						
LC ID	**MLS ID**	**THLS ID**	**# Elements**	**Min MS**	**AID**	**Analysis**
7	106	0	10	.13	144	Composite Strength- Hoffman Interaction
2	101	201	8	.66	144	Composite Strength- Hoffman Interaction
5	104	0	2	.63	144	Composite Strength- Hoffman Interaction
4	103	0	0	-------	-------	-------
6	105	0	0	-------	-------	-------
1	101	0	0	-------	-------	-------

7 Component Dimensions

7.1 One Stack Unstiffened Components



7.1.1 Dimensions

Component Dimensions					
Component ID	t_{tf} (in)	H (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
30	0.243	0.243	1.975	6.25	12.35
58	0.05	0.05	0.4032	5.11	2.06
59	0.03	0.03	0.2419	6.686	1.618
60	0.04	0.04	0.3226	1.357	0.4377
61	0.04	0.04	0.3226	4.877	1.573
62	0.04	0.04	0.3226	8.893	2.868
63	0.2	0.2	1.613	4.499	7.257
64	0.07	0.07	0.5645	4.577	2.584
65	0.04	0.04	0.3226	8.588	2.77
66	0.02	0.02	0.1613	4.499	0.7257
67	0.03	0.03	0.2419	4.035	0.9762
69	0.18	0.18	1.452	4.499	6.531
70	0.06	0.06	0.4838	3.735	1.807
71	0.02	0.02	0.1613	6.757	1.09
72	0.03	0.03	0.2419	2.074	0.5018
75	0.02	0.02	0.1613	2.074	0.3345
76	0.09	0.09	0.7258	3.083	2.237
77	0.19	0.19	1.532	6.25	9.576
78	0.17	0.17	1.371	6.25	8.568

**See Section 7.15 for Variable Descriptions.*

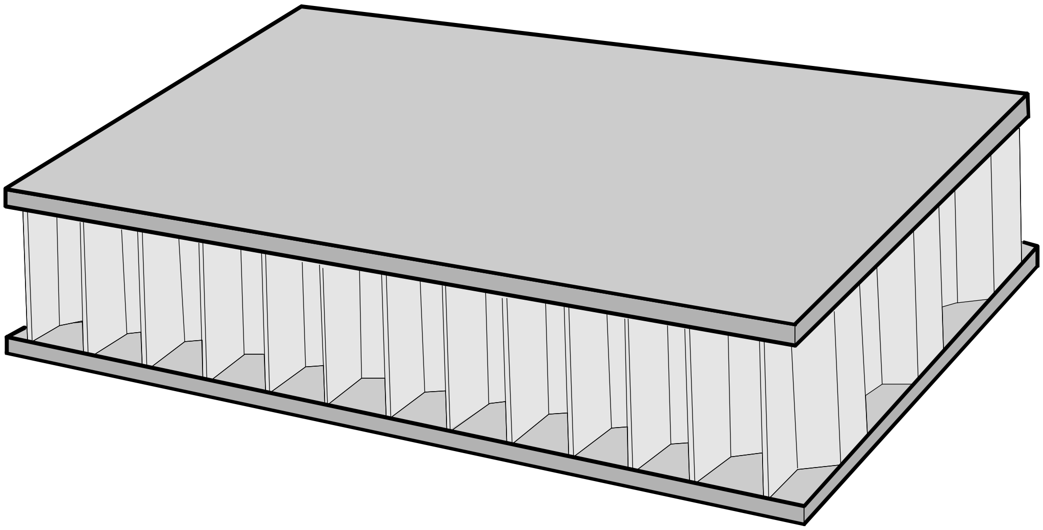
7.1.2 Materials

Component Materials		
Component ID	Stack 1 / Top Face Material Type	Stack 1 / Top Face Material ID
30	Layup Orthotropic	1 2
58	Orthotropic	3

Component Materials		
Component ID	Stack 1 / Top Face Material Type	Stack 1 / Top Face Material ID
59	Orthotropic	3
60	Orthotropic	3
61	Orthotropic	3
62	Orthotropic	3
63	Orthotropic	3
64	Orthotropic	3
65	Orthotropic	3
66	Orthotropic	3
67	Orthotropic	3
69	Orthotropic	3
70	Orthotropic	3
71	Orthotropic	3
72	Orthotropic	3
75	Orthotropic	3
76	Orthotropic	3
77	Orthotropic	3
78	Orthotropic	3

**See Section 7.16 for Material Names.*

7.2 Honeycomb Sandwich Components



7.2.1 Dimensions

Component Dimensions							
Component ID	t_{tf} (in)	t_c (in)	H (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)	
33	0.075	1	1.15	2.347	113.5	266.4	
34	0.02	0.5	0.02	0.54	0.6568	63.26	41.55
38	0.035	3	0.02	3.055	1.798	31.25	56.18
39	0.0405	1.25	1.331	1.141	84.38	96.24	
40	0.0567	1.25	1.363	1.404	14.04	19.72	
41	0.0405	1.25	1.331	1.141	8.887	10.14	
42	0.0405	1.25	1.331	1.141	30.39	34.66	
57	0.02	0.875	0.02	0.915	1.053	10.5	11.05

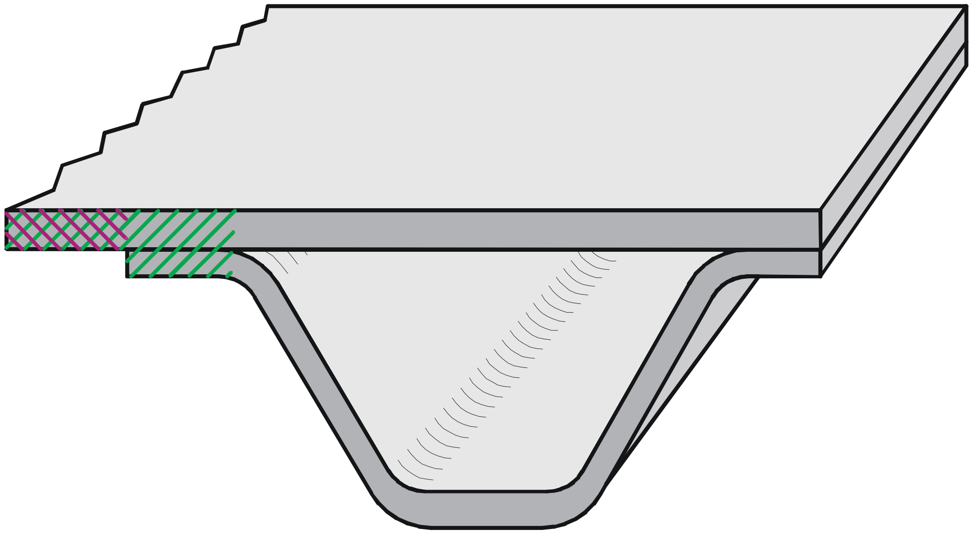
**See Section 7.15 for Variable Descriptions.*

7.2.2 Materials

Component Materials				
Component ID	Stack 1 / Top Face Material Type	Stack 1 / Top Face Material ID	Stack 2 / Core Material Type	Stack 2 / Core Material ID
33	Isotropic	4	Honeycomb	5
34	Isotropic	4	Honeycomb	6
38	Isotropic	4	Honeycomb	7
39	Layup Orthotropic	8 2	Honeycomb	9
40	Layup Orthotropic	10 2	Honeycomb	9
41	Layup Orthotropic	11 2	Honeycomb	9
42	Layup Orthotropic	11 2	Honeycomb	9
57	Isotropic	12	Honeycomb	6

**See Section 7.16 for Material Names.*

7.3 Bonded Hat Stiffened Components



7.3.1 Dimensions

Component Dimensions													
Component ID	t_{tf} (in)	t_w (in)	H (in)	S (in)	w_b (in)	θ (°)	w_t (in)	w_{cs} (in)	t_{fl} (in)	t_{cr} (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
35	0.05	0.085	2.5	7	2.25	80	2	1.928	0.12	0.12	2.637	78	205.7
37	0.04079	0.045	2.5	6	1	85	2	2.578	0.03	0.06	1.635	97.96	160.2
80	0.08333	0.085	3	7	1.75	80	2	2.278	0.12	0.2	3.451	9.911	34.2
81	0.08333	0.07	1.5	7	2.25	80	2	2.293	0.12	0.12	2.679	10.05	26.93

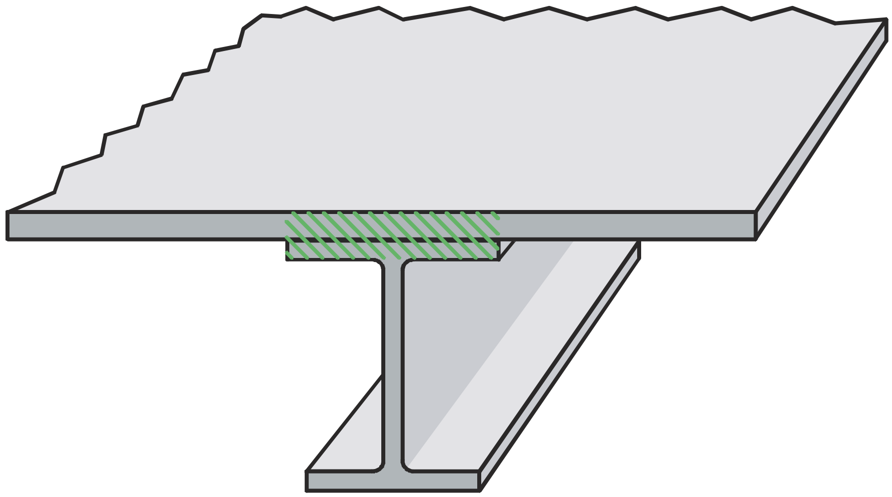
**See Section 7.15 for Variable Descriptions.*

7.3.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	Core Web Material Type	Core Web Material ID
35	Isotropic	13	Isotropic	4
37	Layup Orthotropic	14 15	Isotropic	12
80	Isotropic	13	Isotropic	4
81	Isotropic	13	Isotropic	4

**See Section 7.16 for Material Names.*

7.4 Bonded I Components



7.4.1 Dimensions

Component Dimensions													
Component ID	t_{tf} (in)	t_w (in)	H (in)	S (in)	w_b (in)	θ (°)	w_t (in)	w_{cs} (in)	$t_{fl\ top}$ (in)	$t_{fl\ bottom}$ (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
44	0.0824	0.09	1.125	4	0.75	90	0.6	3.4	0.09	0.09	3.046	6.055	18.44
50	0.1216	0.09	1.125	4	0.375	90	0.6	3.4	0.09	0.09	3.734	5.999	22.4
51	0.0824	0.09	1.125	4	1.125	90	0.6	3.4	0.09	0.09	3.24	5.972	19.35
52	0.04	0.05	0.5	2.05	0.75	90	0.75	1.3	0.05	0.05	1.61	8.6	13.84
53	0.1	0.0375	1	1.6	0.75	90	0.75	0.85	0.0375	0.0375	2.667	8.606	22.95
54	0.08	0.0375	1	2.5	0.75	90	0.75	1.75	0.0375	0.0375	1.939	8.612	16.7
55	0.08	0.025	1	2.05	0.75	90	0.75	1.3	0.025	0.025	1.795	8.618	15.47
56	0.08	0.025	1	2.05	0.75	90	0.75	1.3	0.025	0.025	1.795	8.622	15.48
68	0.1049	0.04661	1.174	5	1.5	90	2	3	0.05061	0.06992	1.291	7.301	9.427
73	0.1049	0.04661	1.174	3	1.5	90	2	1	0.05061	0.06992	1.574	4.148	6.53
74	0.1049	0.04661	1.174	5	1.5	90	2	3	0.05061	0.06992	1.291	4.148	5.357

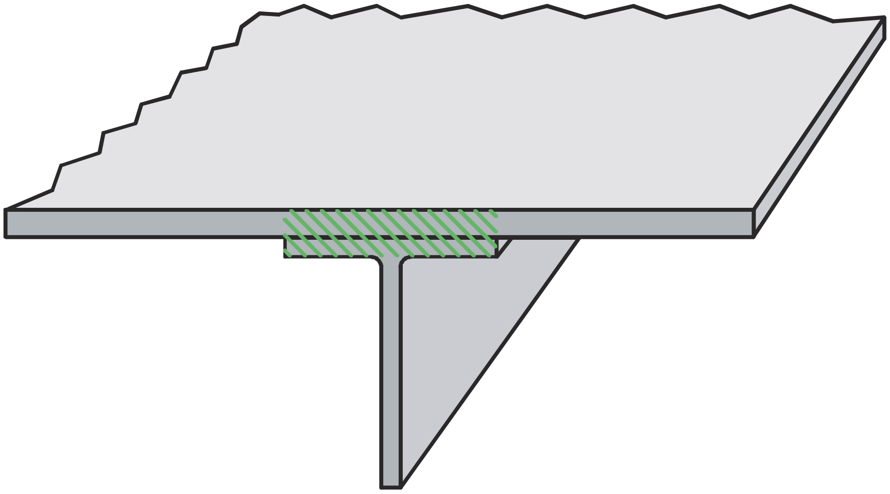
**See Section 7.15 for Variable Descriptions.*

7.4.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	Web Material Type	Web Material ID
44	Isotropic	12	Isotropic	12
50	Isotropic	12	Isotropic	12
51	Isotropic	12	Isotropic	12
52	Isotropic	16	Isotropic	12
53	Isotropic	16	Isotropic	12
54	Isotropic	16	Isotropic	12
55	Isotropic	16	Isotropic	12
56	Isotropic	16	Isotropic	12
68	Laminate	17	Laminate	18
73	Laminate	17	Laminate	18
74	Laminate	17	Laminate	18

**See Section 7.16 for Material Names.*

7.5 *Bonded T Components*



7.5.1 Dimensions

Component Dimensions											
Component ID	t_{tf} (in)	t_w (in)	H (in)	S (in)	θ (°)	w_t (in)	w_{cs} (in)	$t_{fl\ top}$ (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
36	0.0832	0.029	0.75	4	90	1	3	0.033	0.8086	103.6	83.81

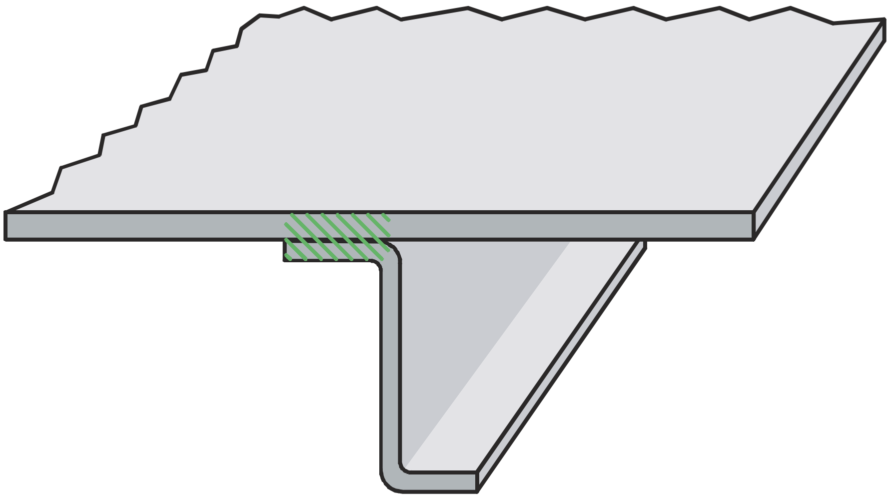
**See Section 7.15 for Variable Descriptions.*

7.5.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	Web Material Type	Web Material ID
36	Layup Orthotropic	19 20	Laminate	21

**See Section 7.16 for Material Names.*

7.6 Bonded Z Components



7.6.1 Dimensions

Component Dimensions													
Component ID	t_{tf} (in)	t_w (in)	H (in)	S (in)	w_b (in)	θ (°)	w_t (in)	w_{cs} (in)	$t_{fl\ top}$ (in)	$t_{fl\ bottom}$ (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
43	0.0824	0.09	0.9583	4	0.375	90	0.6	3.4	0.09	0.09	2.765	6.083	16.82
45	0.0824	0.09	1.125	4	1.125	90	1.2	2.8	0.09	0.09	3.551	6.027	21.4

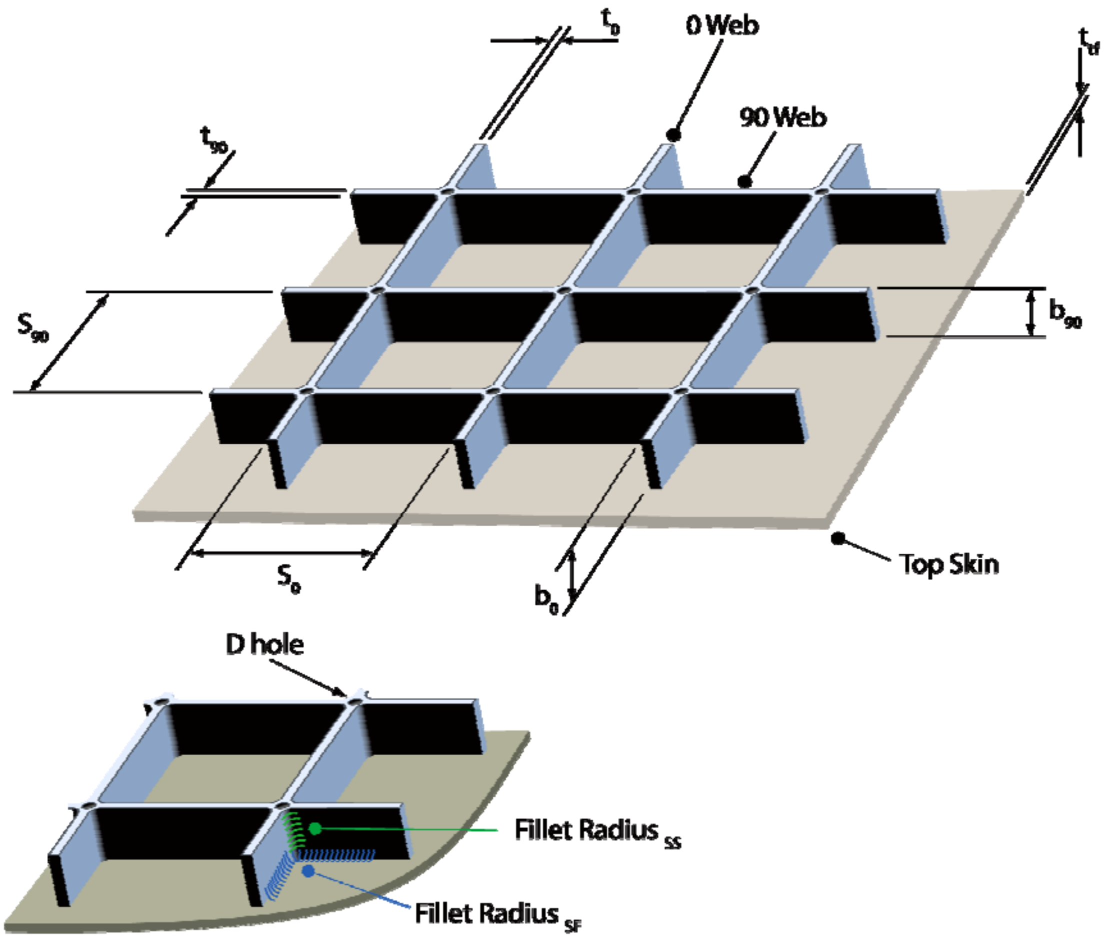
**See Section 7.15 for Variable Descriptions.*

7.6.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	Web Material Type	Web Material ID
43	Isotropic	12	Isotropic	12
45	Isotropic	12	Isotropic	12

**See Section 7.16 for Material Names.*

7.7 *OrthoGrid Components*



7.7.1 Dimensions

Component Dimensions													
Component ID	t_{tf} (in)	t_0 (in)	t_{90} (in)	S_0 (in)	θ (°)	b_0 (in)	b_{90} (in)	S_{90} (in)	S_θ (in)	H (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
47	0.12	0.07	0.06	4	30	1.25	1.25	6.928	8	1.37	3.514	10.21	35.89

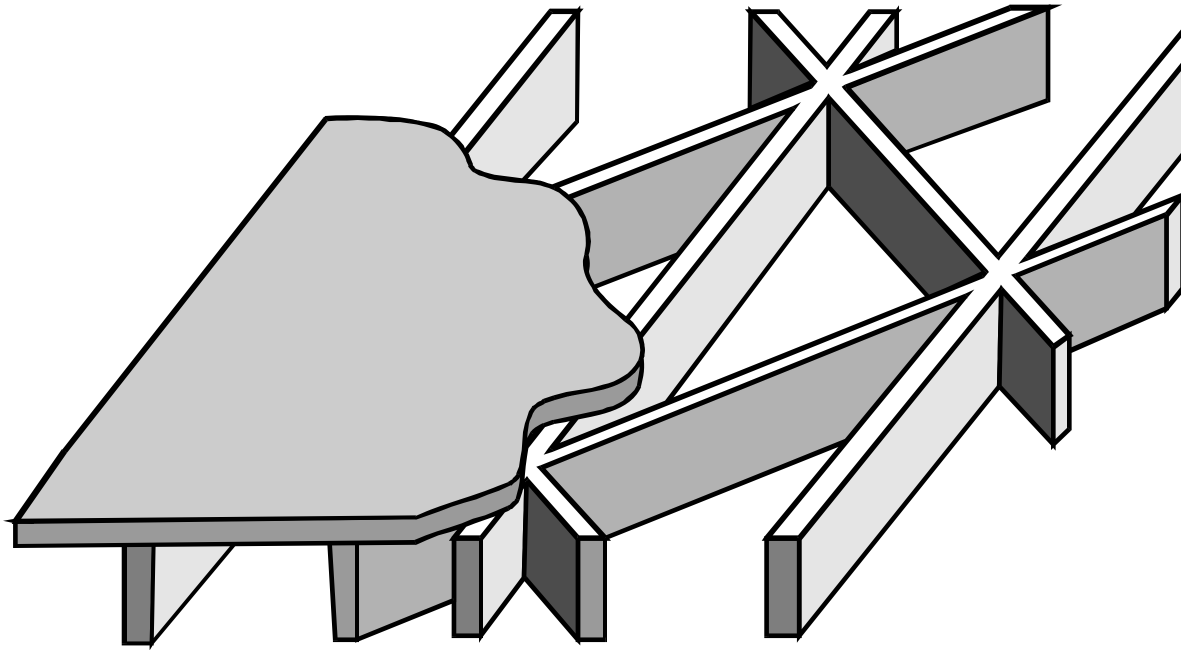
**See Section 7.15 for Variable Descriptions.*

7.7.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	0° Web Material Type	0° Web Material ID
47	Isotropic	12	Isotropic	12

**See Section 7.16 for Material Names.*

7.8 AngleGrid Components



7.8.1 Dimensions

Component Dimensions													
Component ID	t_{tf} (in)	t_0 (in)	t_θ (in)	S_0 (in)	θ (°)	b_0 (in)	b_θ (in)	S_{90} (in)	S_θ (in)	H (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
48	0.12	0.14	0.12	3	30	1.3	1.3	5.196	6	1.42	5.481	10.21	55.98
49	0.06	0.14	0.09333	4	30	1.3	1.1	6.928	8	1.36	3.09	5.106	15.78

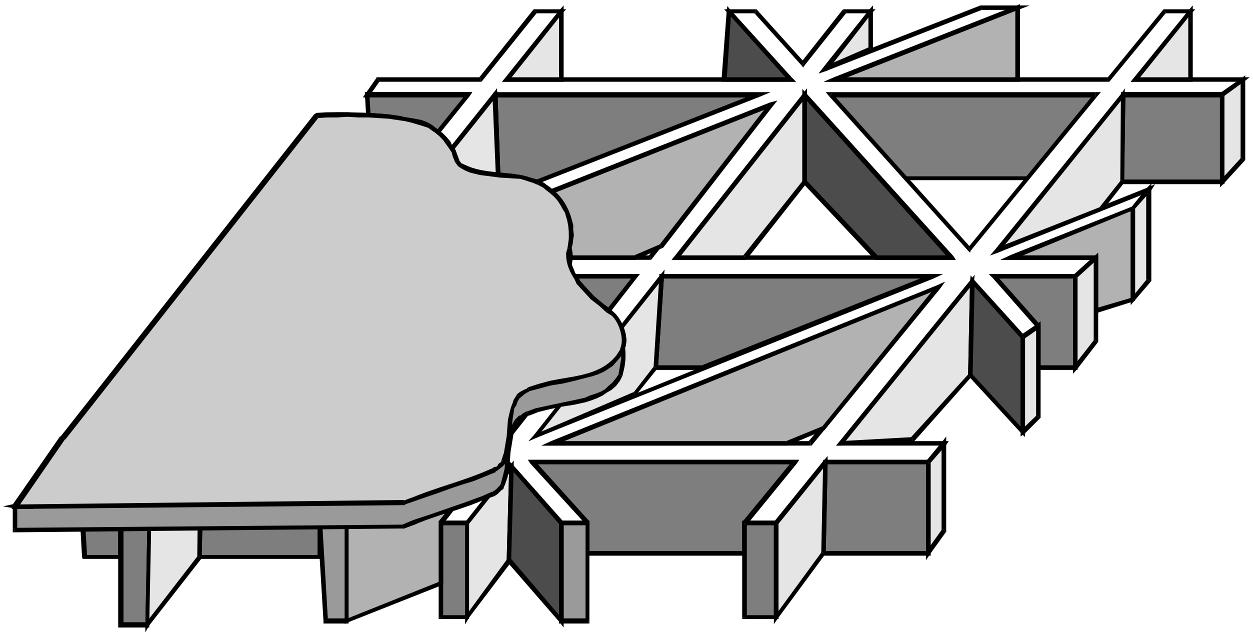
**See Section 7.15 for Variable Descriptions.*

7.8.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	0° Web Material Type	0° Web Material ID
48	Isotropic	12	Isotropic	12
49	Isotropic	12	Isotropic	12

**See Section 7.16 for Material Names.*

7.9 GeneralGrid Components



7.9.1 Dimensions

Component Dimensions															
Component ID	t_{tf} (in)	t_0 (in)	t_{90} (in)	t_θ (in)	S_0 (in)	θ (°)	b_0 (in)	b_θ (in)	b_{90} (in)	S_{90} (in)	S_θ (in)	H (in)	Unit Weight (lb / ft^2)	Area (ft^2)	Weight (lb)
46	0.06	0.14	0.12	0.09333	3	30	1.3	1.1	1.4	5.196	6	1.46	4.36	5.12	22.32

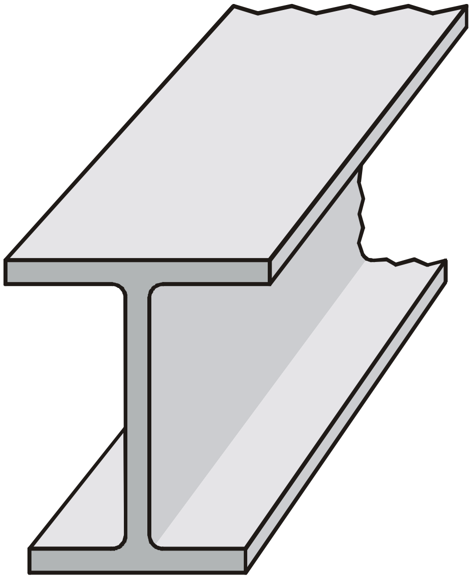
**See Section 7.15 for Variable Descriptions.*

7.9.2 Materials

Component Materials				
Component ID	Top Face Material Type	Top Face Material ID	0° Web Material Type	0° Web Material ID
46	Isotropic	12	Isotropic	12

**See Section 7.16 for Material Names.*

7.10 I Beam Components



7.10.1 Dimensions

Component Dimensions										
Component ID	t_w (in)	H (in)	w_b (in)	θ (°)	w_t (in)	$t_{fl\ top}$ (in)	$t_{fl\ bottom}$ (in)	Unit Weight (lb / ft)	Area (ft^2)	Weight (lb)
17	0.15	7	4	90	4	0.15	0.15	4.234	31.68	134.1
18	0.15	6	4	90	3	0.15	0.15	3.658	44.22	161.7
19	0.15	7	2	90	2	0.15	0.15	3.082	22.32	68.79
20	0.15	7	4	90	4	0.15	0.15	4.234	18.3	77.47
21	0.095	6	2	90	2	0.095	0.095	1.789	16.65	29.8
22	0.095	6	2	90	2	0.095	0.095	1.789	5.879	10.52
23	0.095	6	2	90	2	0.095	0.095	1.789	1.567	2.804
24	0.1225	6	2	90	2	0.1225	0.1225	2.294	1.567	3.596

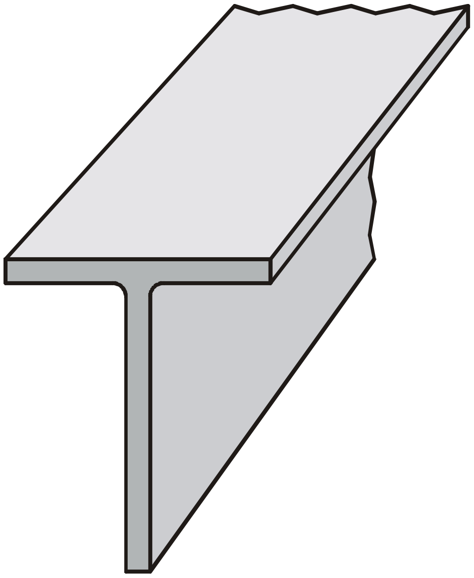
**See Section 7.15 for Variable Descriptions.*

7.10.2 Materials

Component Materials		
Component ID	Web Material Type	Web Material ID
17	Isotropic	12
18	Isotropic	12
19	Isotropic	12
20	Isotropic	12
21	Isotropic	12
22	Isotropic	12
23	Isotropic	12
24	Isotropic	12

**See Section 7.16 for Material Names.*

7.11 T Beam Components



7.11.1 Dimensions

Component Dimensions								
Component ID	t_w (in)	H (in)	θ (°)	w_t (in)	$t_{fl\ top}$ (in)	Unit Weight (lb / ft)	Area (ft^2)	Weight (lb)
7	0.25	4	90	4	0.25	2.348	87.5	205.5
8	0.25	4	90	4	0.25	2.348	58.34	137
9	0.25	4	90	4	0.25	2.348	50.01	117.4
11	0.145	2.714	90	4	0.145	1.154	50.01	57.73

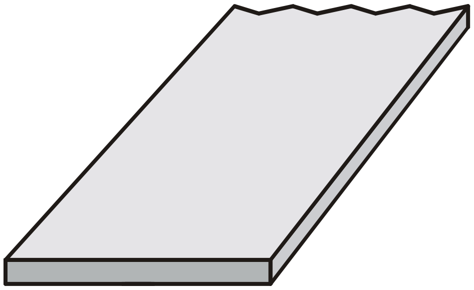
**See Section 7.15 for Variable Descriptions.*

7.11.2 Materials

Component Materials		
Component ID	Web Material Type	Web Material ID
7	Isotropic	4
8	Isotropic	4
9	Isotropic	4
11	Isotropic	4

**See Section 7.16 for Material Names.*

7.12 Cap Beam Components



7.12.1 Dimensions

Component Dimensions							
Component ID	H (in)	θ (°)	w_t (in)	$t_{fl\ top}$ (in)	Unit Weight (lb / ft)	Area (ft^2)	Weight (lb)
13	0.9612	90	4.2	0.9612	7.751	9	69.76
14	0.2048	90	3	0.2048	1.18	36.34	42.88
15	0.2533	90	3	0.2533	1.459	36.2	52.82

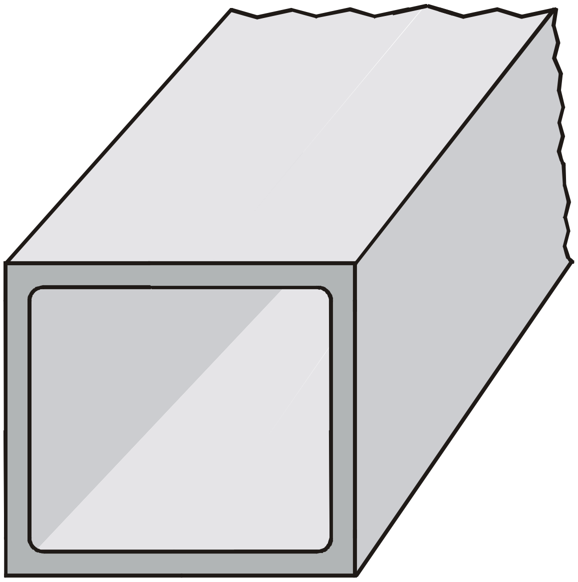
**See Section 7.15 for Variable Descriptions.*

7.12.2 Materials

Component Materials		
Component ID	Web Material Type	Web Material ID
13	Isotropic	12
14	Isotropic	12
15	Isotropic	12

**See Section 7.16 for Material Names.*

7.13 Rectangular Beam Components



7.13.1 Dimensions

Component Dimensions								
Component ID	t_{sw} (in)	H (in)	W (in)	t_{tw} (in)	t_{bw} (in)	Unit Weight (lb / ft)	Area (ft^2)	Weight (lb)
2	0.095	3.857	2	0.1025	0.1025	1.338	39.43	52.75
12	0.1225	2.429	2	0.085	0.085	1.083	9	9.745

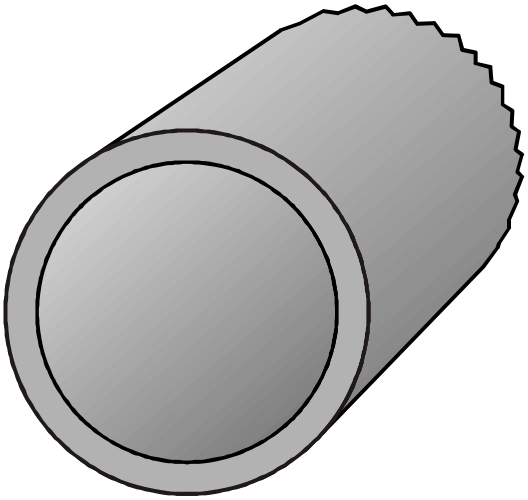
**See Section 7.15 for Variable Descriptions.*

7.13.2 Materials

Component Materials		
Component ID	Side Wall Material Type	Side Wall Material ID
2	Isotropic	4
12	Isotropic	4

**See Section 7.16 for Material Names.*

7.14 *Circular Tube Components*



7.14.1 Dimensions

Component Dimensions						
Component ID	t (in)	H (2a) (in)	W (2b) (in)	Unit Weight (lb / ft)	Area (ft^2)	Weight (lb)
3	0.028	12	12	2.022	18.2	36.8

**See Section 7.15 for Variable Descriptions.*

7.14.2 Materials

Component Materials		
Component ID	Thickness Material Type	Thickness Material ID
3	Isotropic	12

**See Section 7.16 for Material Names.*

7.15 Variable Symbols

Variable Symbols	
Variable Symbol	**Variable Description**
t_{tf}	Top Face - Thickness
H	Panel - Height
t_c	Core - Thickness
t_w	Core Web - Thickness
S	Corrugation - Spacing
w_b	Bottom Crown - Width
θ	Core Web - Angle
w_t	Top Flange - Width, hat only
w_{cs}	Top Clear Span - Free Width
t_{fl}	Top Flange - Thickness
t_{cr}	Bottom Crown - Thickness
$t_{fl\ top}$	Top Flange - Thickness
$t_{fl\ bottom}$	Bottom Flange - Thickness
t_0	0° Web - Thickness
t_{90}	90° Web - Thickness
S_0	0° Web - Stiffener Spacing
b_0	0° Web - Height
b_{90}	90° Web - Height
S_{90}	90° Web - Stiffener Spacing
S_θ	Angle Web - Stiffener Spacing
t_θ	Angle Web - Thickness
b_θ	Angle Web - Height
t_{sw}	Side Wall - Thickness
W	Beam - Width
t_{tw}	Top Wall - Thickness
t_{bw}	Bottom Wall - Thickness
t	Tube - Thickness
H (2a)	Tube - Height (2a)
W (2b)	Tube - Width (2b)

7.16 Material Key

Honeycomb Materials	
ID	**Material Name**
5	Honeycomb "ECK-3/16-2.0 (1.8)"
6	Honeycomb "ECA-1/8-1.8 (1.5)"
7	Honeycomb "ECK-3/16-4.0 (2.8)"
9	Honeycomb "Hexcel 5052 3.1pcf KMAT8%555"

Isotropic Materials	
ID	**Material Name**
4	Aluminum "Al 2024"
12	Titanium "Ti-6Al-4V, commonly used aerospace Titanium"
13	Aluminum "Al 2219"
16	Aluminum "Al 6061"

Laminate Materials	
ID	**Material Name**
17	User Laminates "[45/-45/0/90/0/90/45/-45/0]s IM7/8552 with layup allowables"
18	User Laminates "[0/90/45/-45/-45/45/90/0] IM7/8552 w adhesive ply_S with layup design allowables"
21	Collier Laminates "[+-45] Graphite BMI - adhesive added"

Layup Materials	
ID	**Material Name**
1	User Layups "Fabric CCM 30_[45/0/0 series]s"
8	User Layups "Fabric CCM 5_[45/0/45/0/45]"
10	User Layups "Fabric CCM 7_[45/0/0/45/0/0/45]"
11	User Layups "Fabric CCM 5_[45/0/0/0/45]"
14	7-8 plies; Unsymm; 0/45/90; 10% rule "7_[45/-45/0/45/0/-45/90]"
19	14&16 plies; Symm; 0/45/90; 45/-45 outside "16_[45/-45/0/0/0/0/45/-45]s"

Orthotropic Materials	
ID	**Material Name**
2	Graphite/Epoxy "CCM Fabric IM7 4HS 3K wet 113F KMAT8%250"
3	Equivalent Orthotropic, Ply Angle Percentages "Gr/Pi: Gen/PMR-15 Max Strain Failure Criteria (20% 45), (70% 0), (10% 90)"
15	Graphite/Epoxy "IM7/8552_Tape, Design and Layup Allowables"
20	Graphite/BMI "IM7-5250-4_Unnotched"

8 Weights of Groups

Group Panel Weight Summary				
Group	**Area (ft^2)**	**Unit Weight (lb / ft^2)**	**Weight (lb)**	**% Total Weight**
1 "Sandwich Composite CCM materials, Wing Skin" (Unstiffened Plate/Sandwich Panel Family)	137.7	1.167	160.8	11.11%
2 "Sandwich metallic, offline pressure, Wing Skin, bot. Carry thu" (Unstiffened Plate/Sandwich Panel Family)	31.25	1.798	56.18	3.88%
3 "Sandwich FWT analysis, Fuselage Barell Fwd" (Unstiffened Plate/Sandwich Panel Family)	73.75	0.7132	52.6	3.64%
4 "Sandwich Wrinkling, Nose Cone" (Unstiffened Plate/Sandwich Panel Family)	113.5	2.347	266.4	18.42%
5 "Laminate Equiv Orthotropic Optz, Wing Spars Ribs" (Unstiffened Plate/Sandwich Panel Family)	87.84	0.6092	53.51	3.70%
6 "Laminate Bolted Joint BJSFM, Wing Skin, Carry thu" (Unstiffened Plate/Sandwich Panel Family)	6.25	1.975	12.35	0.85%
1000001 "Starting Honeycom" (Unstiffened Plate/Sandwich Panel Family)	0	0	0	0.00%
1000002 "Starting Point for Honeycomb" (Unstiffened Plate/Sandwich Panel Family)	0	0	0	0.00%
7 "Hat Beam Column pressure effect" (Corrugated Stiffened Panel Family)	97.96	2.724	266.8	18.44%
8 "Hat Hybrid composite metallic panel" (Corrugated Stiffened Panel Family)	97.96	1.635	160.2	11.07%
1000004 "Beam Column Group, Hat" (Corrugated Stiffened Panel Family)	0	0	0	0.00%
1000005 "Beam Column Group, Hat (2)" (Corrugated Stiffened Panel Family)	0	0	0	0.00%
1000006 "Beam Column Group, Hat (3)" (Corrugated Stiffened Panel Family)	0	0	0	0.00%
1000007 "Beam Column Group, Hat (4) cp47" (Corrugated Stiffened Panel Family)	0	0	0	0.00%
1000008 "Beam Column pressure effect, Hat" (Corrugated Stiffened Panel Family)	0	0	0	0.00%
10 "Uniaxial, linked and statistical optz, Fuselage" (Uniaxial Stiffened Panel Family)	30.14	3.266	98.41	6.80%
11 "Uniaxial Local Buckling, Crippling and interaction, Fuselage" (Uniaxial Stiffened Panel Family)	43.06	1.961	84.44	5.84%
12 "Tee Panel, Joint Bond analysis and optz demo, Fuselage" (Uniaxial Stiffened Panel Family)	103.6	0.8086	83.81	5.79%
13 "Advance Composite Analyses, Wing Substructure Spars and Ribs" (Uniaxial Stiffened Panel Family)	15.6	1.366	21.31	1.47%
1000003 "adv opz 01Uniaxial Fuelage Five TOP Components" (Uniaxial Stiffened Panel Family)	0	0	0	0.00%
15 "OrthoGrid Panel Analysis, Wing Skin Root" (Grid Stiffened Panel Family)	10.21	3.514	35.89	2.48%
16 "IsoGrid, AngleGrid, General Grid, Wing Skin Root" (Grid Stiffened Panel Family)	20.44	4.603	94.08	6.50%
1000009 "Grid Stiffened" (Grid Stiffened Panel Family)	0	0	0	0.00%
1000010 "Grid stiffened including IsoGrid junk" (Grid Stiffened Panel Family)	0	0	0	0.00%
Totals	**869.3**	**1.664**	**1447**	

Group Beam Weight Summary				
Group	**Length (ft)**	**Unit Weight (lb / ft)**	**Weight (lb)**	**% Total Weight**
20 "Longerons" (Open Beam Family)	245.9	2.105	517.6	40.72%
21 "Ringframes" (Open Beam Family)	142.2	3.438	488.8	38.45%
22 "Spar Caps" (Open Beam Family)	81.54	2.029	165.5	13.02%
24 "Not Sized, rigid beams" (Open Beam Family)	0	0	0	0.00%
25 "Rectangular Beam Group" (Rectangular Beam Family)	48.43	1.291	62.5	4.92%
30 "Circular Beam Group" (Circular Beam Family)	18.2	2.022	36.8	2.90%
Totals	**536.2**	**2.371**	**1271**	

9 Weights of Components

Component Panel Weight Summary						
Group / Component	Area (ft^2)	Unit Weight (lb / ft^2)	Weight (lb)	Lowest MS	Controlling Load Case	Controlling Failure Mode
1 "Sandwich Composite CCM materials, Wing Skin"	**137.7**	**1.167**	**160.8**			
39 "Wing skin, bottom, Sandwich All Analyses"	84.38	1.141	96.24	0.008136	2	Composite Strength, Hoffman Interaction
40 "Wing skin, top, Composite Hoffman Ply Analysis, element loads"	14.04	1.404	19.72	0.1269	7	Composite Strength, Hoffman Interaction
41 "Wing skin, top, Composite Tsai-Wu Ply Analysis"	8.887	1.141	10.14	0.207	6	Composite Strength, Tsai-Wu Interaction
42 "Wing skin, top, Composite Laminate Analysis"	30.39	1.141	34.66	0.5051	2	Composite Strength, Laminate, Longitudinal Strain Compression
2 "Sandwich metallic, offline pressure, Wing Skin, bot. Carry thu"	**31.25**	**1.798**	**56.18**			
38 "Offline pressure, Flat panel, nearly square, user loads"	31.25	1.798	56.18	0.2409	6	Shear Strength, Y (Transverse) direction {Hexcel}
3 "Sandwich FWT analysis, Fuselage Barell Fwd"	**73.75**	**0.7132**	**52.6**			
34 "Fuselage, fwd, Sandwich Flat Wise Tension Analysis"	63.26	0.6568	41.55	1.66	7	Sandwich Face/Core Flatwise Tension
57 "Fuselage, fwd bottom"	10.5	1.053	11.05	0.07126	4	Wrinkling, Eqn 2, Honeycomb Core, X, Y & Interaction
4 "Sandwich Wrinkling, Nose Cone"	**113.5**	**2.347**	**266.4**			
33 "Nose Cone, Sandwich Wrinkling Analysis"	113.5	2.347	266.4	0.2992	7	Wrinkling, Eqn 1, Isotropic or Honeycomb Core, X, Y & Interaction
5 "Laminate Equiv Orthotropic Optz, Wing Spars Ribs"	**87.84**	**0.6092**	**53.51**			
58 "Wing ext spar, aft IB"	5.11	0.4032	2.06	0.4575	6	Composite Strength, Max Strain 1 Direction
59 "Wing ext spar, aft OB"	6.686	0.2419	1.618	0.04559	2	Composite Strength, Max Strain 1 Direction
60 "Wing ext spar, fwd IB"	1.357	0.3226	0.4377	0.04278	7	Composite Strength, Max Strain 1 Direction
61 "Wing ext spar, fwd MID"	4.877	0.3226	1.573	0.2483	4	Composite Strength, Max Strain 1 Direction
62 "Wing ext spar, fwd OB"	8.893	0.3226	2.868	0.4241	6	Composite Strength, Max Strain 1 Direction
63 "Wing int spar, fwd IB"	4.499	1.613	7.257	0.08828	7	Composite Strength, Max Strain 1 Direction

Component Panel Weight Summary						
Group / Component	Area (ft^2)	Unit Weight (lb / ft^2)	Weight (lb)	Lowest MS	Controlling Load Case	Controlling Failure Mode
64 "Wing int spar, fwd MID"	4.577	0.5645	2.584	0.08145	7	Composite Strength, Max Strain 1 Direction
65 "Wing int spar, fwd OB"	8.588	0.3226	2.77	0.4641	6	Composite Strength, Max Strain 1 Direction
66 "Wing int spar, mid IB"	4.499	0.1613	0.7257	0.6294	7	Composite Strength, Max Strain 1 Direction
67 "Wing int spar, mid MID"	4.035	0.2419	0.9762	0.02121	6	Composite Strength, Max Strain 1 Direction
69 "Wing int spar, aft IB"	4.499	1.452	6.531	0.1035	6	Composite Strength, Max Strain 1 Direction
70 "Wing int spar, aft MID"	3.735	0.4838	1.807	0.07955	4	Composite Strength, Max Strain 1 Direction
71 "Wing int spar, aft OB"	6.757	0.1613	1.09	0.4747	5	Composite Strength, Max Strain 1 Direction
72 "Wing rib, fwd OB"	2.074	0.2419	0.5018	0.6803	7	Composite Strength, Max Strain 1 Direction
75 "Wing rib, aft OB"	2.074	0.1613	0.3345	1.619	4	Composite Strength, Max Strain 1 Direction
76 "Wing rib, fwd IB"	3.083	0.7258	2.237	0.1184	7	Composite Strength, Max Strain 1 Direction
77 "Wing rib, mid fwd IB"	6.25	1.532	9.576	0.03815	6	Composite Strength, Max Strain 1 Direction
78 "Wing rib, mid aft IB"	6.25	1.371	8.568	0.01176	5	Composite Strength, Max Strain 1 Direction
6 "Laminate Bolted Joint BJSFM, Wing Skin, Carry thu"	**6.25**	**1.975**	**12.35**			
30 "CCM Sleight Parachute Bolted Joint BJSFM, Solid Laminate"	6.25	1.975	12.35	0.005804	6	Joint, Bolted, Single Hole, BJSFM, loaded and far field
1000001 "Starting Honeycom"	**0**		**0**			
1000002 "Starting Point for Honeycomb"	**0**		**0**			
7 "Hat Beam Column pressure effect"	**97.96**	**2.724**	**266.8**			
35 "beam-column pressure, offline pressure calc for bending"	78	2.637	205.7	0.01345	6	Crippling - Buckling interaction, Johnson-Euler
80 "beam-column pressure, FEA loads for bending"	9.911	3.451	34.2	0.1472	6	Isotropic Strength, Von Mises Interaction Yield Criterion
81 "beam-column initial imperfection"	10.05	2.679	26.93	0.08934	6	Isotropic Strength, Von Mises Interaction Yield Criterion
8 "Hat Hybrid composite metallic panel"	**97.96**	**1.635**	**160.2**			

Component Panel Weight Summary						
Group / Component	Area (ft^2)	Unit Weight (lb / ft^2)	Weight (lb)	Lowest MS	Controlling Load Case	Controlling Failure Mode
37 "Fuselage, aft, composite and metallic analysis"	97.96	1.635	160.2	2.198	5	Composite Strength, Max Stress 1 Direction
1000004 "Beam Column Group, Hat"	**0**		**0**			
1000005 "Beam Column Group, Hat (2)"	**0**		**0**			
1000006 "Beam Column Group, Hat (3)"	**0**		**0**			
1000007 "Beam Column Group, Hat (4) cp47"	**0**		**0**			
1000008 "Beam Column pressure effect, Hat"	**0**		**0**			
10 "Uniaxial, linked and statistical optz, Fuselage"	**30.14**	**3.266**	**98.41**			
43 "Fuselage, Sta 180, T0"	6.083	2.765	16.82	0.2039	7	Isotropic Strength, Longitudinal Direction
44 "Fuselage, Sta 210, T0"	6.055	3.046	18.44	0.02729	4	Panel Buckling, Curved or Flat, All BC
45 "Fuselage, Sta 240, T0"	6.027	3.551	21.4	0.06277	4	Panel Buckling, Curved or Flat, All BC
50 "Fuselage, Sta 270, T0"	5.999	3.734	22.4	0.02895	4	Panel Buckling, Curved or Flat, All BC
51 "Fuselage, Sta 300, T0"	5.972	3.24	19.35	0.05843	4	Local Buckling, Interaction
11 "Uniaxial Local Buckling, Crippling and interaction, Fuselage"	**43.06**	**1.961**	**84.44**			
52 "I concept with local bucking of spacing span"	8.6	1.61	13.84	0.214	2	Local Buckling, Interaction
53 "I concept"	8.606	2.667	22.95	0.003402	2	Isotropic Strength, Transverse Direction
54 "Fuselage, Sta 240, T45, I concept"	8.612	1.939	16.7	0.01125	2	Isotropic Strength, Transverse Direction
55 "Fuselage, Sta 270, T45, Tee Concept"	8.618	1.795	15.47	0.05124	4	Panel Buckling, Curved or Flat, All BC
56 "Fuselage, Sta 300, T45, Tee Concept"	8.622	1.795	15.48	0.001187	6	Local Buckling, Longitudinal Direction
12 "Tee Panel, Joint Bond analysis and optz demo, Fuselage"	**103.6**	**0.8086**	**83.81**			
36 "Fuselage, mid aft, Joint optz Tee panel"	103.6	0.8086	83.81	0.08782	1	Joint, Bonded, Adhesive, Longitudinal & Transverse Shear Strain
13 "Advance Composite Analyses, Wing Substructure Spars and Ribs"	**15.6**	**1.366**	**21.31**			
68 "Wing int spar, mid OB"	7.301	1.291	9.427	2.891	5	Composite Strength, Laminate, Open Hole Compression (OHC) after impact

Component Panel Weight Summary

Group / Component	Area (ft^2)	Unit Weight (lb / ft^2)	Weight (lb)	Lowest MS	Controlling Load Case	Controlling Failure Mode
73 "Wing rib, mid fwd OB, Bonded Joint, Fracture"	4.148	1.574	6.53	-0.7764	7	Joint, Bonded, Fracture, Principal Transverse
74 "OHT/OHC ply only"	4.148	1.291	5.357	0.1279	5	Composite Strength, Open Hole Compression (OHC) after impact
1000003 "adv opz 01Uniaxial Fuelage Five TOP Components"	**0**		**0**			
15 "OrthoGrid Panel Analysis, Wing Skin Root"	**10.21**	**3.514**	**35.89**			
47 "Wing skin, Crippling Analysis"	10.21	3.514	35.89	0.1788	2	Local Buckling, Interaction
16 "IsoGrid, AngleGrid, General Grid, Wing Skin Root"	**20.44**	**4.603**	**94.08**			
46 "Wing skin, forward top"	5.12	4.36	22.32	0.0508	7	Isotropic Strength, Von Mises Interaction Yield Criterion
48 "Wing skin, Local Buckling Analysis"	10.21	5.481	55.98	0.06259	6	Isotropic Strength, Von Mises Interaction Yield Criterion
49 "Wing skin, aft top"	5.106	3.09	15.78	0.03964	6	Isotropic Strength, Von Mises Interaction Yield Criterion
1000009 "Grid Stiffened"	**0**		**0**			
1000010 "Grid stiffened including IsoGrid junk"	**0**		**0**			
Totals	**869.3**	**1.664**	**1447**			

Component Beam Weight Summary

Group / Component	Length (ft)	Unit Weight (lb / ft)	Weight (lb)	Lowest MS	Controlling Load Case	Controlling Failure Mode
20 "Longerons"	**245.9**	**2.105**	**517.6**			
7 "Longerons, forward"	87.5	2.348	205.5	-0.8269	4	Beam Buckling, Column Plane 2, I2
8 "Longerons, mid forward"	58.34	2.348	137	-0.6348	4	Beam Buckling, Column Plane 2, I2
9 "Longerons, over the wing"	50.01	2.348	117.4	-0.8519	6	Beam Buckling, Column Plane 2, I2
11 "Longerons, aft"	50.01	1.154	57.73	0.0708	2	Local Buckling, Longitudinal Direction
21 "Ringframes"	**142.2**	**3.438**	**488.8**			
17 "Ringframes, T0"	31.68	4.234	134.1	-0.5332	4	Local Buckling, Interaction
18 "Ringframes, T45"	44.22	3.658	161.7	0.007635	6	Local Buckling, Longitudinal Direction
19 "Ringframes, T90"	22.32	3.082	68.79	0.05828	7	Crippling - Buckling interaction, Johnson-Euler
20 "Ringframes, T115"	18.3	4.234	77.47	-0.4222	7	Local Buckling, Interaction
21 "Ringframes, T140"	16.65	1.789	29.8	0.2256	4	Local Buckling, Interaction
22 "Ringframes, T165"	5.879	1.789	10.52	0.4793	4	Local Buckling, Interaction
23 "Ringframe, nose upper"	1.567	1.789	2.804	0.4814	6	Local Buckling, Interaction
24 "Ringframe, nose lower"	1.567	2.294	3.596	0.1866	7	Local Buckling, Interaction

Component Beam Weight Summary						
Group / Component	**Length (ft)**	**Unit Weight (lb / ft)**	**Weight (lb)**	**Lowest MS**	**Controlling Load Case**	**Controlling Failure Mode**
22 "Spar Caps"	**81.54**	**2.029**	**165.5**			
13 "Spar caps top, carry thru"	9	7.751	69.76	0.009623	7	Isotropic Strength, Longitudinal Direction
14 "Spar caps, bottom"	36.34	1.18	42.88	0.1498	4	Local Buckling, Interaction
15 "Spar caps, top"	36.2	1.459	52.82	0.02862	5	Crippling, Isotropic, method Niu, formed and extruded sections
24 "Not Sized, rigid beams"	**0**		**0**			
25 "Rectangular Beam Group"	**48.43**	**1.291**	**62.5**			
2 "Rectangular Beam Component"	39.43	1.338	52.75	0.04745	7	Isotropic Strength, Longitudinal Direction
12 "Spar caps bot, carry thru"	9	1.083	9.745	0.0226	7	Isotropic Strength, Longitudinal Direction
30 "Circular Beam Group"	**18.2**	**2.022**	**36.8**			
3 "Circular Beam Component"	18.2	2.022	36.8	0.049	4	Beam Buckling, Cylindrical, Axial and Bending, NASA SP-8007
Totals	**536.2**	**2.371**	**1271**			

10 Calculations for Stress and Strain Analyses

10.1 Analysis Approach

Analysis Methods:
00= Analysis Approach

10.1.1 Approach Introduction

The Stress Report distinguishes the analysis methods into two broad classes. The first is the method used to compute detailed stresses and strains throughout a panel or beams cross section depth. This is covered breifly in the beginning of the stress report. The second major class of analysis methods is for predicting failure, and much more equations are presented. Therefore, the bulk of this Calculation section is devoted to the methods used to predict failure modes and loads.

10.1.2 Panel Sign Convention

Each FEM package has a slightly different sign convention. HyperSizer handles all of these small differences automatically. All HyperSizer analyses are performed and reported using the sign convention of Fig. 1.

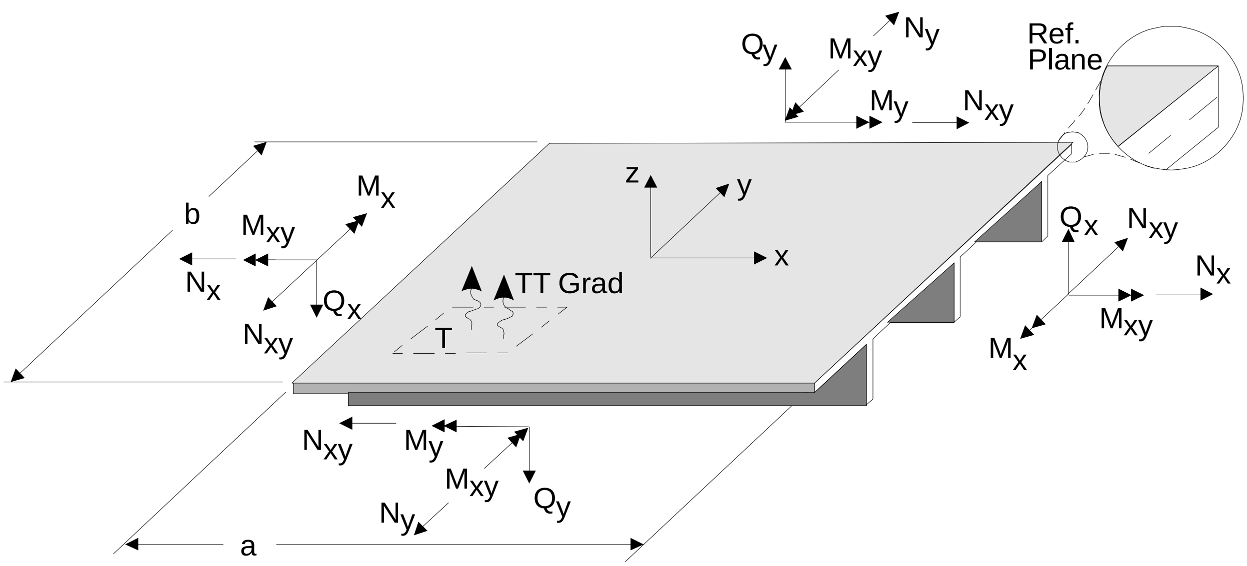


Fig. 1, Panel sign convention follows that of NASTRAN FEA.

10.1.3 Beam Sign Convention

Each FEM package has a vastly differing sign conventions for beam elements. HyperSizer handles all of these small differences automatically. All HyperSizer analyses are performed and reported using the sign convention of Fig. 2.

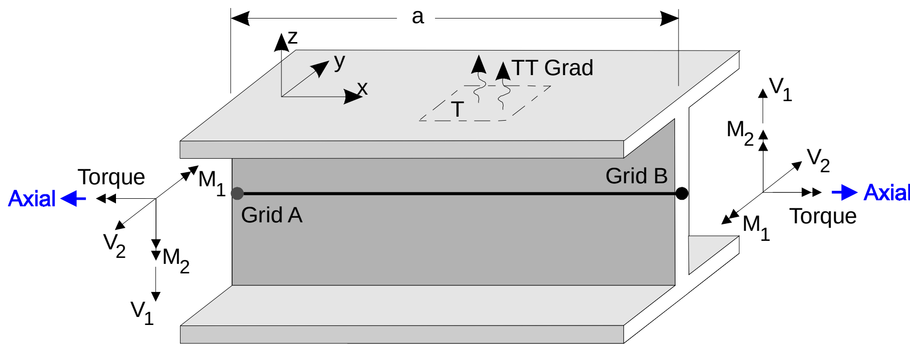


Fig. 2. Beam sign convention follows that of NASTRAN FEA.

11 Calculations for Failure Analyses

11.1 Ply Properties & Allowable Correction Factors

Ply property and allowable correction factor data is generated only for single component analysis.

11.2 *Material Strength, Composite, Ply and Laminated Based In-Plane Failure Criteria*

Analysis Methods: Undamaged or notched allowables can be used	
Ply Based	
135 = Max Strain 1 Direction	**141 = Tsai-Hill Interaction**
136 = Max Strain 2 Direction	**142 = Tsai-Wu Interaction**
137 = Max Strain 12 Direction	**143 = Tsai- Hahn Interaction**
138 = Max Stress 1 Direction	**144 = Hoffman Interaction**
139 = Max Stress 2 Direction	
140 = Max Stress 12 Direction	
146 = Hashin Matrix Cracking	
147 = Hashin Fiber Failure	**151 = Open Hole Tension (OHT)**
148 = LaRC03 Matrix Cracking	**152 = Open Hole Compression (OHC) after impact**
149 = LaRC03 Fiber Failure	**203 = Interlaminar Shear**
Laminate Based	
153 = Longitudinal Strain Tension	**156 = Open Hole Tension (OHT)**
154 = Longitudinal Strain Compression	**157 = Open Hole Compression (OHC) (after impact)**
155 = Shear Strain	

Three primary approaches are provided by HyperSizer for quantifying composite material strength. The **first** and mostly frequently used approach is based on ***macro*** analysis that establishes failure on the ply level. The typical failure criteria such as max strain, max stress, Tsai-Wu, etc are applied. Also provided are ply level ***physical based*** theories such as Hashin's and the recently developed NASA Langley LaRC03 criteria which identifies a piecewise smooth quadratic failure surface where each piece of the surface represents seven unique failure modes such as tensile fiber mode, compressive fiber mode, tensile matrix mode, compressive matrix mode, shear matrix mode, etc. The **second** approach which is purely empirical establishes failure on the laminate level and not to an individual lamina. One of the provided methods is called ***Angle-Minus-Load (AML)***, or sometimes referred to as Angle-Minus-Longitudinal and defines strength as a percentage of plies in the fundamental 0/±45/90 directions. The AML theory was developed within the aircraft industry by Boeing. It attempts to define strength with an assumed amount of damage. This laminate failure theory is advantageous for design due to its ease of use. However, the AML criterion only defines failure of the laminate without identifying the ply level failure or the mode of failure. Also laminate based allowable approaches require more test data, and that test data is not typically publicly available. The **third** approach which is more computational is based on ***micro*** analysis and establishes failure on the fiber/matrix constituent level. A generalized method of cells formulation is implemented that models the fiber and matrix as a repeating unit cell, hence pure fiber and pure matrix data is input. With all three approaches provided by HyperSizer, a balance is achieved between ease of structural design and detailed analysis.

An extension to the first described ply based analysis is to provide correction factors to account for fabrication effects such as co-cure, environmental effects such as temperature and humidity, and hard vs. soft laminate effects such as the percent of plies in the different 0/±45/90 directions. The ply based failure criteria are performed using corrected factors multiplied to the stress/strain allowables.

11.2.1 Ply Based Failure Prediction

This section lists the margins-of-safety for all of the user selected failure criteria turned on from the HyperSizer failure tab.

11.2.1.1 Symbols

X_t = Tension Stress allowable of ply in the longitudinal (1) direction

X_c = Compression Stress allowable of ply in the longitudinal (1) direction

Y_t = Tension Stress allowable of ply in the transverse (2) direction

Y_c = Compression Stress allowable of ply in the transverse (2) direction

S = Compression Stress allowable of ply in the transverse (2) direction

Fundamental Failure Theories

Analysis ID = 135-137 Max Strain
Analysis ID = 138-140 Max Stress

Quadratic Failure Theories

Analysis ID = 141 Tsai-Hill Interaction
Analysis ID = 142 Tsai-Wu Interaction

The Tsai-Wu criterion, [142.1] unlike directional criteria such as max stress or max strain, is based on a single relationship for a biaxial with in-plane shear stress field. Under plane stress conditions (i.e. using classical lamination theory), for a unidirectional ply, this criterion predicts failure when,

[Equation Removed] **(142.1)**

where X_t and X_c are the tensile and compressive strengths in the fiber direction, Y_t and Y_c are the tensile and compressive strengths in the transverse direction, and *S* is the in-plane shear strength.

The interaction term, F_{12}, that involves σ_{11} and σ_{22} cannot be determined via a uniaxial ply level test as can the other strengths and typically has only a minor effect on the criterion's prediction. Therefore, F_{12} it is often set to zero. This results in the following criterion which is implemented in the HyperSizer software as the 'Tsai-Wu' criterion,

[Equation Removed] **(142.2)**

Analysis ID = 143 Tsai-Hahn Interaction

Component ID:	30
Load Case:	6: Mechanical Load Set #105 (Run Deck #1) "
Object ID:	Top Stack
Ply Number:	1

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1400.	700.0	1243.	0.	0.	0.	700.0	490.0

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	1400.	700.0	1243.	0.	0.	0.	72.0	0.

A modification to the Tsai-Wu theory was proposed by Tsai and Hahn [143.1], which estimates the F_{12} coefficient as,

[Equation Removed] **(143.1)**

This results in an interaction criterion that takes the form:

[Equation Removed] **(143.2)**

Equation 143.2 is implemented in HyperSizer as the Tsai-Hahn criterion.

Variable	Value, units
X_t	57.25 klb/in^2
X_c	39.36 klb/in^2
Y_t	57.25 klb/in^2
Y_c	39.36 klb/in^2
S	12.84 klb/in^2
σ_{11}	16.14 klb/in^2
σ_{22}	-7.4935 klb/in^2
τ_{12}	-0.3006 klb/in^2
MS	**1.4496**

Analysis ID = 144 Hoffman Interaction
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "
Object ID: Top Face
Ply Number: 1

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	-267.6	-1264.	-1119.	-158.5	-740.1	-585.4	-54.98	-75.04

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-144.8	-690.3	-665.0	0.004003	0.01069	0.02793	209.9	0.

The Hoffman criterion predicts failure using a the same five terms as Tsai-Wu and Tsai-Hahn, only the last term is different

[Equation Removed] **(144.1)**

Equation 144.2 is implemented in HyperSizer as Hoffman criterion.

[Equation Removed] **(144.2)**

This interaction equation is rewritten as a margin of safety and performed for each ply in the laminate. The lowest MS for any given ply is reported.

[Equations Removed] **(144.3)**

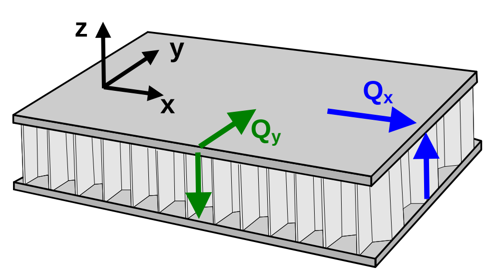
Variable	Value, units
X_t	57.25 klb/in^2
X_c	39.36 klb/in^2
Y_t	57.25 klb/in^2
Y_c	39.36 klb/in^2
S	12.84 klb/in^2
σ_{11}	-34.39 klb/in^2
σ_{22}	13.72 klb/in^2
τ_{12}	-0.7696 klb/in^2
MS	**0.008136**

The above equation applies if the ply is stronger in the 1 direction. If the ply is stronger in the 2 direction, that is if Yt > Xt, then F12=-F22/2.

11.3 Material Strength, Composite, Ply Based Out-of-Plane Failure Criteria

This analysis is for either solid laminates, laminates of stiffened panels, or laminates of sandwich panels that have out-of-plane Qx and Qy loads. This analysis does not quantify σ_{33}. σ_{33} peel stresses are computed and used in the strength margins for bonded joint analyses.

Out-of-plane Qx and Qy loads developed on a sandwich panel will most likely be more critical for the core material and not the interlaminar strength of the laminate.



Out-of-plane shear forces, Q_x and Q_y in a honeycomb sandwich panel.

To assess the interlaminar shear stress failure of a laminate subjected to out-of-plane shear loading, Q_x and Q_y, HyperSizer determines ply-by-ply interlaminar shear stresses. Classical Lamination Theory (CLT), or even higher order theories (such as first order shear deformable), do not provide the correct interlaminar stress distribution through the laminate thickness. In fact, CLT provides identically zero interlaminar shear stresses, whereas the higher-order plate theories provide piece-wise profiles that are discontinuous at ply interfaces.

HyperSizer resolves out-of-plane shear forces Q_x and Q_y into ply-by-ply shear stresses τ_{xz} and τ_{yz} using a newly developed technique called the "***Simplified Shear Solution" (SSS)***, based on composite beam theory. The SSS is extremely efficient as it provides a closed form rather than a numerical solution which can run thousands of times per second. The method works in concert with CLT, which provides the in-plane ply-by-ply stresses, to provide a better approximation of the true multi-axial stress state within laminates and panels. The details of the SSS analysis method are presented in [205.1].

The generality of SSS allows it to be applied to any of HyperSizer's panel concepts including unstiffened laminates, sandwiches, uniaxial and grid stiffened concepts. The SSS assumes that the force, moment, and shear resultants are known at a particular location in a panel and is independent of the source of these known resultant quantities, permitting HyperSizer to read these from FEA or from user typed values from the FBD tab.

11.3.1.1 Symbols

A_{ij}	= Components of the classical lamination theory ABD stiffness matrix
E_x, E_y	= Elastic moduli of an individual ply in global laminate coordinates
E_1, E_2	= Elastic moduli of an individual ply in local ply coordinates
$\overline{E}_x, \overline{E}_y$	= Equivalent elastic modulus of a laminate in the analysis direction
n_k	= Ratio of ply modulus to laminate modulus
Q_x, Q_y	= Out-of-plane resultant shear forces
$\overline{Q}_{ij}$	= Ply reduced stiffness matrix from classical lamination theory
S_x	= Panel stiffener spacing
τ_{xz}, τ_{yz}	= Out-of-plane shear stresses in global laminate coordinates
τ_{13}, τ_{23}	= Out-of-plane shear stresses in local ply coordinates
t_{ply}	= Ply thickness

11.3.1.2 Solution Equations

The Simplified Shear Solution (SSS), presented in detail in the reference, calculates the shear stress at any location in a composite laminate from the applied shear resultant force as:

[Equation Removed]

where *i* = *x* or *y*, *k* is the layer number, $\overline{I}$ is the effective moment of inertia per unit width of the laminate (see appendix), and the coordinate $\hat{z}$ is the distance from the laminate neutral axis. The parameter, n_k, is the ratio between the elastic modulus of ply *k* and the average modulus of the laminate, or:

[Equation Removed]

The SSS then requires only the equivalent elastic modulus of each ply in the analysis direction, E_i^k. These elastic properties are not the fiber and matrix elastic constants, E_1 and E_2, but rather are the moduli of each ply rotated into global laminate coordinates using the individual ply angles. These equivalent properties are found using Classical Lamination Theory (CLT) as:

[Equation Removed]

where $\overline{Q}_{ij}$ are members of the ply reduced stiffness matrix, which are available to HyperSizer for Classical Lamination Theory (CLT) analysis. Note that all three elastic constants given above are needed to treat the laminate as a plate (plane strain) rather than a beam (plane stress).

The SSS returns ply-level shear stresses at any point in the laminate in global xy laminate coordinates. However, most failure analysis methods require stress allowables in terms of ply local (12) coordinates. Therefore, stresses in global coordinates for any ply must be transformed into global coordinates according to:

[Equation Removed]

where θ is the ply angle.

Out-of-Plane , Z Axis Failure Theories

Analysis ID = 203 Interlaminar Shear	
Component ID:	41
Load Case:	6: Mechanical Load Set #105 (Run Deck #1) "
Object ID:	Top Face
Ply Number:	5

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1166.	3333.	166.5	752.0	2151.	107.4	1667.	832.5

Once the ply local stresses are known, failure is assumed to occur if the following interaction equation is violated:

[Equation Removed] **(203.1)**

where Fsu_{13} and Fsu_{23} are the ply material stress allowables in the 13 and 23 directions respectively, which can be found in references such as MIL-HDBK-17. This interaction equation is rewritten as a margin of safety and performed for each ply in the laminate. The lowest MS for any given ply is reported.

[Equation Removed] **(203.2)**

Variable	Value, units
Fsu_{13}	8120. lb/in^2
Fsu_{23}	8120. lb/in^2
τ_{xz}	1291. lb/in^2
τ_{yz}	644.9 lb/in^2
θ	45.0 °
τ_{13}	1369. lb/in^2
τ_{23}	-456.9 lb/in^2
MS	**4.6266**

11.3.2 References

- 142.1 Tsai, S.W. and Wu, E.M. (1971) "A General Theory of Strength for Anisotropic Materials"
- *Journal of Composite Materials*, 58-80.
- 143.1 Tsai, S.W. and Hahn, H.T. (1980) *Introduction to Composite Materials*, Technomic Pub. Co., 1980.
- 203.1 Collier Research Corporation, 2007, "A Simplified Shear Solution for Laminate Interlaminar Shear"

11.4 Material Strength, Isotropic (Metals) Failure Criteria

Analysis Methods:

110 = Isotropic Strength, Longitudinal Direction
111 = Isotropic Strength, Long Transverse Direction
112 = Isotropic Strength, Shear Direction
113 = Isotropic Strength, Von Mises Interaction Yield Criteria

11.4.1 Approach Summary

HyperSizer's failure criteria for isotropic, and in particular ductile metallic materials, are based on various well-accepted industry practices and recommendations from MIL-HDBK-5J [1], which is the industry standard for metallic material properties and failure prediction.

The material properties shown in Table 1, provided by MIL-HDBK-5J, are entered into the HyperSizer database for the isotropic failure mechanisms. For two of these properties, the material properties are not available from the handbook directly, but are calculated from other properties based MIL-HDBK-5J recommendations.

Table 1. Material properties available from MIL-HDBK-5J and entered into HyperSizer.

Limit/ Yield	Ultimate	Description
F_{tyL}	F_{tuL}	Tensile strength in the longitudinal (parallel to grain) direction
F_{tyLT}	F_{tuLT}	Tensile strength in the long transverse (perpendicular to grain) direction. "Long" refers to the longest dimension parallel to the grain, also called the "width" direction. There is a "short transverse" direction as well (e.g. FtyST), but it is not widely available for many materials in MIL-HDBK-5 and is not included in HyperSizer
F_{cyL}	F_{cuL}*	Compressive strength in the longitudinal (parallel to grain) direction. *F_{cuL} and F_{cuLT} are not listed for materials in MIL-HDBK-5, and are not entered into HyperSizer directly. HyperSizer implements F_{cuL}=F_{tuL} and F_{cuLT}=F_{tuLT} as recommended by MIL-HDBK-5J.
F_{cyLT}	F_{cuLT}*	Compressive strength in the long transverse direction.
F_{sy}**	F_{su}	Shear strength. **F_{sy} is not given for most materials in MIL-HDBK-5 and is not entered into HyperSizer directly. The procedure implemented in HyperSizer for calculating this property was recommended by MIL-HDBK-5J and is discussed below.

11.4.2 Allowables

MIL-HDBK-5J provides material allowables for ultimate shear stress, however for most materials does not provide a shear yield stress allowable. For a purely isotropic material, without preference between tension and compression allowables, the theoretical allowable for pure shear is $F_{\underline{sy}} = F_{ty}/2$ where F_{ty} is the normal yield stress determined from a uniaxial tensile test. Most real materials, however, exhibit strength preferences both between tension and compression and between the longitudinal and long transverse direction, as seen in MIL-HDBK-5. The handbook recommendation for estimating the proper shear yield stress allowable is (Section 9.8.4.6.2) given on page 9-202:

[Equation Removed] **(D.1)**

Variable	Value, units
F_{tyL}	36.0 klb/in^2
F_{tyLT}	35.0 klb/in^2
F_{cyL}	35.0 klb/in^2
F_{cyLT}	36.0 klb/in^2
F_{su}	27.0 klb/in^2
F_{tuL}	42.0 klb/in^2
F_{tuLT}	42.0 klb/in^2
F_{sy}	**22.82 klb/in^2**

11.4.3 Equations

Analysis ID = 110 Isotropic Strength, Longitudinal Direction
Component ID: 9
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "
Object ID: Web

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-69600.	-15400.	-64200.	-21250.	-6906.	-1354.	-92540.	-799.5

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-17850.	0.	-1842.	-16.91	0.	-26.86	72.0	0.

Limit Loads — Ultimate Loads

[Equation Removed] ***[Equation Removed]*** **(110.1)**

Limit loads:

Variable	Value, units
F_{cyL}	59.0 klb/in^2
$\sigma_x < 0$	-77.18 klb/in^2
MS	**-0.2356**

Ultimate Loads:

Variable	Value, units
F_{cuL}	67.0 klb/in^2
$\sigma_x < 0$	-115.8 klb/in^2
MS	**-0.4213**

Analysis ID = 111 Isotropic Strength, Long Transverse Direction
Component ID: 81
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "
Object ID: Open Span

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	-3666.	889.1	-289.7	-2744.	-28.72	-112.9	-101.3	-6.1584

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-901.9	889.1	-289.7	-10.89	-28.72	-0.1095	72.0	0.

[Equation Removed] **(111.1)**

Limit Loads:

Variable	Value, units
F_{tyLT}	53.0 klb/in^2
$\sigma_y > 0$	35.48 klb/in^2
MS	**0.4936**

Ultimate Loads:

Variable	Value, units
F_{tuLT}	65.0 klb/in^2
$\sigma_y > 0$	0. klb/in^2
MS	**0.2212**

Analysis ID = 112 Isotropic Strength, Shear Direction
Component ID: 54
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "
Object ID: Top Clear Span

Panel Loads:

	Nx	Ny	Nxy	Mx	My	Mxy	Qx	Qy

	lb/in	lb/in	lb/in	lb in/in	lb in/in	lb in/in	lb/in	lb/in
Strength Loads	155.9	1775.	929.2	99.5	3.654	8.1698	107.8	0.3003

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	137.9	1775.	929.2	1.5579	4.409	0.567	72.0	0.

[Equation Removed] **(112.1)**

Limit Loads:

Variable	Value, units
F_{sy}	22.82 klb/in^2
τ_{xy}	8.0976 klb/in^2
MS	**1.8183**

Ultimate Loads:

Variable	Value, units
F_{su}	27.0 klb/in^2
τ_{xy}	12.15 klb/in^2
MS	**1.2229**

Analysis ID = 113 Isotropic Strength, Von Mises Interaction Yield Criteria
Component ID: 9
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "
Object ID: Web

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-69600.	-15400.	-64200.	-21250.	-6906.	-1354.	-92540.	-799.5

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-17850.	0.	-1842.	-16.91	0.	-26.86	72.0	0.

The margin of safety for the Von Mises interaction criteria (Eq. 5 and 6) is found by following the procedure outlined in [6], Chapter 15.

Limit Loads:

[Equation Removed] **(113.1)**

Variable	Value, units
F_{tyL}	59.0 klb/in^2
F_{tyLT}	58.0 klb/in^2
F_{cyL}	59.0 klb/in^2
F_{cyLT}	58.0 klb/in^2
F_{sy}	34.93 klb/in^2
σ_y	0. klb/in^2
σ_x	-77.18 klb/in^2
τ_{xy}	-6.6301 klb/in^2
MS	**-0.2435**

Ultimate Loads:

[Equation Removed] **(113.2)**

Variable	Value, units
F_{tuL}	67.0 klb/in^2
F_{tuLT}	67.0 klb/in^2
F_{cuL}	67.0 klb/in^2
F_{cuLT}	67.0 klb/in^2
F_{su}	40.0 klb/in^2
σ_x	-115.8 klb/in^2
σ_y	0. klb/in^2
τ_{xy}	-9.9452 klb/in^2
MS	**-0.4272**

11.4.4 References

- Department of Defense Handbook, Metallic Materials And Elements For Aerospace Vehicle Structures, MIL-HDBK-5J, April 2003.
- Beer, F. and Johnston, E., Mechanics of Materials. McGraw Hill Book Company, New York, 1981.
- Hill, R. "A Theory of the Yielding and Plastic Flow of Anisotropic Metals". Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences, Vol. 193, Issue 1033, 1948.
- Higdon et. al., Mechanics of Materials. John Wiley and Sons, New York, 1985.
- NASA Preferred Reliability Practices, "Structural Stress Analysis", PRACTICE NO. PD-AP-1318, April, 1996.
- Collier Research Corporation, SBIR Final Report: Consistent Structural Integrity and Efficient Certification with Analysis, Air Force Research Lab (AFRL) SBIR Phase II contract # F33615-02-C-3216, October 2004, Volume 3.

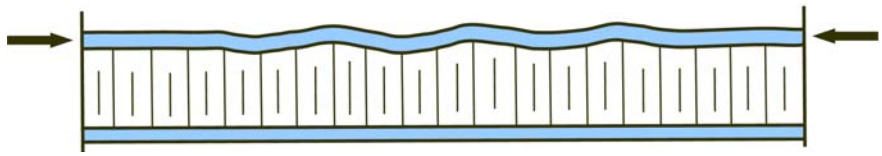
11.5 Sandwich Panel Facesheet Wrinkling

Analysis Methods:

090 = Wrinkling, Eqn 1, Isotropic or Honeycomb Core, X, Y & Interaction

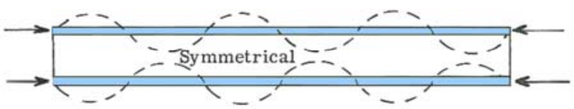
091 = Wrinkling, Eqn 2, Honeycomb Core, X, Y & Interacton

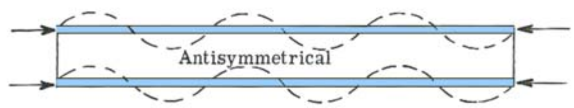
11.5.1 Approach Summary



Sandwich structures with thin facesheets and lightweight cores are prone to a type of local failure known as facesheet wrinkling. The term wrinkling refers to local, short wavelength buckling phenomenon of the facesheet, with mode shapes having wavelengths up to the thickness of the core. The small buckling wavelength of the wrinkling mode results in the allowable load being insensitive to structural boundary conditions and curvature. Sandwich structures exhibit little or no post-wrinkling load carrying capability, therefore failure of these structures by wrinkling is typically catastrophic. As a consequence, accurate prediction of wrinkling is important to quantifying structural integrity of sandwich structures.

There are two distinct wrinkling modes: symmetrical and antisymmetrical. The following equations handle both.





11.5.2 Equations

There are two primary mathematical models from which wrinkling equations are derived. The first is general and based on solid isotropic cores [1, 4]

[Equation Removed] **(E.1)**

The second is suitable for honeycomb cores (anti-plane math model), [1, 3].
[Equation Removed] **(E.2)**

Table 3 of reference 1 provides the suggested factors to use with the wrinkling allowable stress equations, which are:

$$k_1 = 0.63$$

$$k_2 = 0.82$$

The factors k1 and k2 are not empirically derived from test data but are rather theoretically based values that are derived from the physics of sandwich facesheet wrinkling.

11.5.3 Composite Materials

Wrinkling equations are based on isotropic materials. Reference 1, equation (15) includes terms for specially orthotropic composite facesheets. However, as implied in Reference 1, a better approach is to use the validated isotropic equations and substitute for Ef, not an equivalent membrane Young's modulus,

[Equation Removed] **(E.3)**

but rather an equivalent flexural modulus based on the Dij of the laminate, (Ref. 1, eqn (18)). HyperSizer computes this term using the D11-1 term from the inverted 6x6 A, B, D matrix.

[Equation Removed] **(E.4)**

11.5.4 Effect of Adhesive

Reference 2, shows that including the effect of a 0.005" thick adhesive layer on the theoretical wrinkling stress of a 0.010" thick facesheet on a 1.0" thick core was to increase this wrinkling stress by 50%. Since analyses do not include the effect of an adhesive layer (if such a layer exists) the wrinkling stress of sandwich panels with very thin facesheets are likely overly conservative.

11.5.5 Combined Loads

Most test data validation is performed on a "strut" specimen loaded uniaxially. Little research or test data exists for combined biaxial loading with or without shear. One procedure to predict wrinkling caused by combined loads is to use the maximum principal facesheet compressive stress. This approach requires that not only major and minor principal stresses be computed, but also the major and minor principal stress allowables. Another procedure is to rotate the actual stresses into a coordinate system with axes parallel to the core ribbon and transverse directions. This approach takes on the interaction equation of the following form:

[Equation Removed] **(E.5)**

HyperSizer implements this approach but extends to it by determining allowable wrinkling stresses in the two directions: $\sigma_{x,wr}$ and $\sigma_{y,wr}$. The Equation section and Appendix 1 of this document describes the procedure in more detail, and how to properly determine the margin-of-safety (MS).

11.5.6 Symbols

E_c	= Through-the-thickness elastic modulus of core
E_f	= Elastic modulus of facesheet
$E_{f,x}$	= x direction, elastic modulus of facesheet
$E_{f,y}$	= y direction, elastic modulus of facesheet
k_1	= Wrinkling factor for equation 1
k_2	= Wrinkling factor for equation 2
σ	= Stress

σ_{wr}	= Wrinkling stress allowable
τ_{xy}	= In-plane shear stress of facesheet
t_f	= Facesheet thickness
t_c	= Core thickness
R_a	= Membrane stress ratio (applied / allowable) of facesheet
R_s	= Shear stress ratio (applied / allowable) of facesheet
Fcw_x	= Wrinkling stress allowable due to biaxial loads
Fcw	= Wrinkling stress allowable due to loads in the ribbon direction
Fsw	= Wrinkling allowable stress due to inplane shear loads
K	= Factor on wrinkling allowable due to ribbon vs. transverse strength of core

Note, unless otherwise noted, x direction is the core ribbon direction.

11.5.7 Equations

Equations 1 and 2 show the wrinkling stress as an allowable. Below are the corresponding margin-of-safety equations (MS). MS equations for combined stresses are presented in the Appendix.

Analysis_ID = 90: Eqn #1, wrinkling isotropic cores
Component ID: 33
Load Case: 7: Mechanical Load Set #106 (Run Deck #1) "
Object ID: Bottom Face

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	-5556.	-202.2	-158.2	-3266.	-178.7	-162.8	-180.8	-19.12

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-3038.	-166.1	-151.3	-0.2267	-0.05668	-0.06293	72.0	0.

[Equation Removed] **(90.1)**

Variable	Value, units
D^{-1}_{11}	0.002658 $(\text{lb in})^{-1}$
D^{-1}_{22}	0.002658 $(\text{lb in})^{-1}$
t_f	0.075 in
$E_{f,x}$	10700. klb/in^2
$E_{f,y}$	**10700. klb/in^2**

***x* direction**

[Equation Removed] **(90.2)**

[Equation Removed] **(90.3)**

Variable	Value, units
σ_x	-40.5 klb/in^2
$\sigma_{wr,x}$	-53.11 klb/in^2
k_1	0.63
$E_{f,x}$	10700. klb/in^2
E_c	8.0 klb/in^2
G_{cl}	7.0 klb/in^2
MS_x	**0.3113**

***y* direction**

[Equation Removed] **(90.4)**

[Equation Removed] **(90.5)**

Variable	Value, units
σ_y	-2.2146 klb/in^2
$\sigma_{wr,y}$	-46.18 klb/in^2
K_{hc}	1.0
k_1	0.63
$E_{f,y}$	10700. klb/in^2
E_c	8.0 klb/in^2
G_{cw}	4.6 klb/in^2
MS_y	**18.81**

where:
$K_{hc} = 1$ isotropic (e.g. foam) core
$K_{hc} = 0.95$ honeycomb core

Interaction

The form of the MS equation depends on whether σy is compressive or tensile.

[Equation Removed] **(90.6)**

Variable	Value, units
σ_x	-40.5 klb/in^2
σ_y	-2.2146 klb/in^2
$\sigma_{wr,x}$	-53.11 klb/in^2
$\sigma_{wr,y}$	-46.18 klb/in^2
σ_{wr}	**-52.75 klb/in^2**

σ_x and σ_y compressive:

[Equation Removed] **(90.7)**

Variable	Value, units
σ_x	-40.5 klb/in^2
σ_y	-2.2146 klb/in^2
σ_{xy}	-2.0171 klb/in^2
σ_{wr}	-52.75 klb/in^2
MS_i	**0.2992**

If either σ_x or σ_y are positive (i.e. in tension), the tensile term is not used in the interaction equation,

σ_x or σ_y tension: *[Equation Removed]* **(90.8)**

Analysis_ID = 91: Eqn #2, wrinkling honeycomb cores
Component ID: 57
Load Case: 4: Mechanical Load Set #103 (Run Deck #1) "
Object ID: Top Face

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1931.	-957.6	289.6	763.4	-168.2	121.4	-2.8182	-1.2048

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	1078.	-769.6	154.0	-0.008369	0.02166	-.6835E-03	72.0	0.

[Equation Removed] **(91.1)**

Variable	Value, units
D^{-1}_{11}	0.09146 (lb in)$^{-1}$
D^{-1}_{22}	0.09146 (lb in)$^{-1}$
t_f	0.02 in
$E_{f,x}$	16400. klb/in^2
$E_{f,y}$	**16400. klb/in^2**

***x* direction**

[Equation Removed] **(91.2)**

[Equation Removed] **(91.3)**

Variable	Value, units
σ_x	Tensile or 0
$\sigma_{wr,x}$	-44.9 klb/in^2
k_2	0.82
$E_{f,x}$	16400. klb/in^2
Ec	8.0 klb/in^2
t_f	0.02 in
t_c	0.875 in
MS_x	**N.A.**

y direction

[Equation Removed] **(91.4)**

[Equation Removed] **(91.5)**

Variable	Value, units
σ_y	-38.48 klb/in^2
$\sigma_{wr,y}$	-44.9 klb/in^2
K_{hc}	0.95
k_2	0.82
$E_{f,y}$	16400. klb/in^2
E_c	8.0 klb/in^2
t_f	0.02 in
t_c	0.875 in
MS_y	**0.1087**

where:
$K_{hc} = 1$ isotropic (e.g. foam) core
$K_{hc} = 0.95$ honeycomb core

Interaction

The form of the MS equation depends on whether σ_y is compressive or tensile.

[Equation Removed] **(91.6)**

Variable	Value, units
σ_x	Tensile or 0
σ_y	-38.48 klb/in^2
$\sigma_{wr,x}$	-44.9 klb/in^2
$\sigma_{wr,y}$	-44.9 klb/in^2
σ_{wr}	**-44.9 klb/in^2**

σ_y and σ_x compressive:

[Equation Removed] **(91.7)**

If either σ_x or σ_y are positive (i.e. in tension), then that term is not used in the interaction equation,

σ_x or σ_y tension: *[Equation Removed]* **(91.8)**

Variable	Value, units
σ_x	Tensile or 0
σ_y	-38.48 klb/in^2
σ_{xy}	7.6994 klb/in^2
σ_{wr}	-44.9 klb/in^2
MS_i	**0.07126**

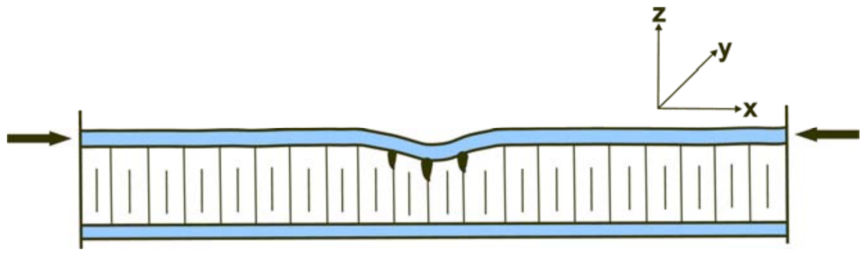
11.5.8 References

- Ley, R. P., Lin, W., Mbanefo, U., "Facesheet Wrinkling in Sandwich Structures", NASA CR-1999-208994, 1999.
- Gutierrez, A.J. and Webber, J.P.H., "Flexural Wrinkling of Honeycomb Sandwich Beams with Laminated Faces," International Journal of Solids and Structures, Vol. 16, 1980, pp. 645-651.
- Hexcel, "The Basics of Bonded Sandwich Construction," TSB 124, 1982.
- Bruhn, E.F., "Analysis & Design of Flight Vehicle Structures", January 1965, C12.5.3.
- Zhang, J.Z., Collier, C.S., "Margin of Safety from Interaction Equation", Collier Research Internal Document, February 12, 2004.

11.6 Sandwich Panel Facesheet Intracell Dimpling

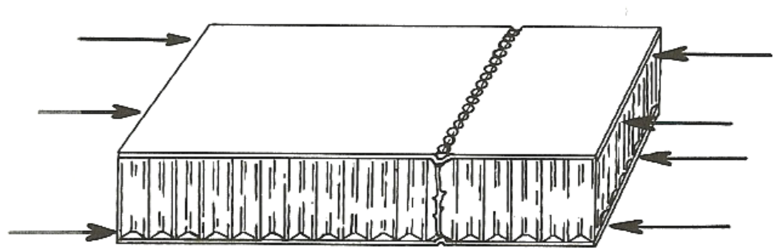
Analysis Methods:
94 = Intracell Dimpling, X, Y & Interaction

11.6.1 Summary Approach



Intracell Dimpling, or Intracell buckling, is a failure specific to honeycomb sandwich concepts that is caused by local instability of the facesheets. If the face thickness of a honeycomb sandwich is reduced while cell size and material are held constant, a thickness will eventually be reached at which the facesheet will buckle between the cell walls. A typical Intracell dimpling failure is shown in the figure below.

11.6.2 Allowable Equation



The Intracell dimpling allowable stress is primarily a function of facesheet material stiffness, thickness and honeycomb cell size. The equation used to determine this stress is from [1].

$$\sigma_{dp} = \frac{2E_f}{\left(1-\nu^2\right)}\left(\frac{t_f}{S}\right)^2 \qquad \textbf{(E.6)}$$

11.6.3 Composite Materials

The above defined Intracell dimpling equation is based on isotropic materials and is based on the facesheet elastic modulus, E_f. However, in order to use the validated isotropic equation (1) for orthotropic materials, the assumption is that it is a better approach to substitute for E_f, not an equivalent membrane Young's modulus,

[Equation Removed] **(E.7)**

but rather an equivalent flexural modulus based on the Dij of the laminate. HyperSizer computes this term using the D_{ii}^{-1} terms from the inverted 6x6 ABD matrix.

[Equation Removed] **(E.8)**

In addition, to account for the orthotropic nature of the material, the Poison's ratio term (1- ν_2) is replaced by the equivalent Poisson's ratio term for orthotropic materials, (1-ν_{12} ν_{21}).

11.6.4 Combined Loads

Most test data validation is performed on a "strut" specimen loaded uniaxially. Little research or test data exists for combined biaxial loading with or without shear. The approach used here is suggested by Bruhn [2]. The procedure is to rotate the actual stresses into a coordinate system with axes parallel to the core ribbon and transverse directions. This approach takes on the interaction equation of the following form:

[Equation Removed] **(E.9)**

where *n* is a function of the ratio, (S/t_f). HyperSizer implements this approach but extends it by determining allowable dimpling stresses in the two directions: $\sigma_{dp,x}$ and $\sigma_{dp,y}$. The Equation section describes the procedure in more detail, and how to properly determine the margin-of-safety (MS) from the interaction equation.

11.6.5 Symbols

ν	= Poisson's Ratio
t_f	= Thickness of facesheet
S	= Cell size
σ	= Normal stress in facesheet
$E_{f,x}$	= x direction, flexural modulus of facesheet
$E_{f,y}$	= y direction, flexural modulus of facesheet
σ_x	= Facesheet stress in x (ribbon) direction
σ_y	= Facesheet stress in y (transverse) direction
A_{11}, A_{22} D_{11}, D_{22}	= Stiffness terms from A, B, D matrix for facesheet

11.6.6 Equations

Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "
Object ID: Top Face

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	-267.6	-1264.	-1119.	-158.5	-740.1	-585.4	-54.98	-75.04

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-144.8	-690.3	-665.0	0.004003	0.01069	0.02793	209.9	0.

[Equation Removed] **(94.1)**

Variable	Value, units
D^{-1}_{11}	0.03549 $(lb\ in)^{-1}$
D^{-1}_{22}	0.03549 $(lb\ in)^{-1}$
t_f	0.0405 in
$E_{f,x}$	5090. klb/in^2
$E_{f,y}$	**5090. klb/in^2**

x direction

[Equation Removed] **(94.2)**

[Equation Removed] **(94.3)**

Variable	Value, units
σ_x	-3.5758 klb/in^2
$\sigma_{dp,x}$	-215.7 klb/in^2
$E_{f,x}$	5090. klb/in^2
ν_{12}	0.3739 in
ν_{21}	0.3739 in
t_f	0.0405 in
S	0.3 in
MS_x	**59.32**

y direction

[Equation Removed] **(94.4)**

[Equation Removed] **(94.5)**

Variable	Value, units
σ_y	-17.04 klb/in^2
$\sigma_{dp,y}$	-215.7 klb/in^2
$E_{f,y}$	5090. klb/in^2
ν_{12}	0.3739 in
ν_{21}	0.3739 in
t_f	0.0405 in
S	0.3 in
MS_y	**11.65**

Biaxial Loads with Shear

In this case, *x* is the direction (either ribbon or transverse) of greatest ***compressive*** stress and *y* is the direction with least compressive (or tensile) stress. The form of the MS equation depends on whether σ_y is compressive or tensile. Margins of safety for biaxial and shear loads are given in terms of the dimpling allowable stress, σ_{dp}, which is computed independently in the *x* and *y* directions from Equation (1).

[Equation Removed] **(94.6)**

Variable	Value, units
σ_x	-3.5758 klb/in^2
σ_y	-17.04 klb/in^2
$\sigma_{dp,x}$	-215.7 klb/in^2
$\sigma_{dp,y}$	-215.7 klb/in^2
σ_{dp}	**-215.7 klb/in^2**

σ_x and σ_y compressive:

[Equation Removed] **(94.7)**

Variable	Value, units
σ_x	-3.5758 klb/in^2
σ_y	-17.04 klb/in^2
σ_{xy}	-16.42 klb/in^2
σ_{dp}	-215.7 klb/in^2
MS_i	**6.0066**

If either σ_x or σ_y are positive (i.e. in tension), the tensile term is not used in the interaction equation,

σ_x or σ_y tension: ***[Equation Removed]*** **(94.8)**

11.6.7 References

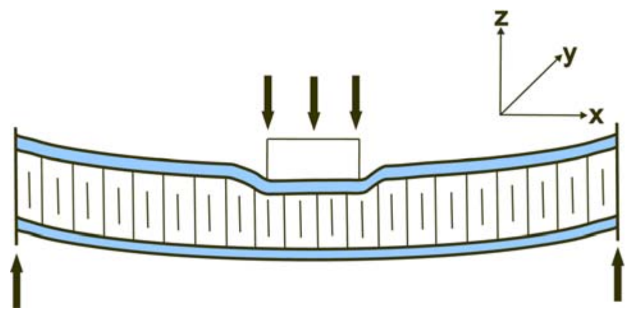
- Hexcel, "The Basics of Bonded Sandwich Construction," TSB 124, 1982.
- Bruhn, E.F., "Analysis & Design of Flight Vehicle Structures", January 1965, C12.5.3.
- Zhang, J.Z., Collier, C.S., "Margin of Safety from Interaction Equation", Collier Research Internal Document, February

11.7 Sandwich Panel Core Crushing

Analysis Methods:
- **100 = Core Crushing Concentrated Load (not yet implemented)**
- **101 = Core Crushing Flexural Bending Load (not yet implemented)**
- **102 = Core Crushing Joint Support Load**

11.7.1 Approach Summary

There are three different types of loadings that can cause core crushing. The first is from a concentrated load. The second is caused by flexural bending moments. The third is caused by joint support loads.



For all three types of loadings, the choice will be available for the user to compare calculated core compressive stress to either: crush, bar, or stabilized material allowables.

$$User\,Choice\;\;of\,(Fcu_{crush}, Fcu_{bare}, Fcu_{stablized})$$

The HyperSizer default is to take the lowest of these three core material allowables.

11.7.2 Symbols

d	= sandwich depth between facesheet neutral axis (sheet midplanes)
D_{ij}	= sandwich bending stiffness
f_{cc}	= compressive core stress in the normal (Z axis)
Fcu_{crush}	= compressive, through-thickness crush strength of core (strength after exceeding initial ultimate failure, a constant post failure load allowable)
Fcu_{bare}	= compressive, through-thickness bare strength of core (higher than the crush strength but lower than facesheet stabilized strength)
$Fcu_{stabilized}$	= compressive, through-thickness stabilized strength of core (higher than crush or bare allowable)
p	= unit pressure loading
Q_x	= out-of-plane shear load in x
Q_y	= out-of-plane shear load in y
W_{eff}	= effective width of support bearing upon sandwich panel
Z_c	= compressive, through-thickness strength of core (general term)

11.7.3 Equations

Core Crushing Concentrated Load, Analysis ID = 100

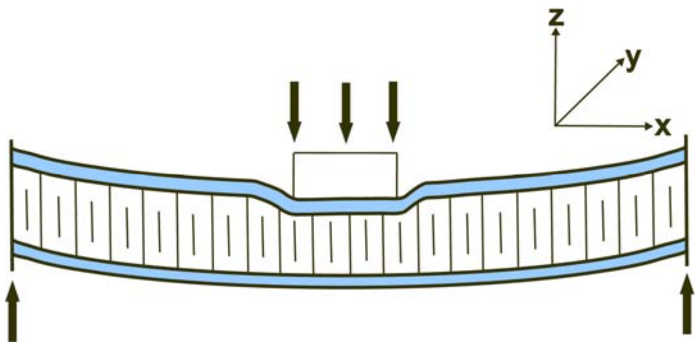
This type of failure occurs when a concentrated load (pressure) bears down in the normal direction, reference 1.

Generic **(100.1)**

[Equation Removed]

HyperSizer

[Equation Removed]



* Software will calculate MS from minimum of Fcu_{crush} or $Fcu_{stabilized}$. User controls which allowable is considered by creating alternate materials with either crush or stabilized properties removed.

Core Crushing Flexural Bending Load, Analysis ID = 101
Component ID: 39
Load Case: 7: Mechanical Load Set #106 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	2486.	1210.	951.1	1177.	339.7	577.5	38.56	68.55

This type of failure occurs when bending moment is high and the facesheets thick and the core is weak. This type of failure is uncommon in airframe structures [2].

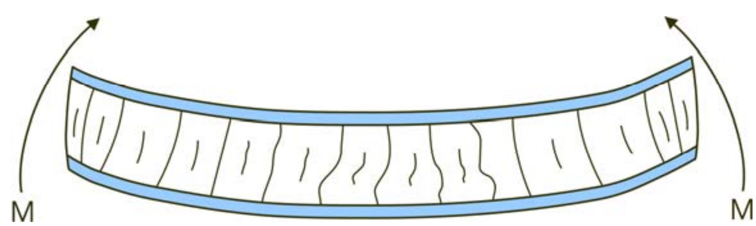
Generic

[Equation Removed]

[Equation Removed]

HyperSizer

[Equation Removed]



(100.2)

Variable	Value, units
M_x	-426.9 lb in/in
M_y	-441.0 lb in/in
d	1.2905 in
D_{11}	536300. lb in^2/in
D_{11}	536300. lb in^2/in
f_{cc}	**0.2634 lb/in^2**

[Equation Removed] **(100.3)**

Variable	Value, units
f_{cc}	0.2634 lb/in^2
Fcu_{crush}	165.0 lb/in^2
$Fcu_{stabilized}$	280.0 lb/in^2
Fcu_{min}	165.0 lb/in^2
MS_{crush}	**625.5**

* Software will calculate MS from minimum of Fcu_{crush} or $Fcu_{stabilized}$. User controls which allowable is considered by creating alternate materials with either crush or stabilized properties removed.

Core Crushing Joint Support Load, Analysis ID = 102

This type of loading occurs when a sandwich panel is continuous across and supported by underlying substructure such as wing spars and ribs, or by fuselage ringframes or shape control members, Fig 1. This is in contrast to joint designs that have closeouts or rampdowns at the sandwich panel edges, such as in Fig 2, in which case the out-of-plane Qx and Qy shear loads do not cause concentrated compressive core stresses.

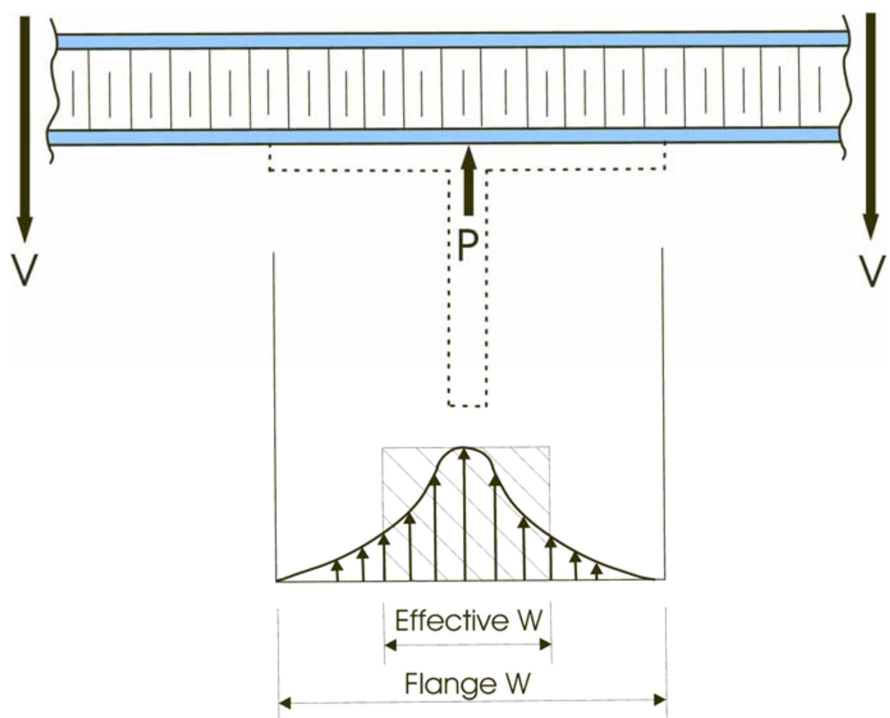


Fig 1. Compressive stress on sandwich panel from support bearing load P.

Failure mode of Fig 1 is HyperSizer a default toggled on analysis. It uses the FEA computed shell element Q_x and Q_y forces to obtain an appropriate concentrated load. In many cases of high out-of-plane (normal) pressure loading, this failure mode will control the optimization. If this failure mode is toggled off, consideration should be given to the extra weight required to obtain a panel edge closeout, such as Fig 2. HyperSizer does not yet perform rampdown closeout analysis.

Panel out-of-plane edge forces Q_x and Q_y (V) are equivalent to a running (unit) "P"

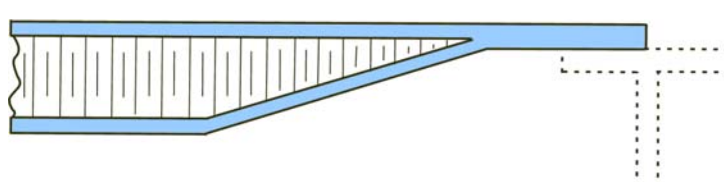


Fig. 2, Sandwich Rampdown closeout design.

force. This force “P” is mostly concentrated in the vicinity of the stiffener, with less force exerted by the more flexible flanges. This applied force distribution is represented by the curve of Fig 1, where the length of the upward pointing vertical lines indicates a relative magnitude. The area under this curve is equal to “P”. Another representation is to assume the “P” force is supported by an effective width of the stiffener. The same peak stress times the effective width is depicted by the rectangle, and this area also equals “P”. The key being that the peak compressive stress equals the same for both representations, and is the value used for failure prediction. An appropriate effective width for aerospace joints is assumed to be 1”.

Generic

[Equation Removed]

[Equation Removed]

HyperSizer

Component ID: 38
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1500.	150.0	450.0	3192.	1705.	495.4	295.5	295.5

[Equation Removed] **(102.1)**

Variable	Value, units
W_{eff}	1.0 in
Q_x	295.5 lb/in
Q_y	295.5 lb/in
Q_{max}	295.5 lb/in
f_{cc}	**295.5 lb/in^2**

[Equation Removed] **(102.2)**

Variable	Value, units
f_{cc}	295.5 lb/in^2
Fcu_{crush}	471.0 lb/in^2
$Fcu_{stabilized}$	471.0 lb/in^2
Fcu_{min}	471.0 lb/in^2
MS_{crush}	**0.5941**

* Software will calculate MS from minimum of Fcu_{crush} or $Fcu_{stabilized}$. User controls which allowable is considered by creating alternate materials with either crush or stabilized properties removed.

11.7.4 References

- Hexcel, “The Basics of Bonded Sandwich Construction,” TSB 124, 1982.

- Boeing, “Advanced Composite Design Handbook,” Rev B, p 4.6-12.

11.8 Sandwich Core Flatwise Tension Analysis

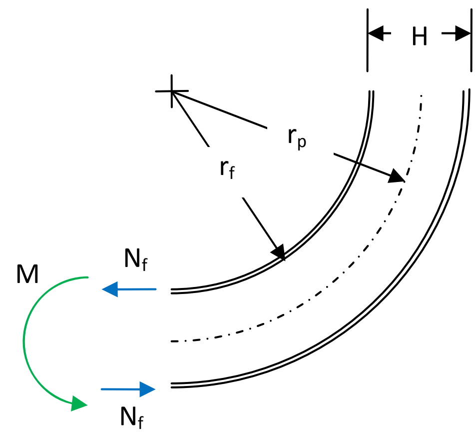
Analysis Methods:
103 = Sandwich Core Flatwise Tension Analysis

11.8.1

Summary approach

The flatwise tension analysis is implemented for the case of pure bending moment applied to a curved sandwich panel. This effect can become significant for cases where the ratio of panel radius of curvature to panel thickness becomes small. The flatwise tension stress is a function of the radius of each facesheet (the radius of the inner facesheet is smaller than that of the outer facesheet) and is calculated seperately for each face.

There are additional flatwise tension effects due to out-of-plane shear that are not accounted for by this method. These effects will be included in future HyperSizer releases.



11.8.2 Symbols

M = Bending moment
N_f = Force on facesheet in direction of curvature
r_p = Radius of curvature of curved panel
r_f = Radius of curvature at facesheet midplane
σ_{rr} = Flatwise tensile stress
Ftu_{core} = Flatwise tensile strength of core
H = Total panel height

11.8.3 Equations

For any curved composite in pure bending moment, the radial stress, σ_{rr} and any radial location can be expressed [1] as:

[Equation Removed] **(E.10)**

Where H is the total panel height. For a sandwich panel, assuming that the in-plane stiffness of the core is negligible compared to the stiffness of the facesheets, the moment, M can be expressed approximately as:

[Equation Removed] **(E.11)**

Where N is the force in each facesheet due to the imposed bending moment. For a positive bending moment, the sign of N will be compression in the upper face and tension in the lower face. If the panel is curved, substituting Equation E.11 into Equation E.10 gives:

[Equation Removed] **(E.12)**

Analysis_ID = 103: Sandwich Core Flatwise Tension Analysis
Component ID: 34
Load Case: 7: Mechanical Load Set #106 (Run Deck #1) "
Object ID: Bottom Face
Panel Radius: 44.99 in
Facesheet Radius: 44.74 in
Object is the Inner face of the sandwich with respect to the curvature.

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1716.	1729.	768.0	673.1	682.4	315.2	28.37	16.1

The radial stress is:

[Equation Removed] **(103.1)**

Variable	Value, units
N_f	1312. lb/in
r_f	44.74 in
σ_{rr}	**29.33 lb/in^2**

The margin of safety is:

[Equation Removed] **(103.2)**

Variable	Value, units
σ_{rr}	29.33 lb/in^2
Ftu_{core}	78.0 lb/in^2
MS	**1.6596**

11.8.4 References

- Keith Kedward, “Flexure of Simply Curved Composite Shapes,” Composites volume 20, number 6, November 1989.

11.9 Sandwich Panel Shear Crimping

Analysis Method:
104 = Shear Crimping, X, Y & Interaction (Hexcel)

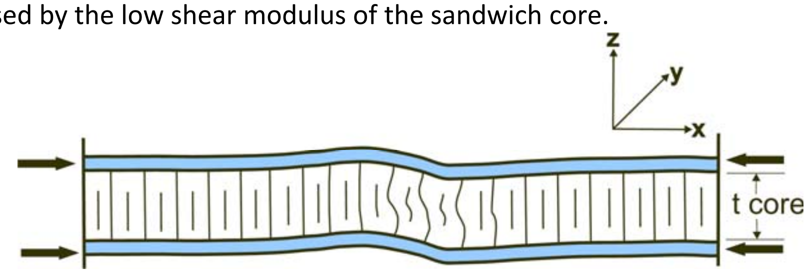
Component: 57
Load Case ID: 4: Mechanical Load Set #103 (Run Deck #1) " (Ultimate loads)
Minimum MS: 0.5534

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1931.	-957.6	289.6	763.4	-168.2	121.4	-2.8182	-1.2048

11.9.1 Summary Approach

Shear Crimping is a general panel instability failure mode which manifests as a short wavelength buckle. This failure is caused by the low shear modulus of the sandwich core.



11.9.1.1 Allowable Equation

The shear crimping allowable force is a function of the core depth (t_{core}), the shear stiffness in the direction of loading and the facesheet thickness. There are several different approaches to shear crimping in the literature. Reference [1] uses only the core thickness and shear stiffness in its allowable calculation.

[Equation Removed] **(104.1)**

References [104.2] and [104.3] begin to account for the moment of inertia of both the facesheets and core by providing alternate allowable calculations. Reference [104.2]:

[Equation Removed] **(104.2)**

Reference [104.3]:

[Equation Removed] **(104.3)**

We believe that neither of these forms fully accounts for the effects of facesheet flexibility on the shear crimping strength. Because crimping is a short wave phenomena, it does not seem that Eq 104.2 or 104.3 properly account for the facesheets, especially if their thickness is large in comparison to the core. If this is the case, the bending and shear resistance of the facehseets must be accounted for in the shear crimping calculation. In other words, the energy required for the facesheets to obtain the crimping deformation should be included. This energy is the lesser of the out-of-plane shearing (short-beam) or the cylindrical bending deformation of the facesheet.

We suggest the following equation for shear crimping:

[Equation Removed] **(104.4)**

where K_c and K_f are factors that specify the proportions to which facesheet and core affect the crimping allowable. By inspection, this is a more general equation to account for the facesheet contribution. As we collect shear crimping test data, these factors will be determined. For now, the default calculation in HyperSizer only accounts for the core, therefore K_c = 1 and K_f = 0.

11.9.1.2 Symbols

G	= Out-of-plane shear modulus of core
G_ℓ	= Out-of-plane shear modulus of core in ribbon direction
G_ω	= Out-of-plane shear modulus of core in transverse direction
t_{core}	= Core thickness of core
θ	= Angle between ribbon direction and principal loading direction
N	= Force per unit length
N_x	= Force per unit length in *x* (ribbon) direction
N_y	= Force per unit length in *y* (transverse) direction
N_I	= Force per unit length in principal coordinates

11.9.2 Equations

Component ID: 57
Load Case: 4: Mechanical Load Set #103 (Run Deck #1) "

Generic

[Equation Removed] **(104.5)**

HyperSizer

***X* direction:**

[Equation Removed] **(104.6)**

Variable	Value, units

G_l	13.0 klb/in^2
t_{core}	1.25 in
N_x	-769.9 lb/in
MS	**20.11**

***Y* direction:**

[Equation Removed] **(104.7)**

Variable	Value, units
G_w	1.7 klb/in^2
t_{core}	0.875 in
N_y	-957.6 lb/in
MS	**0.5534**

Biaxial Loads with Shear

If in-plane shear is present, the loads are rotated into principal coordinates using Mohr's circle approach. So from *Nx*, *Ny*, we get the principal stresses, N_I, N_{II}, which are the major and minor principal stresses respectively. Rotate the out-of-plane shear term into the direction of principal coordinates according to:

[Equation Removed] **(104.8)**

where θ is the angle between the ribbon direction and the principal loading direction.

Variable	Value, units
θ	84.33 °
G_ω	3.2 klb/in^2
G_l	**1.7 klb/in^2**
GIZ	**3.1854 klb/in^2**

[Equation Removed] **(104.9)**

Variable	Value, units
G_{IZ}	3.1854 klb/in^2
t_{core}	0.875 in
N_I	-986.3 lb/in
MS	**1.8259**

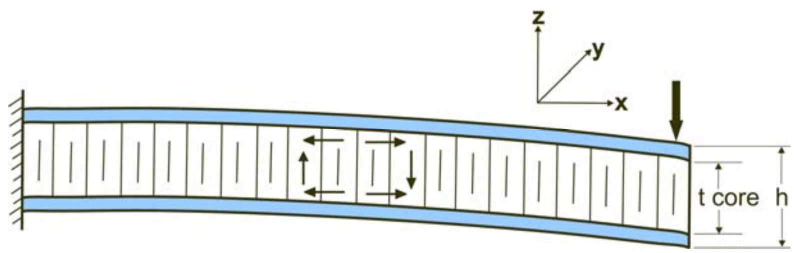
11.9.3 References

- Hexcel, "The Basics of Bonded Sandwich Construction," TSB 124, 1982.
- Bruhn, E.F., "Analysis & Design of Flight Vehicle Structures", January 1965, C12.5.3.
- Boeing, "Advanced Composite Design Handbook," Rev B, p 4.6-12

11.10 Sandwich Panel Shear Strength

Analysis Methods:
105 = Shear Strength, X (longitudinal) direction (Hexcel)
106 = Shear Strength, Y (transverse) direction (Hexcel)
107 = Shear Strength, Interaction

11.10.1 Summary approach



The shear strength failure calculation is a comparison of the shear strength in the core to the out-of-plane shear loads induced by cantilevered loads (as shown here) or pressure loads, which are common in aerospace applications. In the case of coupling of HyperSizer with FEA, the shear loads in the panels, Q_x and Q_y are extracted directly from the element forces of the FEA results.

11.10.2 Allowable Equation

The shear strength allowable is the shear strength in the core multiplied by a shear strength correction factor, which is derived from vendor data. The form of the shear strength allowable is taken from Reference 1.

[Equation Removed] **(E.13)**

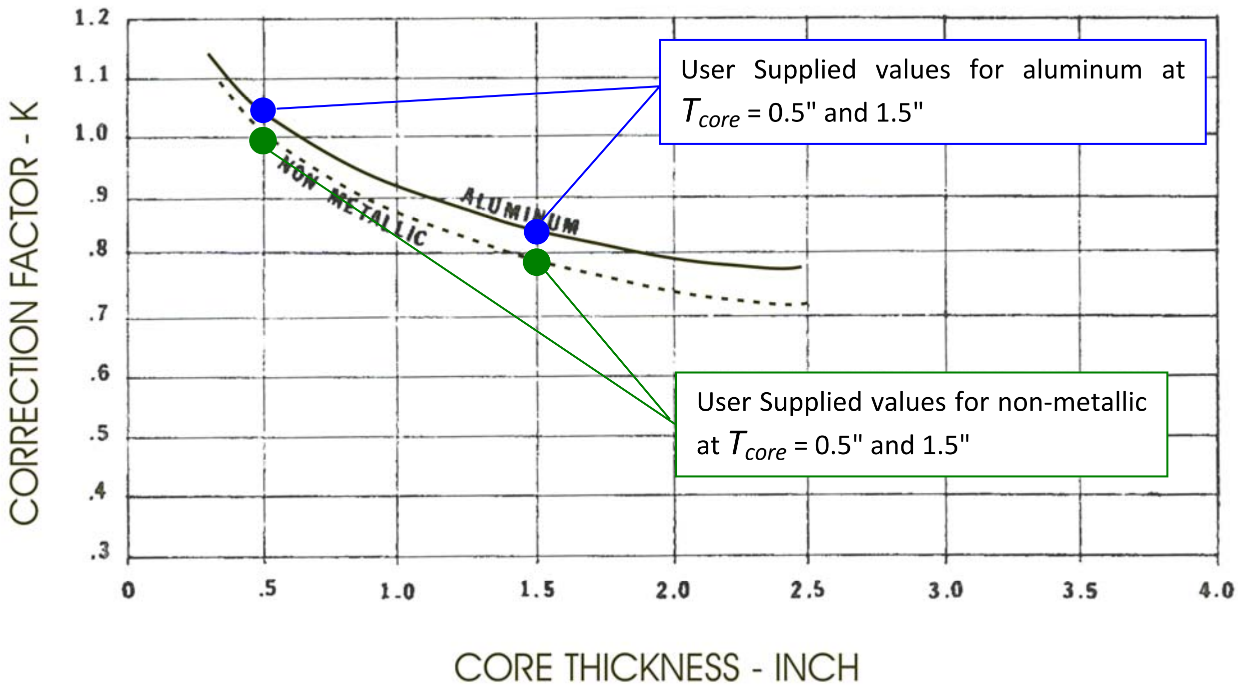
11.10.3 Symbols

R = Out-of-plane shear strength of core
K_{sscf} = Shear strength correction factor
Q = Out-of-plane shear load per unit length
Q_x = Out-of-plane shear load per unit length in x (ribbon) direction
Q_y = Out-of-plane shear load per unit length in y (transverse) direction
h_p = Total panel height
h_{eff} = Effective panel height (core + ½ facesheets)
Fsu_ℓ = Out-of-plane ultimate shear strength of core in ribbon direction
Fsu_ω = Out-of-plane ultimate shear strength of core in transverse direction
t_{core} = Core thickness

11.10.4 Shear Strength Correction Factor

The shear strength correction factor is a modification of the shear strength allowable based on the thickness of the core. The shape of the curve is generic, but the specific curve is material dependent and should be supplied by the honeycomb vendor. The figure below shows some typical curves for aluminum and "non-metallic" taken from Ref. [1]. The important thing to remember is the user has complete freedom to enter a correction factor for their own material systems.

In HyperSizer, the generic material depend curve is approximated with a bilinear curve fit based on user supplied values for shear strength correction factors at two core thicknesses (t_{core} = 0.5" and 1.5").



11.10.5 Equations

Analysis_ID = 105: Shear Strength, X (longitudinal) direction (Hexcel)
Component ID: 57
Load Case: 7: Mechanical Load Set #106 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	3142.	2372.	405.7	1220.	875.7	183.8	44.08	13.12

The sandwich shear strength calculation is based on the panel effective stiffness, h_{eff}, which is the core thickness plus ½ the facesheet thicknesses.

[Equation Removed] **(105.1)**

Equation 105.1 can be rearranged in terms of panel height, h_p, and core thickness, t_c, as:

[Equation Removed] **(105.2)**

Variable	Value, units
h_p	0.915 in
t_{core}	0.875 in
h_{eff}	**0.895 in**

[Equation Removed] **(105.3)**

Variable	Value, units
Q_x	44.08 lb/in
h_{eff}	0.895 in
Fsu_l	75.0 lb/in^2
K_{sscf}	0.925
MS_x	**0.4084**

Analysis_ID = 106: Shear Strength, Y (transverse) direction (Hexcel)
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	-267.6	-1264.	-1119.	-158.5	-740.1	-585.4	-54.98	-75.04

[Equation Removed] **(106.1)**

Variable	Value, units
h_p	1.331 in
t_{core}	1.25 in
h_{eff}	**1.2905 in**

[Equation Removed] **(106.2)**

Variable	Value, units
Q_y	-75.04 lb/in
h_{eff}	1.2905 in
Fsu_w	75.0 lb/in^2
K_{sscf}	0.9
MS_y	**0.1608**

Analysis_ID = 107: Shear Strength, Interaction

[Equation Removed] **(107.1)**

Failure is indicated when the following inequality is true:

[Equation Removed] **(107.2)**

Where:

[Equation Removed] **(107.3)**

This translates into a MS of:

[Equation Removed] **(107.4)**

11.10.6 References

- Hexcel, "The Basics of Bonded Sandwich Construction," TSB 124, 1982.

11.11 Buckling of Flat Anisotropic Panels

Analysis Methods:

02 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial
03 = Panel Buckling, Flat, Simple BC, Shear
05 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial w/ Shear Interaction
07 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial w/ TSF
08 = Panel Buckling, Flat, Simple BC, Shear w/ TSF (Transverse Shear Flexibility)
09 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial w/TSF & Shear Interaction

11.11.1 Flat panel buckling summary

Flat panel buckling is performed with closed form equations. The limitations are:

- Panel must be flat
- Only simple boundary conditions on the four edges

However, the panel can have generality with:

- biaxial loadings with in-plane shear loads (biaxial compression and biaxial tension field stiffening)
- any length to width aspect ratio
- unsymmetric laminates and stiffened panels

11.11.2 Approach Introduction

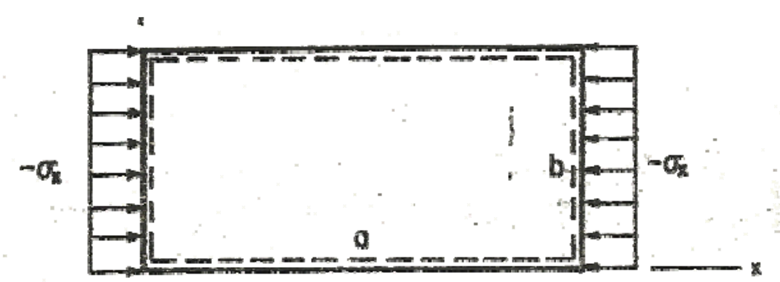
Consider first composite plates whose bifurcation buckling is governed by the differential equation

[Equations Removed] **(F.1)**

This is the classical equation for the buckling of a rectangular plate having orthotropy. It is applicable to:

- A single layer
- A cross-ply plate having multiple layers which are symmetrically arranged with respect to the midplane of the plate (i.e., a symmetrical laminate)

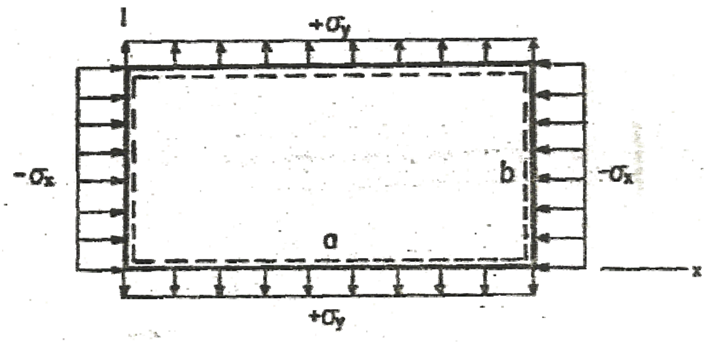
11.11.3 Uniform Uniaxial Loading



Solving Equation (F.1) results in the critical buckling stress resultant, Nx

[Equations Removed] **(F.2)**

11.11.4 Uniform Biaxial Loading



Solving Equation (F.1) with retaining bot σ_x and σ_y, stress resultants of the biaxial loading ratio N_y / N_x

[Equations Removed] **(F.3)**

11.11.5 Unsymmetric Laminates and Stiffened Panels

The equations governing the bifurcation buckling of an unsymmetrical laminate may be expressed in terms of the displacements in matrix operator form as

[Equations Removed] **(F.4)**

where the L_{ij} are differential operators representing the plate stiffness; *F* is a differential operator representing the inplane stress resultants (N_x, N_y, N_{xy}). *u* and *v* are inplane displacements of the midplane during buckling in the *x* and *y* directions; and *w* is the transverse displacement. It is important to note that *u* and *v* are not the inplane displacements which occur with increasing inplane stress resultants, but rather the additional displacements which arise when the buckling load is reached and the plate is deformed in a buckled mode shape of infinitesimal amplitude. These additional inplane displacements characterize the bending-stretching coupling, which exists in the deformation of an unsymmetrical laminate. In Equation (F.4) the bending-stretching coupling is induced by the operators $L_{13}(= L_{31})$ and $L_{23}(= L_{32})$, which vanish only when the B_{ij} bending-stretching stiffness coefficients are all zero. Existence of the bending-stretching coupling in unsymmetrical laminates have been demonstrated and quantified, both theoretically and experimentally. The primary effect is to decrease the stiffness of a plate; therefore, in the case of buckling, critical loads are reduced.

Equations (4) are an eighth order set of differential equations which closely resemble the form of shell buckling equations, which are also eighth order. (Indeed, the coupling between bending and stretching is what links together the two sets of fourth order equations which would otherwise exist for a shell.) Since the equations are of eighth order, four boundary conditions must be specified along each edge to define the problem physically, and to generate a proper mathematical eigenvalue problem.

For an unsymmetrically laminated plate, the meaning of a "simply supported" edge is not clear. Assuming that, as in classical plate theory, the edge must have zero transverse displacement and bending moment, there remain yet four possible combinations of "simple" (i.e., not elastically restrained) boundary conditions, depending upon the inplane constraints. For unsymmetrical cross-ply laminates, $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = 0$ having S2 conditions at *x* = 0, *a* and *y* = 0, *b* the exact solution takes the form

[Equations Removed] **(F.5)**

where

[Equations Removed] **(F.6)**

For a nontrivial solution, the determinant of the coefficient matrix of Equation (F.5) must be set equal to zero, yielding the solution for buckling stresses (N_x *and* N_y are positive in tension):

[Equations Removed] **(F.7)**

It is seen that for a symmetrical laminate $(B_{ij} = 0)$, the right-hand-side of Equation (F.7) reduces to C_{33}, and the equation is the same as Equation (F.3) for the biaxially loaded, orthotropic plate having simple edge conditions on all four sides.

All HyperSizer panel buckling solutions include membrane-coupling stiffness terms, the [B] matrix. This is the 6x6 matrix for the panel stiffness based on using the material's compression modulus of elasticity.

ABD Matrix

644000.	240800.	.1528E-09	415500.	155400.	0.
	644000.	-.1528E-09	0.	415500.	0.
		271600.	0.	0.	175300.
			536300.	200600.	.1272E-09
				536300.	-.1272E-09
					226200.

A term units:lb/in
B term units:lb in/in
D term units:lb in^2/in

Analysis ID = 2 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	**Ny lb/in**	**Nxy lb/in**	**Mx lb in/in**	**My lb in/in**	**Mxy lb in/in**	**Qx lb/in**	**Qy lb/in**
Buckling Loads	14.08	-476.0	-413.8	-16.74	-275.5	-220.7	-19.07	-28.76

Input data for the flat panel buckling solution includes unit loads, the length (a) and width (b), and the above 6x6 stiffness matrix of the panel in terms of C_{ij}. The output includes number of half modes (m) and (n) and the computed buckling critical unit load.

Variable	Value, units
N_x	14.08 lb/in
N_y	-476.0 lb/in
N_{xy}	-413.8 lb/in
a	136.2 in

b	95.76 in
C_{11}	634.8 lb/in
C_{12}	387.7 lb/in
C_{13}	17.65 lb in/in
C_{22}	837.6 lb/in
C_{23}	23.5 lb in/in
C_{33}	1.5206 lb in^2/in
m	1
n	1
$N_{y,crit}$	**-717.0 lb/in**

The buckling eigenvalue is calculated with the buckling knockdown factor, γ,

[Equation Removed] **(2.2)**

Variable	Value, units
N_x	-476.0 lb/in
$N_{x,crit}$	-717.0 lb/in
γ	1.0
$Eigv_y$	**1.5064**

The margin of safety is:

[Equation Removed] **(2.3)**

Variable	Value, units
MS	**0.5064**

Analysis ID = 7 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial (with TSF)
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Buckling Loads	14.08	-476.0	-413.8	-16.74	-275.5	-220.7	-19.07	-28.76

Panels with substantial out-of-plane, or transverse, flexibility exhibit reduced buckling strength over more rigid panels. The transverse shear flexibility is a function of the shear stiffness of the panel. HyperSizer assumes the following transverse stiffness constants for different panel types:

Sandwich (Honeycomb or Foam):

[Equation Removed] **(7.1)**

Stiffened Panels (I, T, Z, Blade, etc.):

[Equation Removed] **(7.2)**

Stiffened Sandwich (Trusscore, I- or Blade Sandwich):

[Equation Removed] **(7.3)**

In addition to the transverse shear stiffness, the following factors account for the relative length of buckling half waves in the x and y directions. The constants a, b, m and n are the lengths and number of buckling waves in each direction from the applicable buckling solution

[Equation Removed] **(7.4)**

Where d_2 is a measure of the length of one buckling half wave.

[Equation Removed] **(7.5)**

The total transverse shear stiffness factor depends on the panel type.

Sandwich (Honeycomb or Foam):
These types of panels exhibit similar relative transverse stiffnesses in the x and y direction,

[Equation Removed] **(7.6)**

Stiffened Panels (I, T, Z, Blade, etc.):
These types of panels are assumed to be infinitely stiff in the y direction and the total transverse stiffness is,

[Equation Removed] **(7.7)**

Stiffened Sandwich (Trusscore, I- or Blade Sandwich):
These types of panels are assumed to have no stiffness in the y direction and the total transverse stiffness is,

[Equation Removed] **(7.8)**

The buckling allowable load with transverse shear flexibility is calculated from the unmodified buckling allowable load and the transverse shear stiffness factor as:

[Equation Removed] **(7.12)**

Variable	Value, units
d_2	232.0 in
$C_{TSF,x}$	16250. lb/in
$C_{TSF,y}$	40000. lb/in
$F_{TSF,x}$	0.4128
$F_{TSF,y}$	0.5872
C_{TSF}	30200. lb/in
$N_{y,cr}$	-717.0 lb/in
$N_{y,cr,TSF}$	**-700.4 lb/in**

The buckling eigenvalue is calculated as

[Equation Removed] **(7.13)**

Variable	Value, units
N_y	-476.0 lb/in
$N_{y,cr,TSF}$	-700.4 lb/in
γ	1.0
$Eigv_y$	**1.4714**

The margin of safety is:

[Equation Removed] **(7.14)**

Variable	Value, units
MS	**0.4714**

Analysis ID = 3 = Panel Buckling, Flat, Simple BC, Shear
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	**Ny lb/in**	**Nxy lb/in**	**Mx lb in/in**	**My lb in/in**	**Mxy lb in/in**	**Qx lb/in**	**Qy lb/in**
Buckling Loads	14.08	-476.0	-413.8	-16.74	-275.5	-220.7	-19.07	-28.76

The form of HyperSizer's calculation of shear flat panel buckling depends on the geometry and stiffnesses of the panel. The parameters θ, β, aa and bb are defined for Eqs. (3.4-3.6).

[Equation Removed] **(3.1)**

[Equation Removed] **(3.3)**

Variable	Value, units
a	109.7 in
b	77.09 in
D_{11}	268200. lb in^2/in
D_{22}	268200. lb in^2/in
D_{33}	113100. lb in^2/in
D_{12}	100300. lb in^2/in
θ	0.8213
β	0.703
aa	-0.1181

bb	1.0629

The form of the used shear buckling equation depends on the values of θ and β,
If $\theta \geq 1$,

If $\beta < 1$,

[Equation Removed] **(3.4)**

If $\beta \geq 1$,

[Equation Removed] **(3.5)**

If $\theta < 1$,

If $\beta < 1$,
[Equation Removed] **(3.6)**

Variable	Value, units
Nxy_{cr}	3300. lb/in

If $\beta \geq 1$,
[Equation Removed] **(3.7)**

The buckling eigenvalue is calculated from Equation (3.7),

[Equation Removed] **(3.8)**

Variable	Value, units
Nxy	-413.8 lb/in
Nxy_{cr}	3300. lb/in
γ	1.0
$Eigv_{xy}$	7.9745

The margin of safety is given as:

[Equation Removed] **(3.9)**

Variable	Value, units
MS_{xy}	6.9745

Analysis ID = 8 = Panel Buckling, Flat, Simple BC, Shear (with TSF)
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Buckling Loads	14.08	-476.0	-413.8	-16.74	-275.5	-220.7	-19.07	-28.76

Panels with substantial out-of-plane, or transverse, flexibility exhibit reduced buckling strength over more rigid panels. The transverse shear flexibility is a function of the shear stiffness of the panel. HyperSizer assumes the following transverse stiffness constants for different panel types:

Sandwich (Honeycomb or Foam):

[Equation Removed] **(8.1)**

Stiffened Panels (I, T, Z, Blade, etc.):

[Equation Removed] **(8.2)**

Stiffened Sandwich (Trusscore, I- or Blade Sandwich):

[Equation Removed] **(8.3)**

In addition to the transverse shear stiffness, the following factors account for the relative length of buckling half waves in the x and y directions. The constants a, b, m and n are the lengths and number of buckling waves in each direction from the applicable buckling solution

[Equation Removed] **(8.4)**

Where d_2 is a measure of the length of one buckling half wave.

[Equation Removed] **(8.5)**

The total transverse shear stiffness factor depends on the panel type.

Sandwich (Honeycomb or Foam):
These types of panels exhibit similar relative transverse stiffnesses in the x and y direction,

[Equation Removed] **(8.6)**

Stiffened Panels (I, T, Z, Blade, etc.):
These types of panels are assumed to be infinitely stiff in the y direction and the total transverse stiffness is,

[Equation Removed] **(8.7)**

Stiffened Sandwich (Trusscore, I- or Blade Sandwich):
These types of panels are assumed to have no stiffness in the y direction and the total transverse stiffness is,

[Equation Removed] **(8.8)**

The buckling allowable load with transverse shear flexibility is calculated from the unmodified buckling allowable load and the transverse shear flexibility factor as:

[Equation Removed] **(8.9)**

Variable	Value, units
d_2	232.0 in
$C_{TSF,x}$	16250. lb/in
$C_{TSF,y}$	40000. lb/in
$F_{TSF,x}$	0.4128
$F_{TSF,y}$	0.5872
C_{TSF}	30200. lb/in
$N_{xy,cr}$	3300. lb/in
$N_{xy,cr,TSF}$	**2975. lb/in**

The buckling eigenvalue is calculated as
[Equation Removed] **(8.10)**

Variable	Value, units
N_{xy}	-413.8 lb/in
$N_{xy,cr,TSF}$	2975. lb/in
γ	1.0
$Eigv_{xy}$	**7.189**

The margin of safety is:

[Equation Removed] **(8.11)**

Variable	Value, units
MS	**6.189**

Analysis ID = 5 Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial w/Shear Interaction
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Buckling Loads	14.08	-476.0	-413.8	-16.74	-275.5	-220.7	-19.07	-28.76

For coupled shear and bi-axial loads, the buckling analysis can be performed as two eigen-problems. "It is well known that this eigen-problem uncouples into two parts".

This interaction equation for coupled shear and biaxial buckling is [3-6]:

[Equation Removed] **(5.1)**

where R_b and R_s are the ratios:

[Equation Removed] **(5.2)**

And the margin of safety is [7]:

[Equation Removed] **(5.3)**

Variable	Value, units
N_x	-476.0 lb/in
$N_{x,cr}$	-717.0 lb/in
R_b	0.6639
N_{xy}	-413.8 lb/in
$N_{xy,cr}$	-3300. lb/in
R_s	0.1254
MS	**0.4561**

11.11.5.1 Interaction Buckling Margin of Safety with Transverse Shear Flexibility

Analysis ID = 9 = Panel Buckling, Flat, Simple BC, Uniaxial or Biaxial w/Shear Interaction (with TSF)
Component ID: 39
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "

Panel Loads:

	Nx lb/in	**Ny lb/in**	**Nxy lb/in**	**Mx lb in/in**	**My lb in/in**	**Mxy lb in/in**	**Qx lb/in**	**Qy lb/in**
Buckling Loads	14.08	-476.0	-413.8	-16.74	-275.5	-220.7	-19.07	-28.76

The form of the interaction equation for coupled biaxial shear buckling with transverse shear flexibility is identical to that for buckling without transverse shear flexibility. However, in this case, the allowables used are those corresponding to biaxial and/or shear buckling.

11.11.6 References

- Leissa, A.W., "Buckling of Laminated Composite Plates and Shell Panels", *Air Force Aeronautical Laboratories Final Report,* Contract #F33615-81-K-3203, June 1985, Report #AD-A162 723.
- Advanced Composites Design Guide, Vol III, U.S. Air Force, 3rd edition 1977 (1983), produced under contract to Rockwell, p. 2.3.5.22.
- Collier, C., 4th International Aerospace Planes conference, AIAA-92-5015, eqn (17).
- Mil Handbook.
- Bruhn P.C1.7 & C5.8.
- LTV composites, p.9.2.6

11.12 Buckling of Curved Anisotropic Panels

Analysis Method:
11 = Panel Buckling, Curved or Flat, All BC

Curved panel buckling is performed with two methods. If the panel is a complete cylinder in axial compression NASA SP8007 applies as does the HyperSizer Raleigh Ritz energy solution. However, the HyperSizer Raleigh Ritz energy solution is more general and also applies to panels that are

- a portion of a complete cylinder
- curved or flat, have biaxial loadings with in-plane shear loads
- boundary conditions that are simple, fixed, or free
- any length to width aspect ratio.

All HyperSizer buckling solutions include membrane-coupling stiffness terms, the [B] matrix. This is the 6x6 matrix for the panel stiffness based on using the material's compression modulus of elasticity.

11.12.1 HyperSizer Raleigh Ritz energy solution

Component ID: 44
Load Case: 4: Mechanical Load Set #103 (Run Deck #1) "

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Buckling Loads	-2912.	-166.9	473.6	-581.8	-0.1547	6.8245	-4.8157	-0.04241

11.12.1.1 Buckling knockdown calculation

This section is omitted if the user has not turned on this option in HyperSizer or if it is not applicable.

11.12.1.2 Panel buckling equations

The ABD Unsymmetric Stiffness Matrix (S) for this panel is

.2328E+07	496900.	496900.	-485800.	-422.7	0.
	.1540E+07	0.	-422.7	-1311.	0.
		554300.	0.	0.	-3742.
			421900.	318.2	0.
				986.3	0.
					668.1

A term units: lb/in
B term units: lb in/in
D term units: lb in^2/in

The Raleigh-Ritz energy method has been chosen for the analysis because of its versatility and speed when compared to finite-element or finite-difference techniques. Many effects may be considered by simply adding their contributions to the total energy of the system, without increasing the size of the resulting set of equations.The basic energy principle involved is the theorem of stationary potential energy. In the present case it may be written as

[Equation Removed] **(11.8)**

For an elastic stability problem, Equation (11.8) becomes

[Equation Removed] **(11.9)**

The Raleigh-Ritz method is then applied to form a set of simultaneous equations for a standard eigenvalue problem for buckling cases. the following assumptions will be implicit in the analysis:
The displacements are small when compared to the thickness
Transverse shear effects are negligible

Input data for the HyperSizer Raleigh Ritz energy solution includes the radius, unit loads, and the length (a) and width (b) of the panel, and the edge boundary condtions. The output includes number of half modes (m) and (n) and the computed buckling critical unit load.

The buckling eigenvalue is calculated with the buckling knockdown factor

[Equation Removed] **(11.10)**

Variable	Value, units
N_x	-2912. lb/in
$N_{x,crit}$	-2991. lb/in
γ	1.0
$Eigv_x$	**1.0273**

The margin of safety is:

[Equation Removed] **(11.12)**

Variable	Value, units
MS	**0.02729**

11.12.2 SP-8007 Cylindrical Panel Buckling

11.12.2.1 Buckling knockdown calculation

This section is omitted if the user has not turned on this option in HyperSizer or if it is not applicable.

11.12.2.2 Panel buckling equations

The SP-8007 Cylindrical Panel Buckling method was not turned on, or the method was not applicable (e.g. the panel is flat or in tension).

11.12.3 References

- NASA-SP-8007, Buckling of thin-walled circular cylinders, NASA SPACE VEHICLE DESIGN CRITERIA (Structures), Washington, DC, United States, Revised August 1968

- Wilkins, D.J., "Anisotropic Curved Panel Analysis," General Dynamics, Fort Worth, TX, May 15, 1973, ADA309250.
- Collier Research Corporation, SBIR Final Report: Consistent Structural Integrity and Efficient Certification with Analysis, Air Force Research Lab (AFRL) SBIR Phase II contract # F33615-02-C-3216, October 2004, Volume 3.

11.13 Local Buckling of Rectangular Orthotropic Members, All Edges Simply Supported

Analysis Methods:
40 = Local Buckling, Biaxial Loading, Longitudinal (x) Dominated
41 = Local Buckling, Biaxial Loading, Transverse (y) Dominated
42 = Local Buckling, Uniform Shear Loading
43 = Local Buckling, Interaction

11.13.1 Approach Summary/Introduction

Consider first composite plates whose bifurcation buckling is governed by the differential equation:

[Equation Removed] **(H.1)**

This is the classical equation for the buckling of a plate having rectangular orthotropy. It is applicable to parallel-fiber composite plates in the following cases:

- A single layer
- A cross-ply plate having multiple layers which are symmetrically arranged with respect to the midplane of the plate (i.e., a symmetrical laminate).

1.

Taking once again the assumed displacement function and substituting it into equation (H.1), retaining both σ_x and σ_y, one obtains the following generalization of Equation (H.1);

[Equation Removed] **(H.2)**

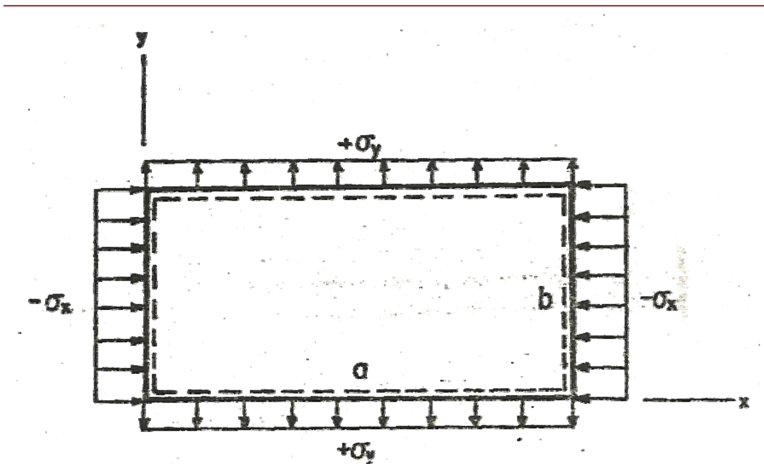


Figure 1: SSSS plate with uniform, biaxial stresses

Analysis ID = 40 Local Buckling, Biaxial Loading, Longitudinal (x) Dominated	
Component ID:	9
Load Case:	4: Mechanical Load Set #103 (Run Deck #1) "
Object ID:	Web

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-45890.	-9696.	-40010.	-14400.	-4949.	-899.1	-62160.	-488.9

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-11920.	0.	-1320.	-10.65	0.	-16.43	72.0	0.

Equation (H.2) is solved for the critical stress resultant in terms of the biaxial stress ratio N_y / N_x.

[Equation Removed] **(40.1)**

Variable	Value, units
N_x	**-11920. lb/in**
Ny	**0. lb/in**
a	**77.5 in**
b	**3.875 in**
m	**20**
n	**1**
D_{11}	**15720. lb in^2/in**
D_{22}	**15720. lb in^2/in**
D_{33}	**5208. lb in^2/in**
D_{12}	**5307. lb in^2/in**
$N_{x,crit}$	-41340. lb/in

[Equation Removed] **(40.2)**

Variable	Value, units
N_x	-11920. lb/in
$N_{x,crit}$	-41340. lb/in
KF	1.0
K_c	1.0
$Eigv_x$	**0.388**

[Equation Removed] **(40.3)**

Buckling Coefficients:

Variable	Description	Value
K_{bc} (One Edge Free)	Accounts for edge boundary conditions	0.1119
K_r (r=36.12)	Accounts for panel curvature	1.0
K_o	Accounts for partial fixity due to orthogrid stiffeners acting on facesheets	1.0
K_{i1}	Accounts for buckling of isogrid triangular region	1.0
K_{i2}	Accounts for buckling of isogrid triangular region	1.0
K_c	**Overall buckling coefficient**	**1.0**

Where KF is the user entered knockdown factor, Kfreeside is the correction factor for sides being free in the buckling solution (such as with a T-panel web), and Kcorrection is an additional correction factor to account for curvature and in the case of a grid stiffened panel, accounts for the fixity provided by the stiffeners (see appendix A for details).
The margin of safety is given as:

[Equation Removed] **(40.4)**

Variable	Value, units
MS_x	-0.612

If the buckling load is dominated by Ny (that is |Ny| > |Nx|), the following form of the buckling equation is used:

Analysis ID = 41 Local Buckling, Biaxial Loading, Transverse (y) Dominated
Component ID: 48
Load Case: 2: Mechanical Load Set #101 (Run Deck #1) "
Object ID: Orthogrid Top Span

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	-1526.	-2978.	-1130.	-280.7	-69.35	-159.5	0.	0.

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-1156.	-2913.	-880.6	-1.6077	-6.7663	0.5956	206.8	0.

[Equation Removed] **(41.1)**

Variable	Value, units
N_x	**-1156. lb/in**
N_y	**-2913. lb/in**
a	**10.39 in**
b	**3.0 in**

m	**1**
n	**1**
D_{11}	**2505. lb in^2/in**
D_{22}	**2505. lb in^2/in**
D_{33}	**849.0 lb in^2/in**
D_{12}	**806.9 lb in^2/in**
$N_{y,crit}$	-3121. lb/in

The buckling eigenvalue is calculated from Equation (41.1),

[Equation Removed] **(41.2)**

Variable	Value, units
N_y	**-2913. lb/in**
$N_{y,crit}$	**-3121. lb/in**
KF	**1.0**
$Eigv_y$	**1.0712**

Where KF is the user entered knockdown factor, Kfreeside is the correction factor for sides being free in the buckling solution (such as with a T-panel web), and Kcorrection is an additional correction factor to account for curvature and in the case of a grid stiffened panel, accounts for the fixity provided by the stiffeners (see appendix A for details).
The margin of safety is given as:

[Equation Removed] **(41.3)**

Variable	Value, units
MS_x	0.07117

Analysis ID = 42 Local Buckling, Uniform Shear Loading
Component ID: 49
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "
Object ID: Orthogrid Top Span

Panel Loads:

	Nx lb/in	**Ny lb/in**	**Nxy lb/in**	**Mx lb in/in**	**My lb in/in**	**Mxy lb in/in**	**Qx lb/in**	**Qy lb/in**
Strength Loads	1000.	999.9	814.1	955.8	31.2	186.1	280.9	38.67

Object Loads:

	Nx lb/in	**Ny lb/in**	**Nxy lb/in**	**Mx lb in/in**	**My lb in/in**	**Mxy lb in/in**	**T °F**	**TT Grad °F/in**
Strength Loads	-37.2	957.1	537.6	3.312	3.6378	0.3129	72.0	0.

[Equation Removed] **(42.1)**

Variable	Value, units
a	**13.86 in**
b	**4.0 in**
D_{11}	**329.5 lb in^2/in**
D_{22}	**329.5 lb in^2/in**
D_{33}	**111.6 lb in^2/in**
D_{12}	**106.3 lb in^2/in**
θ	**1.0**
β	**0.2887**
aa	**-0.085**
bb	**1.08**

If $\theta \geq 1$,
If $\beta < 1$,

[Equation Removed] **(42.2)**

Variable	Value, units
Nxy_{cr}	1171. lb/in

If $\beta \geq 1$,

[Equation Removed] **(42.3)**

If $\theta < 1$,
If $\beta < 1$,

[Equation Removed] **(42.4)**

If $\beta \geq 1$,

[Equation Removed] **(42.5)**

The buckling eigenvalue is calculated from Equation (42.6),

[Equation Removed] **(42.6)**

Variable	Value, units
N_{xy}	**537.6 lb/in**
Nxy_{cr}	**1171. lb/in**

KF	**1.0**
$Eigv_{xy}$	2.178

The margin of safety is given as:

[Equation Removed] **(42.7)**

Variable	Value, units
MS_{xy}	**1.178**

Analysis ID = 43 Local Buckling, Interaction
Component ID: 9
Load Case: 4: Mechanical Load Set #103 (Run Deck #1) "
Object ID: Web

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-45890.	-9696.	-40010.	-14400.	-4949.	-899.1	-62160.	-488.9

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	-11920.	0.	-1320.	-10.65	0.	-16.43	72.0	0.

The interaction failure equation used for biaxial-shear buckling interaction is:

a) ***[Equation Removed]*** **or** **b)** ***[Equation Removed]*** **(43.1)**

for x or y dominated buckling loads respectively.
The eigenvalue from this interaction equation is [reference MS document],
For x dominated buckling loads, Nx < 0 and |Nx| > |Ny|,

[Equation Removed] **(43.2)**

Variable	Value, units
Nx	**-11920. lb/in**
Nx_{cr}	**-41340. lb/in**
Nxy	**-1320. lb/in**
Nxy_{cr}	**55280. lb/in**
KF	**1.0**
$Eigv_i$	0.388

For y dominated buckling loads, Ny < 0 and |Ny| > |Nx|,

[Equation Removed] **(43.3)**

In either case, the margin of safety is then,

[Equation Removed] **(43.4)**

Variable	Value, units
MS_i	-0.612

11.13.2 Appendix: Concept Dependent Correction Factors

11.13.2.1 Correction for Free Panel Edges

11.13.2.2 Correction for Panel Curvature

11.13.2.3 Uniaxial and Hat Stiffened Panel Spacing Span

11.13.2.4 Orthogrid Panels

11.13.2.5 Isogrid Panels

11.13.3 References

- Leissa, A.W., "Buckling of Laminated Composite Plates and Shell Panels", Air Force Aeronautical Laboratories Final Report, Contract #F33615-81-K-3203, June 1985, Report #AD-A162 723.

11.14 *Crippling of Panels and Beams*

Analysis Methods:

50 = Crippling, Isotropic, method Niu, formed and extruded sections
51 = Crippling, Isotropic, method LTV, formed and extruded sections
52 = Crippling, Composite, method Mil-Hdbk-17-3E including Dij

11.14.1 Approach Summary

Crippling is a failure mode that occurs after the individual span segments of a stiffened panel or beam local buckle. As load is increased, load in the buckled spans remain nearly constant, and stresses increase in the corners of the remaining stable cross section, including the effective widths of unbuckled spans. For isotropic materials, failure occurs when concentrated stress exceeds the material yield stress Fcy for limit loads, or the ultimate stress, Fcu for ultimate loads. In composite materials, which do not exhibit yield, failure occurs when stresses exceed ultimate stress allowable.

Crippling analysis is performed by identifying the individual span segments of the cross section and assigning a width and thickness to them, as well as a boundary condition of one or no edge free. Allowable crippling stresses for each segment are determined from material test log-log curves, and their contribution to the entire section is computed by summing their weighted averages, see Eq. (I.1).

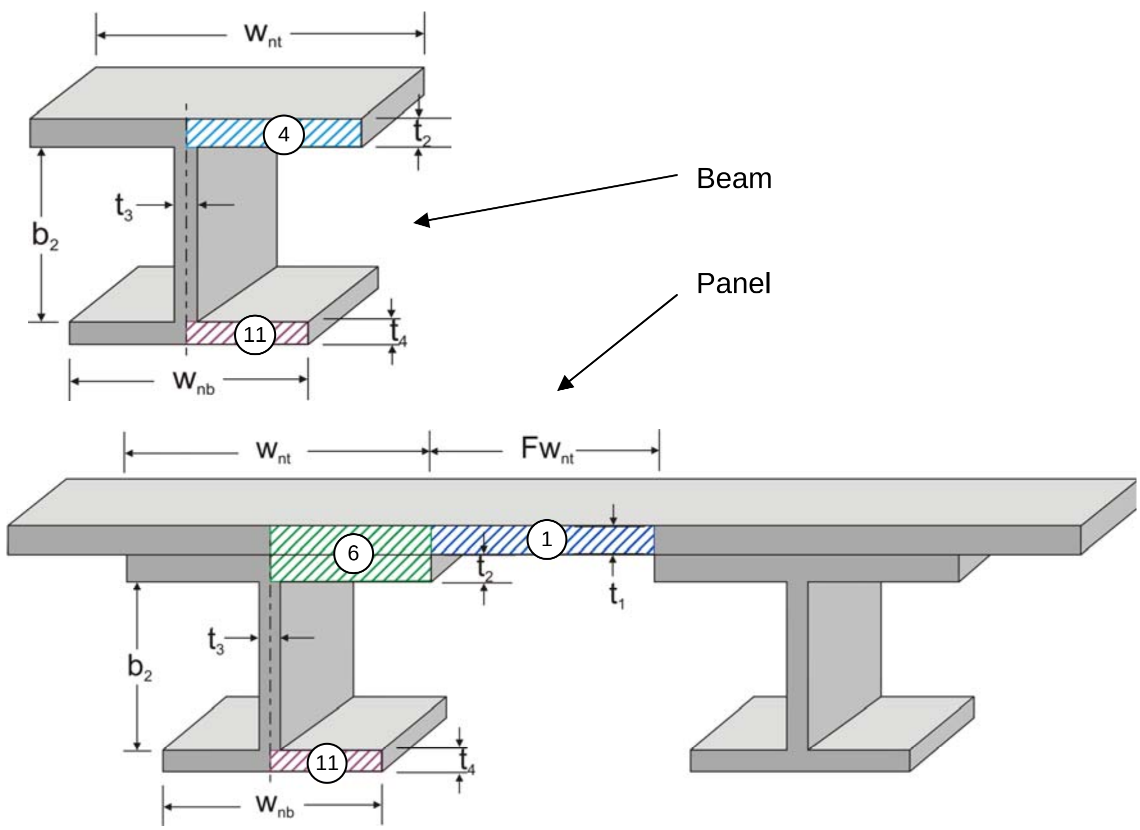


Fig. 1, A quick introduction into the similarities and differences between crippling span widths and thicknesses between beams and panels. This is covered in more detail later.

This technical document first introduces the generally accepted industry crippling analysis method, and then defines specific HyperSizer implementation, including details not normally covered in other publications, such as how to apply this method to panel cross sections, how to handle biaxial buckling loads including bending moments, differences between limit load and ultimate loads, and differences between isotropic and composite materials.

11.14.2 Symbols

p_{cc} = unit cross sectional crippling load
Area = unit cross sectional area
F_{cc} = design crushing or crippling stress of cross section
b_n = width of an individual span segment
t_n = thickness of an individual span segment
F_{ccn} = design crushing or crippling stress of an individual span segment
E_c = isotropic modulus of elasticity in compression in the longitudinal direction
F_{cyL} = design compressive yield stress of the material in the longitudinal direction
F_{cuL} = F_{tuL} = design ultimate stress of the material in the longitudinal direction
$P_{c,Axial}$ = load case dependent, axial load allowable of the cross section, which is in a state of compression
C_{ij} = Full "ABD" 6x6 stiffness matrix

11.14.3 General Equations

Unit cross sectional crippling load:

[Equation Removed] **(I.1)**

Margin-of-safety:

[Equation Removed] **(I.2)**

The design crippling stress, F_{ccn} , of each span segment is determined using Eqs. (3) or (4) for metallic members or Eqs. (9) – (11) for composites.

11.14.4 HyperSizer Metallic Material Implementation

	One free edge	No free edges	
Isotropic Method (AID = 50)	*[Equation Removed]*	*[Equation Removed]*	**(I.3)**
Isotropic Method (AID = 51)	*[Equation Removed]*	*[Equation Removed]*	**(I.4)**

The calculated F_{ccn} is used in Eq. (I.1). These equations represent the log-log curves shown in the figures of the references. For reference, the method 50, [Ref 1, Niu, p. 444] log-log curve is repeated in this document as Fig. 2.

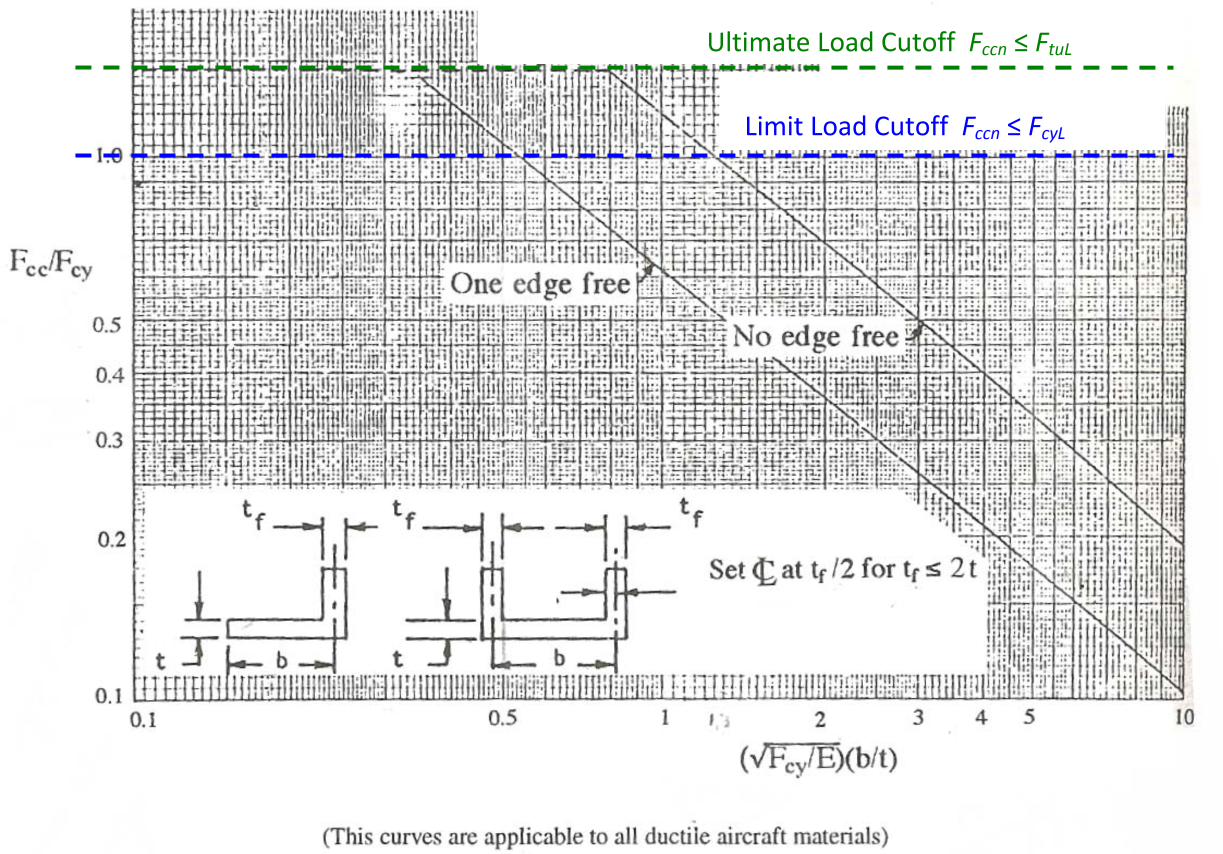


Fig. 2, Log-Log crippling curves from [Ref 1, Niu, p. 444] for metallic sections that fit test data for one-edge-free and no-edge-free.

11.14.4.1 Metallic Notes:

1) The variable, FcyL, is the typical industry material compressive yield stress allowable for limit loads as found in MIL-HDBK-5. For ultimate loads, instead of using the yield allowable, the ultimate stress allowable, FtuL, is used. Note: According to MIL-HDBK-5J (p. 1-11), since the actual failure mode for the highest tension and compression stress is in shear failure, the maximum (ultimate) compression stress is limited to Ftu. In accordance with this, HyperSizer sets FcuL = FtuL and FcuLT = FtuLT.

2) The crippling allowables are limited to: Fccn ≤ FcyL for limit loads, and Fccn ≤ FtuL for ultimate loads.

3) Eqs. (3) and (4) are applicable only for purely uniaxial compressive loads. If bending or biaxial loads are being applied, (or for ultimate loads, FtuL) is replaced with Fc,axial, the axial load that corresponds to biaxial loading failure. This is discussed more thoroughly in the section, "Bending Moments and Biaxial Loadings" and Figs. 7 and 8.

11.14.5 HyperSizer Composite Material Implementation

Plate bending stiffnesses play an important role in determining the initial buckling and crippling loads of span segments. Unlike in metallic plates, however, there exists no direct relationship between the extensional and bending stiffnesses of a composite plate and, therefore, laminates with equal in-plane stiffnesses may buckle at different load levels if their stacking sequences are not identical.

The commonly used procedure for predicting the crippling strength of a metallic stiffener, composed of several one-edge and no-edge-free elements, is to compute the weighted sum of the crippling strengths of the individual elements. Test results appear to indicate that the same procedure can be successfully applied to composite stiffeners of uniform thickness if the element crippling strengths are determined with the aid of two nondimensional parameters defined in Mil-Hdbk-17.

[Equation Removed] **(I.5)**

The left nondimensional parameter of Eq. (5) is the vertical axis, and the right term is the horizontal axis of the log-log curves of Fig. 3. Mil-Hdbk-17 defines an effective flexural stiffness noted as $\bar{E}$.

[Equation Removed] **(I.6)**

Instead of using this Mil-Hdbk-17 symbol, we use the symbol FE_x for flexural stiffness. We define E_x and FE_x as

[Equation Removed] **(I.7)**

Where

[Equation Removed] **(I.8)**

[Fortran ref: *Dprime(i,j) = ABDinv(i+3,j+3)*]

We define a symbol, X_G, for the horizontal *x* axis (right term of Eq. (5)) as

[Equation Removed] **(I.9)**

and we define a symbol, Y_G, for the vertical *y* axis (left term of Eq. (5)) as

One free edge	**No free edges**	
[Equation Removed]	***[Equation Removed]***	**(I.10)**

which represent the Log-Log graph equations of Fig. 3. The Y_G value is used to compute a crippling span objects allowable stress, Eq. (11), to put into Eq. (1).

[Equation Removed] **(I.11)**

11.14.5.1 Composite Notes:

1) Fcu of a laminate is determined by the ultimate compressive strength of the particular laminate panel span segment per reference Mil-Hdbk-17, Vol 3, Ch 4, page 88. HyperSizer's current implementation is first ply failure for each load case dependent state of identified allowable load from any failure criteria turned on by the user.

2) The crippling allowable is limited to: Fccn ≤ Fcu for both limit and ultimate loads.

3) Eq. (I.11) is applicable only for purely uniaxial compressive loads. If bending or biaxial loads are being applied, is replaced with Fc,axial, the axial load that corresponds to biaxial loading failure. This is discussed more thoroughly in the section, "Bending Moments and Biaxial Loadings" and Figs. 6 and 7.

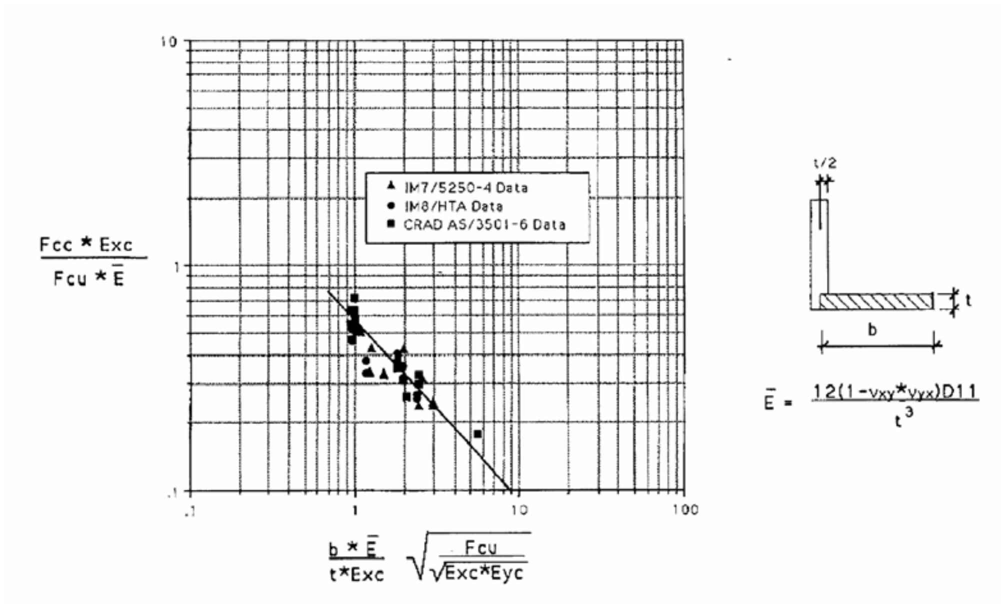


FIGURE 4.7.2.4(a) *One-edge-free crippling test results.*

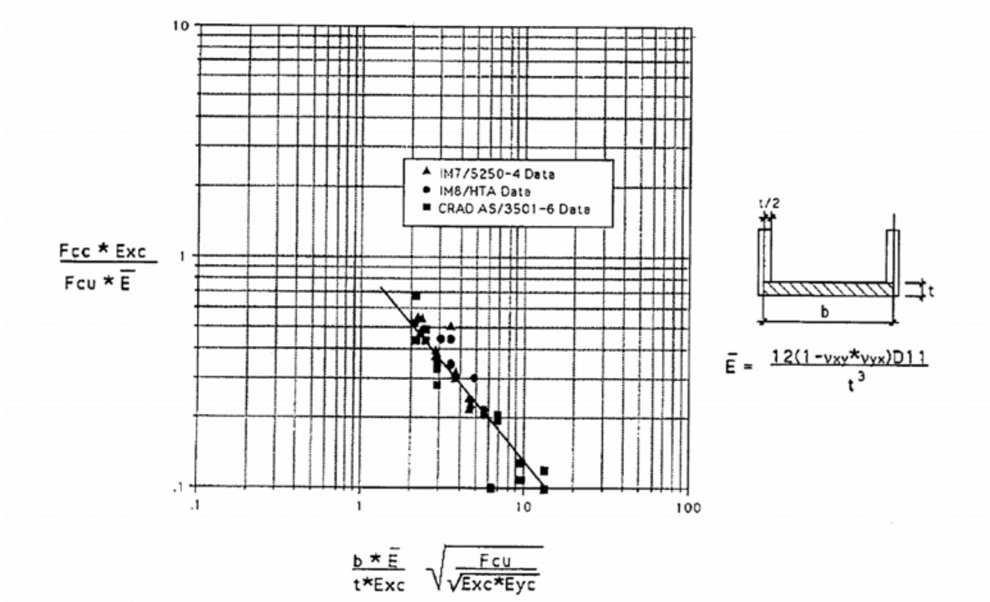


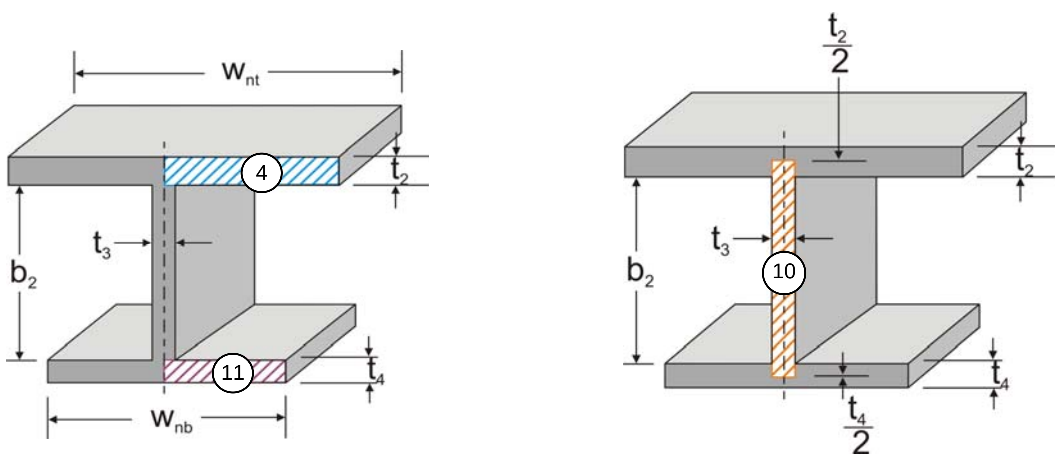
FIGURE 4.7.2.4(b) *No-edge-free crippling test results.*

Fig. 3, Log-Log crippling curves from Mil-Hdbk-17 of composite specimens that fit test data for one-edge-free above, and no-edge-free below.

11.14.6 Panel vs. Beam Implementation

In the crippling of an I-Beam section, there are three identified objects; the upper flange (4), the lower flange (11) and the web object (10). The convention for width of each object when determining the allowable crippling stress, Fccn, is the same convention used by the published log-log curves of Figs. 2 and 3. That is, the width of each crippling object goes ½ of the thickness into each adjacent object. The thicknesses and widths of each object are shown in Fig. 4.

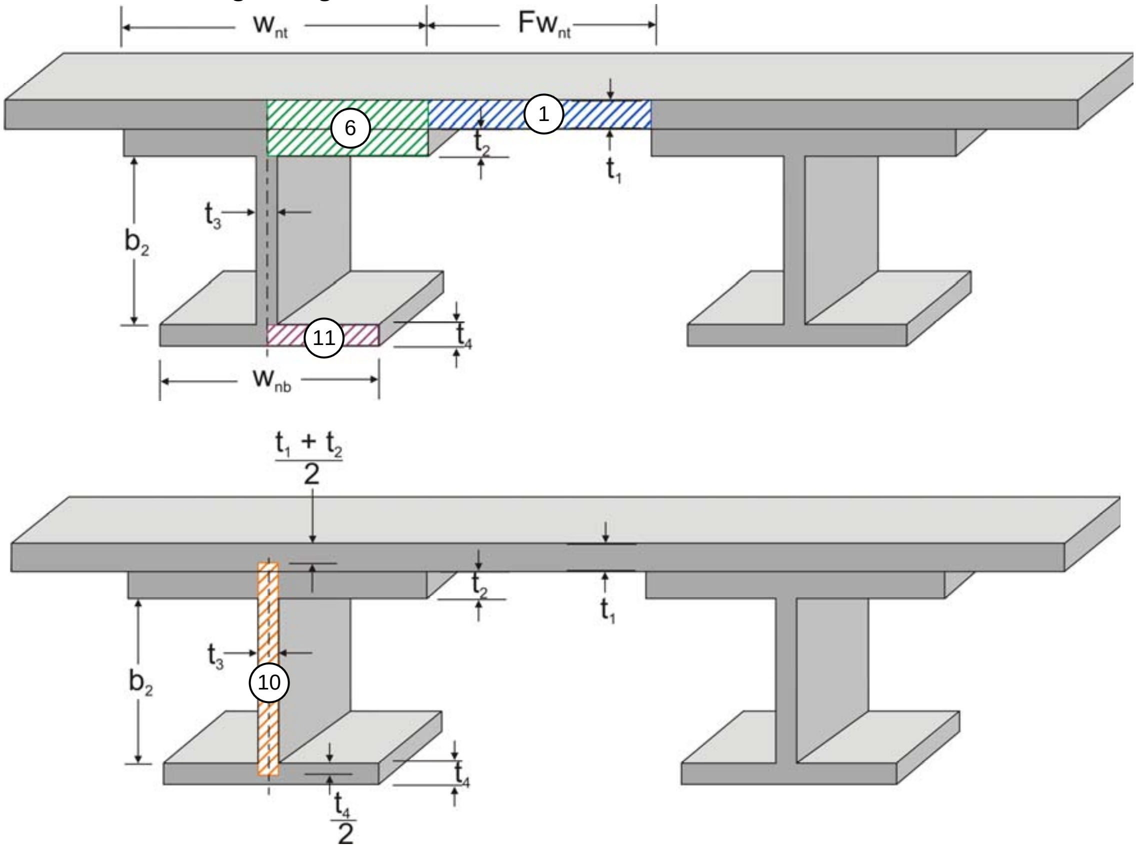
The crippling objects for a stiffened panel follows a similar convention as those for beams. There are two substantial differences between the objects defined for a stiffened panel and those defined for a beam. The first is that the skin between stiffeners must be accounted. This object is defined in Fig. 5 as Object (1). The boundary condition of this object is assumed to be no-edges free with the bonded flange area substantially stiffer than the free span and therefore providing a support for the object.



Object	t_n	b_n	Free Edges
4 10 11	*[Equation Removed]*	*[Equation Removed]*	*[Equation Removed]*

Fig. 4, Crippling span widths and thicknesses for an I-Beam. The width, b, and thickness, t, for each span follow the convention of that shown in Figures 2 and 3 for determining the crippling allowable load.

The second difference is that the bonded region between the flange and skin are treated as a single object, and for a composite, a single laminate when determining the crippling allowable stress. The thickness of this new object (6) is t1 + t2. In addition, the width of the web object is assumed to go ½ the thickness of the combined facesheet and flange, rather than just the thickness of the flange as shown in the lower image of Fig. 5.



Object	t_n	b_n	Free Edges
1	*[Equation Removed]*	*[Equation Removed]*	*[Equation Removed]*
6			
10			
11			

Fig. 5, Crippling span widths and thicknesses of the span objects for I-stiffened panels. The width, b, and thickness, t, for each span follow the convention of that shown in Figures 2 and 3 for determining the crippling allowable load. In contrast to the beam, notice that the flange and face objects have been combined into a single object (6) and that the width of the web object extends midway into this combined object, which is unlike the web width of a beam.

11.14.7 Bending Moments and Biaxial Loadings

A limitation of the industry standard methods that attempt to predict crippling is that the methods assume that the compressive loads that cause crippling are a), only uniaxial; and b), uniform throughout the cross-section. This means that these methods neither account for biaxial compressive-compressive or compressive-tensile loadings, or bending moments that can cause load reversals from the top to the bottom of the cross-section. HyperSizer implementation of the crippling method addresses both of these concerns.

11.14.7.1 Bending Moments

If a panel or beam is in uniaxial compression, the crippling margin of safety is determined by simply summing up the total crippling allowable in the cross-section and comparing it to the total applied load in the cross-section. However, if the panel or beam is undergoing bending, or a combination of bending and compression, a load reversal can occur as illustrated in Fig. 7, where, for illustrative purposes, the lower half of the cross-section is undergoing tension and the upper part is undergoing compression.

HyperSizer handles the load shown in Fig. 7 in two ways. First, any analysis object that is in tension in the axial direction will be excluded from the crippling allowable summation and the corresponding crippling area from Eq. (1). This means that for the loading shown, the bottom flange object will not be included in the crippling analysis. In addition, when determining the crippling applied load, the tensile load that is applied to this object will be ignored in the crippling analysis.

Second, the web object could potentially have compression at one end and tension in the other as shown. If this is the case, HyperSizer will calculate the area, Ac,web, that is in compression from the equation,

[Equation Removed] **(I.12)**

Where F_c is the maximum compressive load in the web, F_t is the maximum tensile load in the web, and A_{web} is the total area of the web. This area, allowable load, and average compressive load are included in the crippling analysis, while the remaining area and tensile load are not.

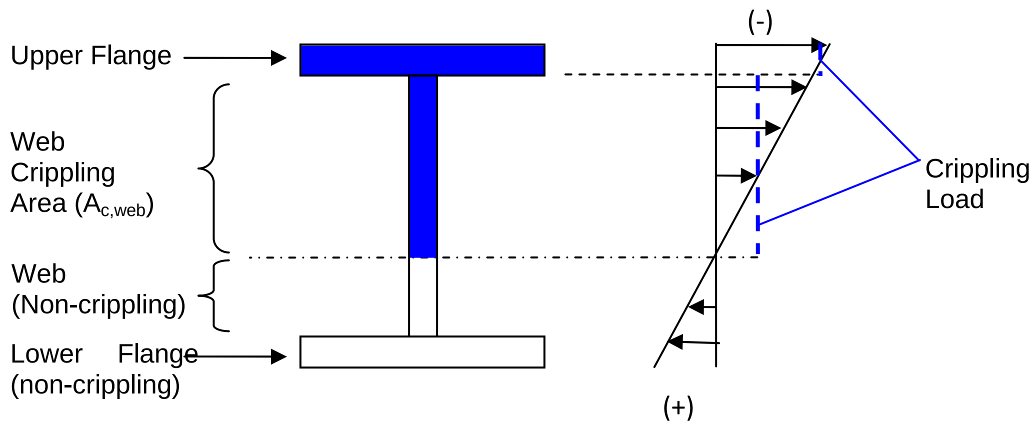


Fig. 6, A combination of bending moment and compression can lead to a load profile as shown to the right. HyperSizer accounts for this by removing any objects that are in pure tension from the crippling analysis. In addition, for the web object, which can be partly in compression and partly in tension, only the compression area and load are included in the crippling analysis.

11.14.7.2 Biaxial Loads

In addition to the load variation due to bending, the ultimate or yield strength of each span, and therefore the crippling strength, can be affected by the biaxial nature of the loads. For example, consider a biaxial load where the compressive axial load is relatively small, but a large tensile transverse load causes the span to be near failure, as portrayed in Fig. 9. If the method ignores the transverse load, the crippling margin of safety would be very large. However, a small increase in load will push the object to a material strength failure, which could contribute to a crippling failure.

This is shown graphically on the failure envelope of Fig. 8. The biaxial load with axial compression and transverse tension is shown as "Load A". The "failure" load is found by projecting out from the applied load to the failure surface. The axial compressive strength, noted as Fc,axial, is then the "x-coordinate" corresponding to this failure load. In this case, Fc,axial is well below the uniaxial strength (Fcy for limit loads or Fcu for ultimate loads) of the span. In the case of another possible loading, "Load B", both the axial and transverse loads are compressive. By projecting out to the failure surface, we see that the Fc,axial can actually be higher than the purely axial strength.

Note that if the axial load is tensile, as is the case for the right half of the failure envelope, the strength and load of this object are not included in the failure analysis. If all objects are in axial tension, crippling analysis is not performed.

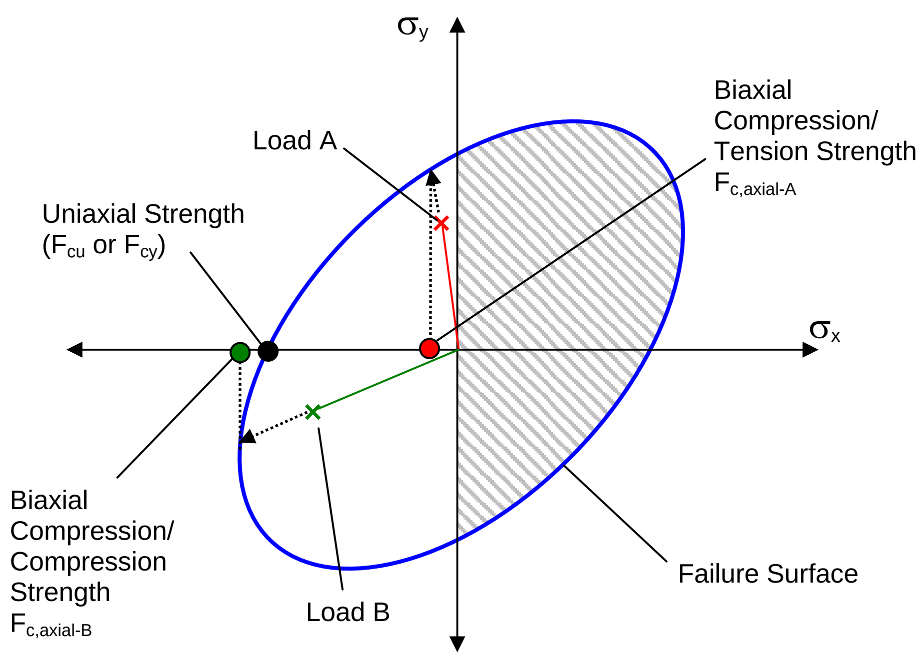


Fig.7, Biaxial loads for each span are taken into account in the crippling analysis by projecting the load state of the span onto the material strength failure envelope and then backing out an axial allowable load, $F_{c,axial}$. This allowable is used to evaluate the crippling allowable load from Eqs. (3) and (4) (metallic) and Eqs. (9)-(11) (composite). In Load A, $F_{c,axial}$ is much lower than the uniaxial strength (F_{cu} or F_{cy}), however in the case of Load B, the allowable axial load can actually be higher than the uniaxial strength.

As an example that will be discussed in more detail later, the facesheet of an aluminum stiffened panel (F_{cy} = 70 ksi) is subjected to a biaxial load where the axial stress is 0.69 ksi compressive and the transverse stress is 2.5 ksi tensile. If the load were purely axial, the strength, $F_{c,axial}$ of the sheet would be equal to the yield stress, 70 ksi. However, the biaxial nature of the loads must be taken into account by considering the minimum material strength margin of safety for this span. For this loading, the minimum margin comes from the von Mises failure criteria, where the equivalent von Mises stress is calculated and compared to the yield stress as shown here,

The margin of safety is: ***[Equation Removed]***

Projecting out to the failure envelope in this case means multiplying the axial loads by (*MS* + 1), which is $F_{c,axial}$ = (24.1)(0.69) = 16.6 ksi. Thus, the material allowable $F_{c,axial}$ is 16.6 ksi, rather than the yield stress of 70 ksi. This value is used in Eqs. 3 and 4 to determine the allowable crippling load (F_{ccn}) for this span.

Analysis ID = 50 = Crippling, Isotropic, method Niu, formed and extruded sections	
Component ID:	9
Load Case:	6: Mechanical Load Set #105 (Run Deck #1) "

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-69600.	-15400.	-64200.	-21250.	-6906.	-1354.	-92540.	-799.5

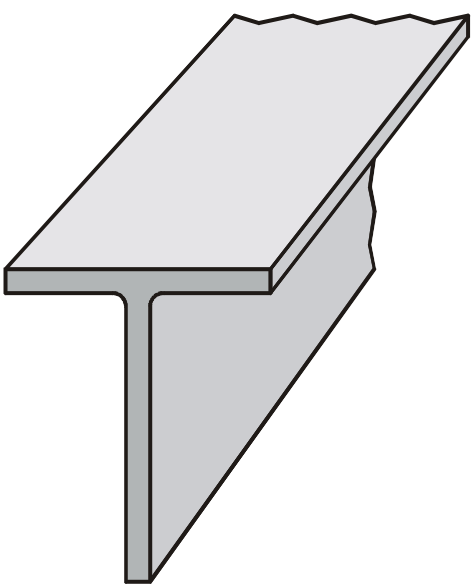
	One free edge	No free edges	
Isotropic method (AID = 50)	***[Equation Removed]***	***[Equation Removed]***	**(50.1)**

Unit cross sectional crippling load:

[Equation Removed] **(50.2)**

Margin-of-safety:

[Equation Removed] **(50.3)**



Allowable Crippling Load

ID	Object	Free Edges	Fc,axial (klb/in^2)	Ec (klb/in^2)	b (in)	t (in)	Fcc (klb/in^2)	Compression Ratio, CR	Fcc*b*t*CR (lb)
7	Bonded Combo Top	1	57.24	10730.	2.0	0.17	57.24	1.0	19460.
10	Web	0	63.85	10700.	4.7319	0.085	42.2	0.9254	15710.
12	Crown Bottom	-	-	-	-	-	(+) Load	-	-
4	Flange Top	1	62.71	10700.	4.0	0.25	56.88	1.0	56880.
17	Web	1	65.49	10700.	3.75	0.25	34.97	1.0	32790.
	TOTAL								89660.

Applied Load

ID	Object	Axial Load (lb/in)	Axial Stress (σ) (klb/in^2)	σ*b*t (lb)
7	Bonded Combo Top	-8724.	-51.32	-17450.
10	Web	-2138.	-25.15	-9362.
12	Crown Bottom	-	-	-
4	Flange Top	-6387.	-25.55	-25550.
17	Web	-17850.	-71.4	-66940.
	TOTAL			92490.

Margin-of-Safety

Variable	Force per Stiffener Span (lb)	Stiffener Spacing (in)	Unit Force (lb/in)
Pcc	89660.	1.0	89660.
Papplied	92490.	1.0	92490.
MS			**-0.03053**

Analysis ID = 51 = Crippling, Isotropic, method LTV, formed and extruded sections
Component ID: 9
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-69600.	-15400.	-64200.	-21250.	-6906.	-1354.	-92540.	-799.5

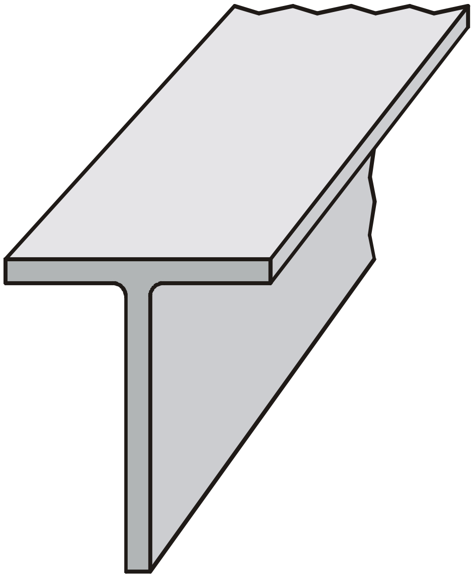
	One free edge	**No free edges**	
Isotropic method (AID = 51)	*[Equation Removed]*	*[Equation Removed]*	**(51.1)**

Unit cross sectional crippling load:

[Equation Removed] **(51.2)**

Margin-of-safety:

[Equation Removed] **(51.3)**



Allowable Crippling Load

ID	Object	Free Edges	Fc,axial (klb/in^2)	Ec (klb/in^2)	b (in)	t (in)	Fcc (klb/in^2)	Compression Ratio, CR	Fcc*b*t*CR (lb)
7	Bonded Combo Top	1	57.24	10730.	2.0	0.17	57.24	1.0	19460.
10	Web	0	63.85	10700.	4.7319	0.085	48.72	0.9254	18130.
12	Crown Bottom	-	-	-	-	-	(+) Load	-	-
4	Flange Top	1	62.71	10700.	4.0	0.25	52.75	1.0	52750.
17	Web	1	65.49	10700.	3.75	0.25	31.72	1.0	29740.
	TOTAL								82490.

Applied Load

ID	Object	Axial Load (lb/in)	Axial Stress (σ) (klb/in^2)	σ*b*t (lb)
7	Bonded Combo Top	-8724.	-51.32	-17450.
10	Web	-2138.	-25.15	-9362.
12	Crown Bottom	-	-	-
4	Flange Top	-6387.	-25.55	-25550.
17	Web	-17850.	-71.4	-66940.
	TOTAL			92490.

Margin-of-Safety

Variable	Force per Stiffener Span (lb)	Stiffener Spacing (in)	Unit Force (lb/in)
Pcc	82490.	1.0	82490.
Papplied	92490.	1.0	92490.
MS			**-0.1081**

11.14.8 Hypersizer Summary Implementation Notes

The isotropic and composite crippling stresses (Fccn) from Eqs. (3) - (4) and Eqs. (9) - (11) are based on compressive elastic stiffness terms.

Isotropic material stress allowable Fcy is used with limit loads, and Ftu with ultimate loads. Composite material stress allowable Fcu is used with both limit and ultimate loads. Refer to more specific information provided in the metallic and composite sections of this document. In particular the uniaxial allowable compressive allowables are used only for pure uniaxial loadings, for other loadings the variable Fc,Axial is used as described in section 5.2.

For more verification examples, refer to the companion document “AID050-052 Crippling of Panels and Beams.HVE”.

11.14.9 References

- HyperSizer “AID050-052 Crippling of Panels and Beams.HVE” Verification Examples
- Niu, C.Y., Airframe Stress Analysis and Sizing, 1st edition October 1997. ISBN 962-712-07-4, p. 444.
- Mil-Hdbk-17-3E, Composite Materials, DOD Coordination Working Draft, 1998.
- MIL-HDBK-5J, Metallic Materials And Elements For Aerospace Vehicle Structures, April 2003.
- NASA Preferred Reliability Practices, “Structural Stress Analysis”, PRACTICE NO. PD-AP-1318, April, 1996.
- Collier Research Corporation, SBIR Final Report: Consistent Structural Integrity and Efficient Certification with Analysis, Air Force Research Lab (AFRL) SBIR Phase II contract # F33615-02-C-3216, October 2004, Volume 3.

11.15 Buckling-Crippling, Johnson-Euler Interaction

Analysis Methods:
53 = Buckling-Crippling, Johnson-Euler Interaction

Component ID: 17
Load Case: 4: Mechanical Load Set #103 (Run Deck #1) "

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Strength Loads	-220900.	-19520.	-239900.	-19940.	-20910.	-3202.	-72110.	-134.5

Beam Loads:

	M1a lb in	M2a lb in	M1a lb in	M2b lb in	Vx lb	Vy lb	Axial Force lb	Torsion lb in
Buckling Loads	-65010.	-5905.	-84750.	-6954.	-4466.	-948.6	-64690.	-35.3

11.15.1 Buckling-Crippling Interaction Summary

Crippling is a failure mode that occurs after the individual span segments of a stiffened panel or beam local buckle. As load is increased, load in the buckled spans remain nearly constant, and stresses increase in the corners of the remaining stable cross section, including effective widths of unbuckled spans. Failure occurs when concentrated stress exceeds the material yield stress, Fcy, for limit loads, or ultimate stress, Ftu , for ultimate loads.

For long unsupported panel or beam lengths, an overall buckling mode will occur. An interaction equation to transition between buckling and crippling is customarily used. The interaction is performed using the Johnson-Euler parabola curve as depicted in the figure below for beams and stiffened panels in the stiffener (longitudinal) direction. This transition curve is defined with equation (H.1).

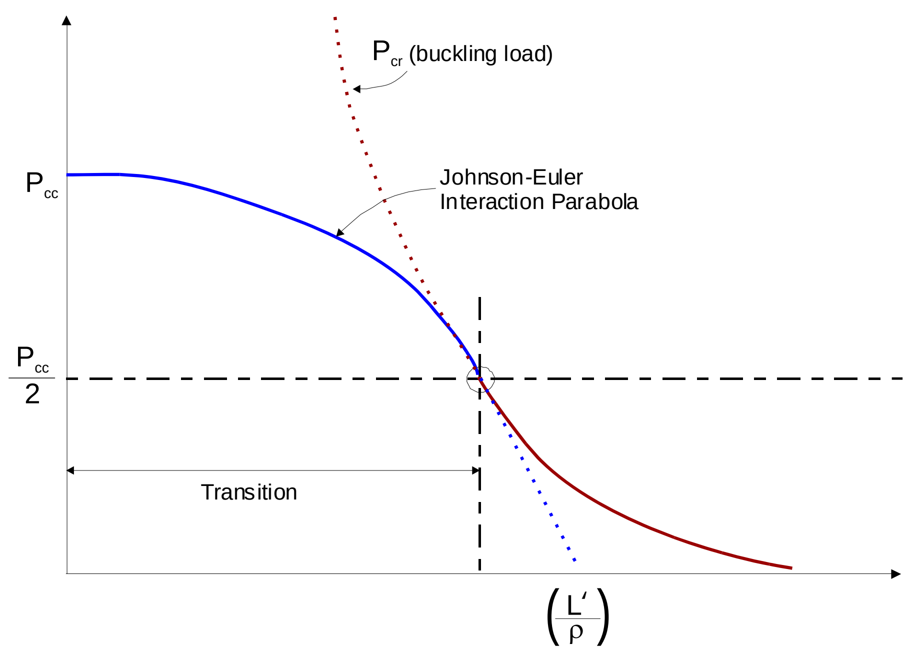


Fig. 1, The Johnson-Euler interaction curve for transitioning allowable crippling load, P_{cc}, (blue color) to allowable buckling load P_{cr} (brown color). When the interaction curve dips below one half the crippling allowable load, Pcc/2, (dashed line indicates the point of tangency of the two curves) then the buckling allowable load is used. The solid line represents the allowable load for a given effective buckling length.

11.15.2 Approach Introduction

This HyperSizer technical document 1st introduces the general accepted industry method, and then 2nd defines specific HyperSizer implementation and includes details not normally covered in other publications such as how to apply this method to panel cross sections, how to handle biaxial buckling loads, differences between limit load and ultimate loads, and differences between isotropic vs. composite materials.

11.15.3 Symbols

p_{cc} = unit cross sectional crippling load (Minimum crippling load of any turned on crippling method)

p_{cr} = unit cross sectional buckling load (Minimum buckling load of any turned on buckling method)

p_{je} = unit cross sectional crippling-buckling Johnson-Euler Interaction load

11.15.4 Equations

[Equation Removed] **(H.1)**

[Equation Removed] **(H.2)**

[Equation Removed] **(H.3)**

The following text is copied directly from p. 4-95 of Mil-Hdbk-17-3E, reference 1. It serves as a useful explanation to the figure shown previously and provides familiarity to equation (H.1).

> As the unsupported length increases, the stiffener may fail in a global buckling mode rather than by local crippling. The usual procedure to account for this is to apply a correction factor to the crippling strength, F_{cc}, based on the slenderness ratio (L'/ρ) of the column. The critical stress for the stiffener now becomes
>
> $$F^{cr} \infty F^{cc}\left[1 - \frac{F^{cc}}{4\pi^2 E_x^c}\left(\frac{L'}{\rho}\right)^2\right] \quad 4.7.2.6(a)$$
>
> The radius of gyration for the cross-section of a composite column is defined as
>
> $$\rho = \sqrt{\frac{(EI)_{st}}{(EA)_{st}}} \quad 4.7.2.6(b)$$
>
> where $(EA)_{st}$ and $(EI)_{st}$ are the extensional and bending stiffnesses of the stiffener.

11.15.5 HyperSizer Implementation Specifics

Isotropic material stress allowable Fcy is used with limit loads, and Ftu with ultimate loads. Composite material stress allowable Fcu is used with both limit and ultimate loads. Both isotropic and composite crippling based on compressive elastic stiffness terms.
Note: According to MIL-HDBK-5J (p. 1-11), since the actual failure mode for the highest tension and compression stress is in shear failure, the maximum (ultimate) compression stress is limited to Ftu. In accordance with this, HyperSizer sets $F_{cuL} = F_{tuL}$ and $F_{cuLT} = F_{tuLT}$.

When the Johnson-Euler interaction curve dips below one half the crippling allowable load, $P_{cc}/2$, the buckling allowable load curve is used in sizing optimizations. That is the Margin-of-Safety (MS) from Johnson-Euler is not reported nor used in controlling the panel or beam sizing. In this case, HyperSizer reports for the **Crippling – Buckling interaction, Johnson-Euler** analysis ID 53 = "NA" for not applicable.

Reference Fig. 2. The point of tangency where the buckling allowable load is appropriate to use occurs at $P_{cc}/2$. If the buckling allowable load is greater than $P_{cc}/2$, than HyperSizer uses the Johnson-Euler curve for allowable load.

HyperSizer's implementation of crippling-buckling interaction, as accomplished with equation (H.1), includes panel buckling methods that account for biaxial loading with in-plane shear, and includes the effects of curvature based on Raleigh-Ritz energy solutions. Thus, for most real world applications and loadings, this implementation is more accurate and robust than the equation presented in MIL-HDBK-17-3E. For pure uniaxial Euler column buckling, HyperSizer's method will return identical results to those of Eq. 4.7.2.6(a).

Reference Fig. 3. Shown are two loading possibilities. The first is a typical uniaxial compression load in the stiffener direction. The second is a biaxial compressive loading. Both loadings are applicable to the buckling-crippling interaction. However, most aerospace manuals and published methods such as Mil-Hdbk-17-3E do not address this interaction for biaxial loadings.

The lowest allowable load of any user selected turned on panel/beam buckling method is paired with the lowest allowable load of any user selected turned on panel/beam crippling method. This is done per limit or ultimate loads. Therefore limit Johnson-Euler MS are based on limit buckling and crippling MS, and ultimate Johnson-Euler MS are based on ultimate buckling and crippling.

Buckling MS are based on HyperSizer processed FEA buckling loads. Crippling and thus Johnson-Euler crippling buckling interaction MS are based on HyperSizer processed FEA strength loads.

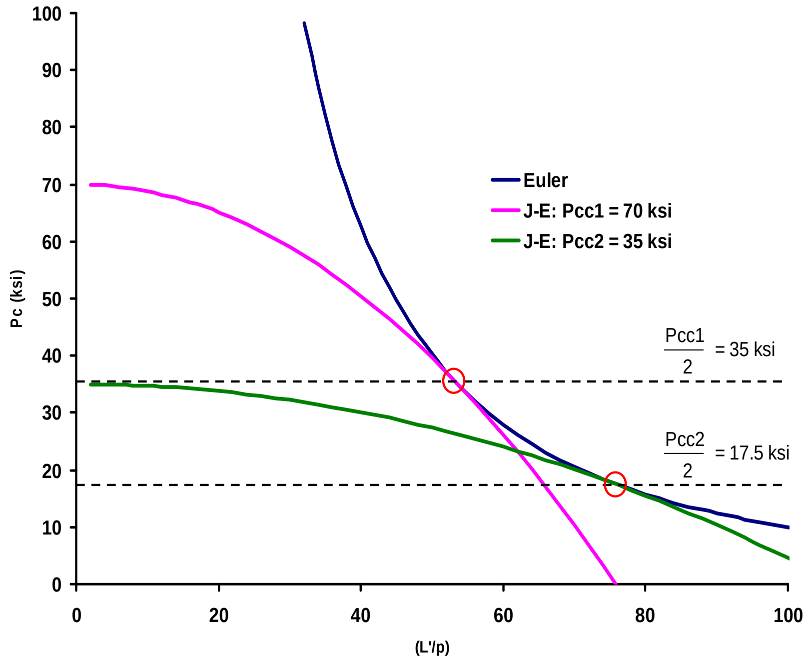
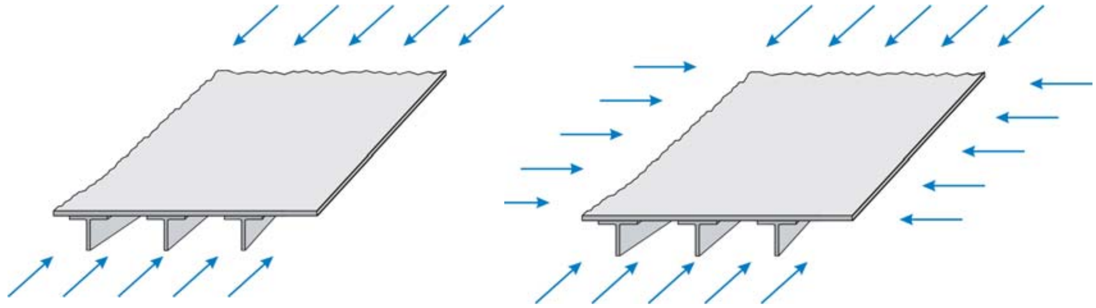


Fig. 2, The Johnson-Euler interaction curve transitions at the point it dips below one half the crippling allowable load, $P_{cc}/2$. This is proven graphically with the dashed line that indicates the point of tangency of the two curves. Shown are two different P_{cc} crippling allowable loads. The pink curve represents a Pcc=70ksi, and the green a P_{cc}=35ksi. In both cases, the Euler buckling curve is tangent at $P_{cc}/2$.



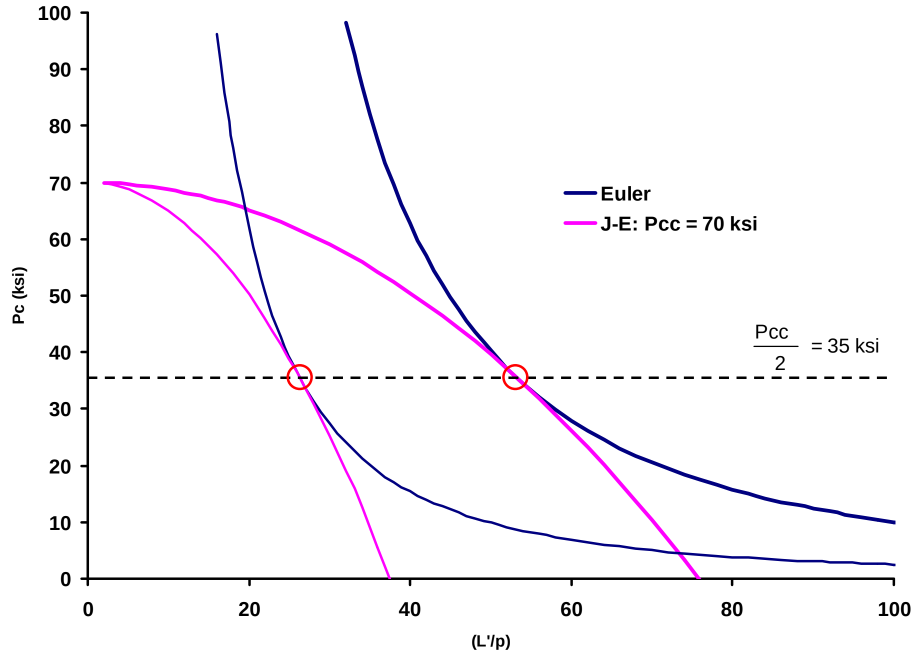


Fig. 3, The Johnson-Euler interaction curve is used for all panel buckling and crippling methods. As illustrated with the two different panel loadings, it is appropriate to implement buckling-crippling interaction for biaxial loadings. Without doing so, the incorrect interaction would be identified for biaxial loading conditions. For instance, the correct uniaxial interaction is identified with the right, blue Euler curve. Using the same crippling allowable Pcc = 70ksi, and the same Longitudinal, X axis, compressive load but this time with an additional transverse, Y axis load, the correct buckling allowable for a given biaxial loading is the left, blue Euler curve, and the JE interaction curve adjusted accordingly. HyperSizer defines these loadcase dependent crippling-buckling interactions on the fly. Also, HyperSizer identifies these load case dependent interactions for all crippling and buckling analysis methods turned on by the user. Therefore a different JE interaction curve is defined for a flat closed form solution vs. curved panel energy solution.

11.15.6 HyperSizer Margins-of-Safety

Component ID: 17
Load Case: 4: Mechanical Load Set #103 (Run Deck #1) "

[Equation Removed] **(53.1)**

[Equation Removed] **(53.2)**

[Equation Removed] **(53.3)**

Variable	Value, units
P_{cr}	311400. lb/in (Buckling Method:Beam Buckling)
P_{cc}	-.1000E+11 lb/in (Crippling Method:Nothing)
P_{je}	-59940. lb/in
P	-64690. lb/in
MS	-0.07341

11.15.7 References

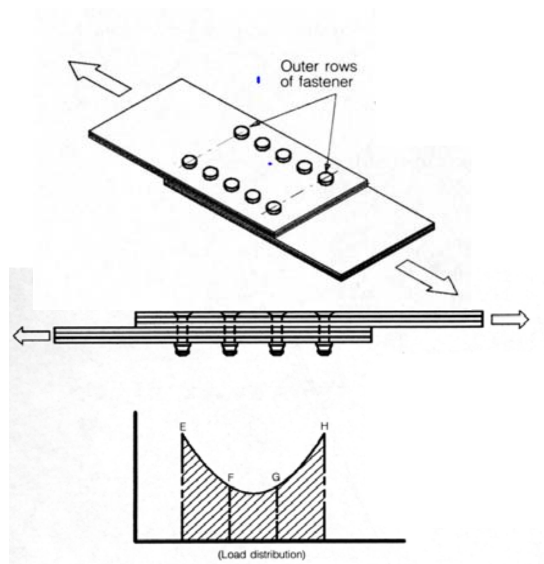
Mil-Hdbk-17-3E, DOD Coordination Working Draft, 1998. Vol3, Ch 4, p. 4-95.

11.16 Bolted Joint Strength, Composite BJSFM Analysis

Analysis Method: Undamaged or notched allowables can be used
<u>Ply Based</u> 190 = BJSFM

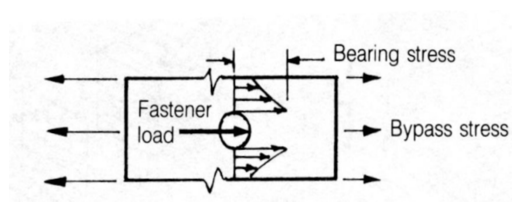
11.16.1 Analysis Approach

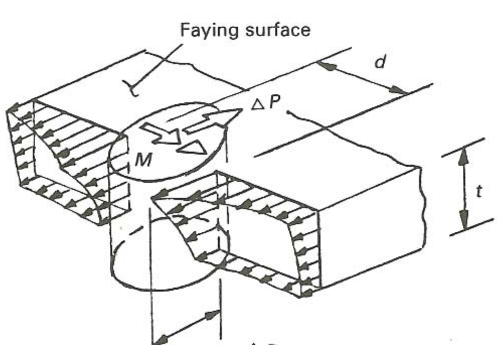
Composite materials, unlike metals, are not ductile and do not yield. The load distribution in a bolted composite joint is non- uniform, as depicted in the figure, where the outer row fasteners pick up more force than the inner row of fasteners. The first step in performing a bolted joint analysis is to quantify forces in each fastener.



This analysis approach is for a single fastener. Proper loads of the controlling fastener in a bolted joint pattern must be quantified. HyperSizer will automatically determine far field loadings of an open hole in a laminate, either from user load input, or FEA load extraction, but requires manual user input of bolt force and angle, per loadcase. Future, HyperSizer versions will be able to extract bolt forces from identified FEM rigid bar elements (RBE2/RBE3) or from CBUSH spring elements that account for fastener flexibility.

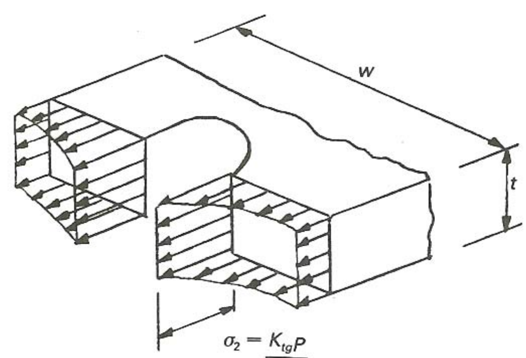
Bearing by-pass is computed from the net result of applying bolt force and far field loading. The two figures illustrate that the applied loading on the left is equal and opposite to the summation of the bolt force, ΔP, and the bearing by-pass force, P. The figure to the bottom left shows local stresses caused by bolt bearing, whereas the bottom right figure shows local stresses from the bypassing load. Both loadings cause stress intensity at the hole edge, that can be amplified with a Kt factor. This analysis method accounts for these stress intensities in the laminate analytically without using Kt factors.





$\sigma_1 = K_{tb} \frac{\triangle P}{dt} \theta$

(a) Local stresses caused by load transfer, ΔP



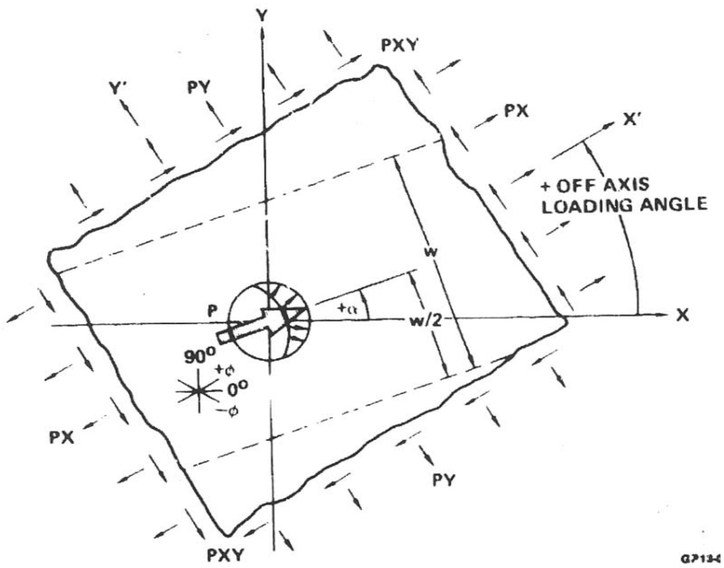
$\sigma_2 = \frac{K_{tg} P}{wt}$

(b) Local stresses caused by bypassing load, P

11.16.1.1 BJSFM Stress Solution

BJSFM is a well-established computational method for analyzing a hole in a composite laminate to general membrane loading fields (Nx, Ny, Nxy) with or without a bolt bearing loads. The program operates by computing the stress/strain field at evenly spaced angular increments in evenly spaced concentric rings around a hole. The bolt force is represented with a cosine function and a loading angle.

The computation of stress/strain field around the circumference of a composite laminate hole with bolt bearing uses Lekhnitski's classical solution of holes through laminated plates [Lekhnitski, S.G., 1968. Anisotropic Plates, Gordon and Breach Science Publishers]. This approach is also used in industry: IBOLT used by Lockheed Martin, and BEARBY used by Boeing. These programs assume a fixed contact zone between the bolt and the composite. BJSFM will compute the same stress/strain distribution as a finely meshed shell FEM, however BJSFM is much faster and more robust in that it is not a function of mesh fineness. Bolt analyses are semi-empirical in that they all require experimental testing to establish the parameters to calibrate stress predictions with tests. So the issue becomes not predicting laminate stress/strain, but rather failure load.



11.16.1.2 Failure Prediction

To resolve the unknown natural relationship between failure and complex stress gradients, the practical approach is to determine an appropriate ***characteristic distance*** away from the free edge/bearing surface to apply typical failure strength criteria. This approach, [Whitney-Nuismer] is deemed to be robust, but requires adequate testing to provide this value for general conditions. The characteristic dimension is a fundamental data entry passed to the bolt hole analysis routine. For composites, the characteristic dimension is primarily a function of the lamina material selection. MIL-HDBK-17-3e, fig 5.3.2.2b, shows a value = 0.016". Average values used in industry = 0.022"

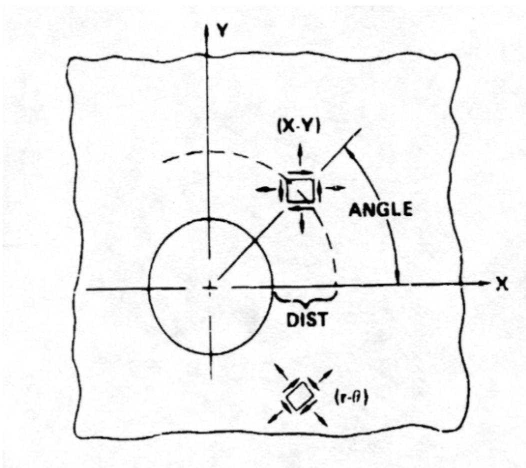
The Characteristic Distance, D_o, is dependent on the type of material allowables used. Unnotched material allowables should use a smaller value, based on calibrations with tests. Notched/ damage tolearnce allowables should use a larger value, meaning the stresses, used for failure, are extracted father away from the hole edge. For a comparison, using unnotched strain allowables of tension = 8000 (µin/in) and compression = 6700 (µin/in), a D_o = 0.022" provides the same effective bearing allowable of 47 ksi as does a Do = 0.120" using notched allowables of tension = 5700 (µin/in) and compression = 3900 (µin/in).

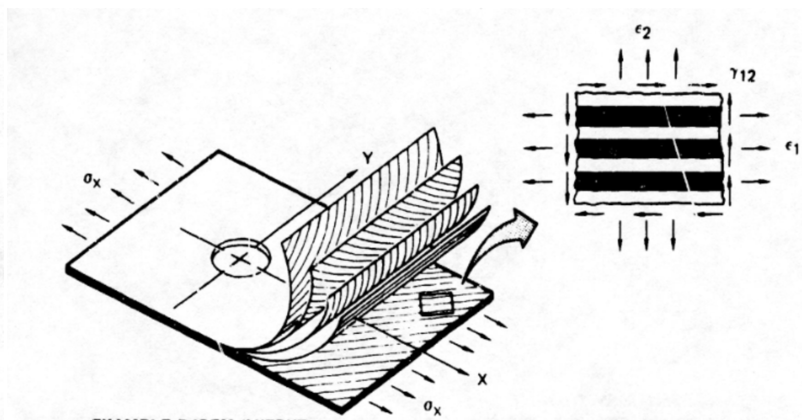
11.16.2 HyperSizer Implementation

HyperSizer automates the execution of BJSFM. The following input is passed from HyperSizer to BJSFM through computer memory, meaning not through ASCII files.

- The laminate layup
- All material properties such as temperature dependent ply stiffness and stress/strain allowables.
- Far field loading
- Hole diameter
- Bolt bearing load and angle

At the prescribed concentric ring, characteristic distance, BJSFM computes the stresses around the hole. The HyperSizer default angular increment is five-degrees (see figure). At each angular point, for the concentric characterstic distance, the stress is computed for each ply, in ply coordinates. One of several failure criteria is applied to determine the critical margin of safety. The margin-of-safety returned to HyperSizer and used for laminate sizing optimization is the minimum margin occurring for any ply, for any angular point, at the characteristic dimension, $D_{o.}$

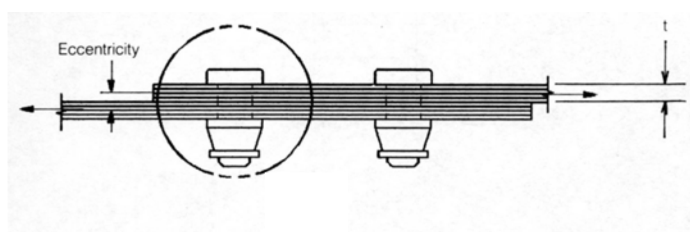




The bolted joint analysis can use a different failure criteria than the other HyperSizer failure analyses. The following ply based failure criteria are available to the BJSFM analysis.

- max strain
- max stress
- Tsai-Hill
- Tsai-Wu
- Hoffman

An innate part of the HyperSizer optimization process is computing the eccentricity of a single lap joint. As a laminate sizes to be thicker, the resulting detrimental eccentricity is accouned for in the analysis. The recommended approach, if using a FEM with shell coplanar elements, is to use HyperSizer's "two stack" concept.



11.16.2.1 Single Bolt Analysis Dimensions and Applied Loading

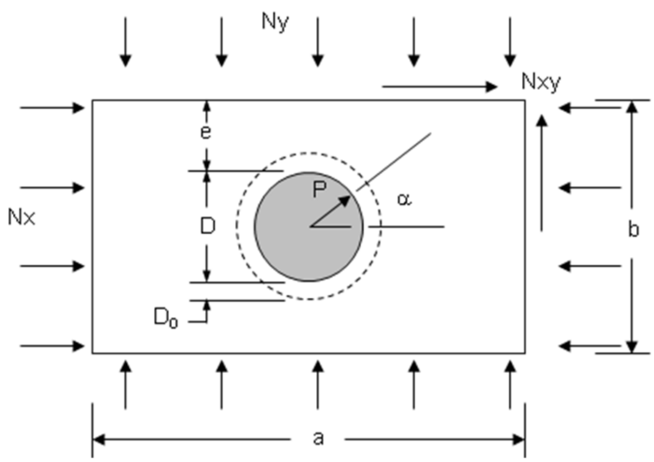
Analysis ID = 190 Joint, Bolted, Single Hole, BJSFM, Loaded and Far Field
Component ID: 30
Load Case: 6: Mechanical Load Set #105 (Run Deck #1) "
Object ID: Top Stack

Panel Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	Qx lb/in	Qy lb/in
Strength Loads	1400.	700.0	1243.	0.	0.	0.	700.0	490.0

Object Loads:

	Nx lb/in	Ny lb/in	Nxy lb/in	Mx lb in/in	My lb in/in	Mxy lb in/in	T °F	TT Grad °F/in
Strength Loads	1400.	700.0	1243.	0.	0.	0.	72.0	0.



Variable	Value, units
Dia	0.3125 in
D_o	0.2075 in
a	1.2568 in
b	1.2568 in
Controlling loadcase	6: Mechanical Load Set #105 (Run Deck #1) "
P	4430. lb
α	0.
Nx	1400. lb/in
Ny	700.0 lb/in
Nxy	**1243. lb/in**

The analysis solution computes bearing by-pass from the net result of applying bolt force and far field loading.

11.16.2.2 Controlling Laminate Results

11.16.2.3 Failure Criteria Symbols

X_t = Tension Stress allowable of ply in the longitudinal (1) direction

X_c = Compression Stress allowable of ply in the longitudinal (1) direction

Y_t = Tension Stress allowable of ply in the transverse (2) direction

Y_c = Compression Stress allowable of ply in the transverse (2) direction

S = Compression Stress allowable of ply in the transverse (2) direction

All of the failure criteria are ply based. The following section lists the margins-of-safety for the user selected failure criteria for BJSFM.

Analysis ID = 190 BJSFM with Max Strain Failure Criteria

Variable	Value, units
X_t	5437. μin/in
X_c	3738. μin/in
Y_t	5437. μin/in
Y_c	3738. μin/in
S	16900. μin/in
ε_{11}	3341. μin/in
ε_{22}	-3716. μin/in
γ_{12}	2336. μin/in
MS	**0.005804**

11.16.2.4 Backed out effective bearing stress allowable

As a sanity check, an effective bearing stress allowable is backed out from the computed margin-of-safety. Keep in mind the HyperSizer margin includes the contribution of the bearing bypass. This far field loading causes the hole to elongate even with out bolt bearing. If far field loading is present, it can have a significant impact on reducing the allowable bearing force, and as such, the backed out effective bearing stress allowable as presented here

[Equation Removed] **(190.4)**

where *Dia* is the hole diameter, t_{lam} is the laminate thickness, and *P* is the bolt force.

Variable	Value, units
Dia	0.3125 in
t_{lam}	0.243 in
P	4430. lb
MS	0.005804
$\sigma_{bearing}$	**58680. lb/in^2**

Method Summary

Computing the stress/strain distribution around the complete circumference of the hole to bearing bolt load and far field by-pass loads is more physics based than using a constant, one-size-fits-all bearing allowable. This is because it captures the variables of the problem such as:

- Diameter
- Layup (% ply angles)
- Bolt loading direction (angle of force to laminate axis)
- Wet and elevated temperature material allowables
    - (doesn't require more testing specifically for bearing)
- Very important bearing by-pass loads